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CHARGE-ODD AND SINGLE-SPIN EFFECTS IN TWO PION PRODUCTION IN $e\vec{p}$ COLLISIONS

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Introduction

Among the possible methods of measuring pion-pion scattering phases at low energies [1] — *e.g.*, the K_{e4} decay of kaons, pionium atom, pion-to-two-pion transition in the scattering of pions by protons (the Chew-Low process) — up to now high-energy processes in which a jet consisting of pions with relative small invariant mass is created has not been considered. The theoretical possibility was considered some time ago by Serbo and Chernyak [2] for a special kinematical region of the produced pions. In this paper we consider the charge-odd contribution to the cross section for charged pion pair production with the kinematics of a jet moving close to the direction of one of the initial particles. We consider the general case, restricted only by requirement that the sum of particle jet momenta transverse to the beam axis be close to zero. This region corresponds to the case in which one of the scattered particles moves close to its initial direction and escapes experimental detection, whereas the components of a jet moving in the opposite direction have measurable scattering angles and are assumed to be detected. The bremsstrahlung mechanism of pion pair creation includes the conversion of a time-like photon into a pion pair via an intermediate ρ-meson state. The Breit-Wigner resonance form of the relevant pion form factor provides an imaginary part, which can give rise to single-spin correlation effects in the differential cross section. These last may also arise as interference effects between the Born and one-loop Feynman diagrams for single-pion production [3] beyond the resonance region due to the presence of intermediate nucleon resonance states [4]. The charge-odd contribution to the cross section has a clear signal: the invariant mass of two pions is equal to the mass of ρ-meson and may thus be separated from even part of the cross section in the two-meson exclusive set-up.

The idea of measuring the distributions in the fragmentation region of one of the colliding beams was first considered in papers of the seventies [5]. In particular, there the odd part of the cross section for the production of a muon pair in a jet moving along initial electron direction was obtained. The energy distributions in a jet moving along, for example, initial electron direction were obtained for jets consisting of two electrons and one positron and a jet consisting of one electron and two photons in the form of a Dalitz-plot distribution. For instance, the charge-odd contribution to the spectrum for muon-pair production has the form was found to be

$$\begin{aligned}\frac{d^2\sigma_{\text{odd}}^{e^+e^-\to e^+e^-\mu^+\mu^-}}{dx_+dx_-} &= \frac{4\alpha^4}{3\pi m_\mu^2}\ln\left(\frac{s}{m_e m_\mu}\right)\frac{(x_+-x_-)(2-x_+-x_-)x_+x_-}{(x_++x_-)^4} \\ &= 0.58\ \text{nb}\ \cdot f_4(x_+,x_-)\,, \qquad (1)\end{aligned}$$

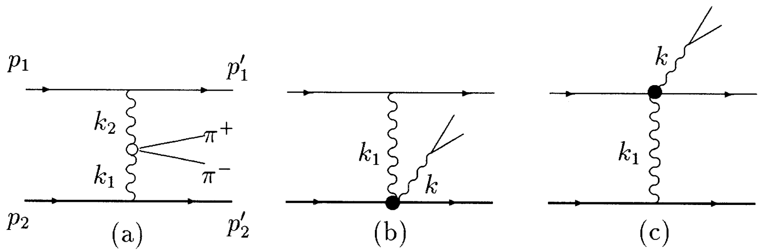

Figure 1: *Diagrams for the pion pair production in e and $\vec{p}$ collisions.*

with

$$x_\pm = \frac{E_\pm + p_\pm^z}{2E}, \qquad s = 4E^2, \tag{2}$$

where E is the electron beam energy in the centre-of-mass reference frame; $x_\pm$ are the muon energy fractions; $E_\pm, p_\pm^z$ — the energies and z-components of muon momenta and m_e and m_μ are the electron and muon masses, respectively. Bearing in mind that even part of the cross section has a similar order of magnitude [6], we see that the asymmetry effects are of order unity. First we consider the jet-2 kinematics (see Fig. 1b) in which a bremsstrahlung photon is emitted from a lower nucleon line. Then the case of jet-1 kinematics (see Fig. 1c) is elaborated in some detail. This is specified by an emission of a bremsstrahlung photon from an upper fermion line.

1 Evaluation of the effects

In this paper the charged pion pair production channel in high-energy collisions of unpolarised electrons and transversely polarised protons is dealt with. First the kinematical region for the creation of a jet moving close to initial proton direction is considered:

$$e(p_1) + p(p_2, a) \to e(p_1') + p(p_2') + \pi^+(q_2) + \pi^-(q_1), \tag{3}$$
$$p_1^2 = p_1'^2 = m_e^2, \quad p_2^2 = p_2'^2 = M^2, \quad q_{1,2}^2 = m^2, \quad p_2 a = 0,$$

where m_e, M and m are the electron, proton and pion masses and a is the proton polarisation vector. Denoting the 4-momentum of the (virtual) exchange photon by k_1 and using the well-known infinite momentum frame approach we write down the matrix element in the following form

$$\begin{aligned} \mathcal{M}_{jet-2} &= \frac{(4\pi\alpha)^2}{k_1^2} \bar{u}(p_1')\gamma_\nu u(p_1) \bar{u}(p_2') J_\mu u(p_2) g^{\mu\nu} \\ &= \mathcal{M}_D + \mathcal{M}_B. \end{aligned} \tag{4}$$

The current receives contributions from the double-photon (D) and bremsstrahlung (B) mechanisms for pion pair production:

$$J^\mu = J^\mu_D + J^\mu_B, \quad J^\mu_D = \frac{1}{k_2^2}\mathcal{M}^{\mu\eta}\gamma_\eta, \quad J^\mu_B = \frac{F(k^2)}{k^2}Q_\eta O^{\eta\mu}, \tag{5}$$

with

$$k_2 = p_2 - p_2', \quad k = q_1 + q_2, \quad Q = q_1 - q_2,$$

and the pion form factor

$$F(k^2) = \frac{m_\rho^2}{k^2 - m_\rho^2 + im_\rho\Gamma_\rho},$$

We have also defined the following tensors (for a correspondence refer to Fig. 2):

$$\mathcal{M}^{\mu\eta} = \frac{1}{\chi_1}(2q_1 - k_1)^\mu(k_2 - 2q_2)^\eta + \frac{1}{\chi_2}(2q_1 - k_2)^\eta(k_1 - 2q_2)^\mu - 2g^{\mu\eta},$$

$$\chi_1 = k_1^2 - 2k_1q_1, \quad \chi_2 = k_1^2 - 2k_1q_2, \quad k_1 + k_2 = q_1 + q_2,$$

$$O^{\eta\mu} = \gamma_\eta\frac{\hat{p}_2 + \hat{k}_1 + M}{(p_2 + k_1)^2 - M^2}\gamma_\mu + \gamma_\mu\frac{\hat{p}_2' - \hat{k}_1 + M}{(p_2' - k_1)^2 - M^2}\gamma_\eta, \tag{6}$$

which are subject to the gauge conditions

$$\mathcal{M}^{\mu\eta}k_{1\mu} = \mathcal{M}^{\mu\eta}k_{2\eta} = 0,$$

$$\bar{u}(p_2')O^{\eta\mu}u(p_2)k_{1\mu} = \bar{u}(p_2')O^{\eta\mu}u(p_2)k_\eta = 0. \tag{7}$$

We introduce the standard Sudakov parametrisation of the 4-momenta in the problem

$$\begin{aligned} q_i &= x_i\tilde{p}_2 + \beta_i\tilde{p}_1 + q_{i\perp}, \\ k_1 &= \alpha\tilde{p}_2 + \beta\tilde{p}_1 + k_{1\perp}, \\ p_2' &= x\tilde{p}_2 + \beta_2'\tilde{p}_1 + p_{2\perp}', \end{aligned} \tag{8}$$

$$\tilde{p}_2 = p_2 - p_1\frac{M^2}{s}, \quad \tilde{p}_1 = p_1 - p_2\frac{m_e^2}{s}, \quad s = 2p_1p_2 \gg M^2.$$

Here

$$x_i = \frac{q_{iz} + E_i}{2E}, \qquad x = \frac{p_{2z}' + E_2'}{2E}$$

are the pion and scattered proton energy fractions ($x + x_1 + x_2 = 1$), $q_{i\perp}$ and $p_{2\perp}'$ are the 4-momenta transverse to the beam axes. The corresponding

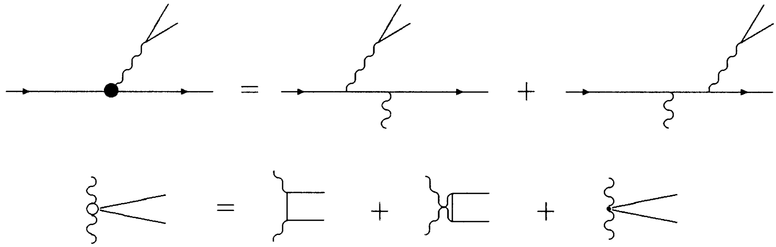

Figure 2: *Decoding of the notations used in Fig. 1*

euclidean 2-vectors are $\mathbf{q}_i$ and $\mathbf{p}_2'$. The small parameters β may be expressed via them

$$\beta_i = \frac{\mathbf{q}_i^2 + m^2}{sx_i}, \qquad \beta_2' = \frac{\mathbf{p}_2'^2 + M^2}{sx}. \tag{9}$$

The cross section and the phase volume in terms of these variables looks

$$\mathrm{d}\sigma = \frac{1}{8s} \sum |\mathcal{M}|^2 d\Gamma,$$

$$\begin{aligned} \mathrm{d}\Gamma &= \frac{\mathrm{d}^3 p_1' \mathrm{d}^3 p_2' \mathrm{d}^3 q_1 \mathrm{d}^3 q_2}{(2\pi)^8 2E_1' 2E_2' 2E_1 2E_2} \delta^4(p_1 + p_2 - p_1' - p_2' - q_1 - q_2) \\ &= \frac{\mathrm{d}^2\mathbf{q}_1 \mathrm{d}^2\mathbf{q}_2 \mathrm{d}^2\mathbf{k}_1 \mathrm{d}x_1 \mathrm{d}x_2}{(2\pi)^8 8sxx_1x_2}. \end{aligned} \tag{10}$$

The kinematics of a jet-2 permits us to use the Gribov representation for the exchange photon Green function (see (4))

$$\frac{g^{\mu\nu}}{k_1^2} = \frac{1}{k_1^2}\left[g_\perp^{\mu\nu} + \frac{2}{s}(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu)\right] = \frac{2}{sk_1^2} p_1^\mu p_2^\nu, \tag{11}$$

and from the gauge condition $J^\mu k_{1\mu} = J^\mu(\beta p_1 + k_{1\perp})_\mu = 0$ it follows

$$J^\mu p_{1\mu} = -\frac{s}{s_2} J^\mu k_{1\perp\mu}. \tag{12}$$

Here we have defined $\tilde{s}_2 = (p_2' + q_1 + q_2)^2 = s_2 + M^2$ and denoted the invariant jet-mass squared as

$$s_2 = s\beta = \frac{\mathbf{p}_1'^2 + (1-x)M^2}{x} + \frac{\mathbf{q}_1^2 + m^2}{x_1} + \frac{\mathbf{q}_2^2 + m^2}{x_2}. \tag{13}$$

For the modulus squared of the matrix element summed over spin states and averaged over the azimuthal angle of the virtual photon, we obtain

$$\sum |\mathcal{M}|^2 = -\frac{(4\pi\alpha)^4}{(k_1^2)^2}\frac{4s^2\mathbf{k}_1^2}{s_2^2}\mathrm{Tr}[(\hat{p}_2' + M)J^{\mu\perp}(\hat{p}_2 + M)(1-\gamma_5\hat{a})\tilde{J}_{\mu\perp}]. \tag{14}$$

This expression contains the charge-even as well as charge-odd contributions and, in addition, the spin correlation term, associated with proton polarisation vector a. The cross section contains the well-known Weizsäcker-Williams (WW) enhancement factor:

$$\int \frac{\mathrm{d}^2 k_1 \mathbf{k}_1^2}{\pi (k_1^2)^2} = L, \tag{15}$$

where the quantity L stands for a *large* logarithm whose value depends on the type of initial particle with momentum p_1. In the case when it is a proton we have

$$\begin{aligned} k_1^2 &= -\left[\mathbf{k}_1^2 + \left(M\frac{s_1}{s}\right)^2\right], \quad L = L_p = \ln\left(\frac{ms}{Ms_1}\right)^2, \\ s_1 &= \frac{\mathbf{p}_1'^2}{x} + \frac{\mathbf{q}_1^2 + m^2}{x_1} + \frac{\mathbf{q}_2^2 + m^2}{x_2}, \quad M = M_p, \quad m = m_\pi. \end{aligned} \tag{16}$$

For the case in which it is an electron we have

$$k_1^2 = -\left\{\mathbf{k}_1^2 + \left(m_e\frac{s_2}{s}\right)^2\right\}, \qquad L = L_e = \ln\left(\frac{ms}{m_e s_2}\right)^2. \tag{17}$$

The main (logarithmic) contribution to the cross section comes from the region $|\mathbf{k}_1| \ll |\mathbf{q}_i|$, which provides the relation between the transverse components of jet particles: $\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{p}_2' = 0$.

The tensor describing conversion of two photons into a pion pair can be presented in a form that may be interpreted as an expansion in pion momenta [1]:

$$\mathcal{M}^{\mu\nu} = a_0 \mathcal{L}_0^{\mu\nu} + a_2 \mathcal{L}_2^{\mu\nu}, \tag{18}$$

$$a_0 = \frac{\chi_1 + \chi_2}{\chi_1\chi_2}, \quad a_2 = \frac{2}{\chi_1\chi_2},$$

[1]Here a_0, a_2 may be associated with partial wave decomposition of pion–pion scattering amplitudes (not to be mixed with the contribution of states with isotopic spin 0 and 2). The quantity a_0 at threshold is connected with a polarizability of a pion $\alpha_\pi = a_0/(8\pi m)$.

$$
\begin{aligned}
\mathcal{L}_0^{\mu\nu} &= k_1k_2g^{\mu\nu} - k_1^\nu k_2^\mu, \\
\mathcal{L}_2^{\mu\nu} &= -k_1k_2Q^\mu Q^\nu - Qk_1(Q^\mu k_1^\nu + Q^\nu k_1^\mu) \\
&\quad +Qk_1Q^\nu(k_1+k_2)^\mu + (Qk_1)^2 g^{\mu\nu}, \\
\mathcal{L}_i^{\mu\nu}k_{1\mu} &= \mathcal{L}_i^{\mu\nu}k_{2\nu} = 0.
\end{aligned}
$$

Upon taking into account the final-state interaction, one expects that these amplitudes, as well as the amplitude for the conversion of a single photon into a pion pair, should acquire the following phases:

$$
a_0 \to a_0\mathrm{e}^{\mathrm{i}\delta_0}, \quad F(k^2) \to F(k^2)\mathrm{e}^{\mathrm{i}\delta_1}, \quad a_2 \to a_2\mathrm{e}^{\mathrm{i}\delta_2}. \tag{19}
$$

The phases $\delta_{0,1,2}$ are associated with states having orbital angular momentum equal to $0, 1, 2$ respectively. It is useful to note[2] that the third possible gauge-invariant structure,

$$
\mathcal{L}_3^{\mu\nu} = Qk_1(k_1^2 g^{\mu\nu} - k_1^\mu k_1^\nu) + Q^\nu(k_1^2 k_2^\mu - k_1k_2k_1^\mu), \tag{20}
$$

which may be thought of as a pair in a state with unit orbital angular momentum, is not realised in the double-photon channel.

Similar considerations may be applied to the jet-1 kinematics: when a jet moving close to initial electron direction consists of a scattered electron and a pion pair. Here we use the following parametrisation of the momenta:

$$
\begin{aligned}
q_i &= \alpha_i\tilde{p}_2 + x_i\tilde{p}_1 + q_{i\perp}, \\
p_1' &= \alpha_1'\tilde{p}_2 + x\tilde{p}_1 + p_{1\perp}', \\
k_1 &= \alpha\tilde{p}_2 + \beta\tilde{p}_1 + k_{1\perp},
\end{aligned} \tag{21}
$$

with

$$
\alpha_i = \frac{\mathbf{q}_i^2 + m^2}{sx_i}, \qquad \alpha_1' = \frac{\mathbf{p}_1'^2}{sx}, \tag{22}
$$

where x, x_i are the energy fractions of a scattered electron and produced pions ($x + x_1 + x_2 = 1$); $\mathbf{p}_1'$, $\mathbf{q}_i$ and are their transverse momenta obeying $\mathbf{p}_1' + \mathbf{q}_1 + \mathbf{q}_2 = 0$.

We thus obtain

$$
\begin{aligned}
\sum|\mathcal{M}_D|^2 &= -\frac{16(4\pi\alpha)^4\mathbf{k}_1^2 s^2}{(k_1^2k_2^2s_1)^2}\frac{1}{4}\mathrm{Tr}[\hat{p}_1'\hat{R}_\perp^\sigma\hat{p}_1\hat{\tilde{R}}_{\sigma\perp}], \\
\sum|\mathcal{M}_B|^2 &= -\frac{16(4\pi\alpha)^4\mathbf{k}_1^2|F(k^2)|^2 s^2}{(k_1^2k^2s_1)^2}\frac{1}{4}\mathrm{Tr}[\hat{p}_1'\hat{B}_\perp^\sigma\hat{p}_1\hat{\tilde{B}}_{\sigma\perp}], \\
2\mathrm{Re}\sum(\mathcal{M}_B\mathcal{M}_D^*) &= -\mathrm{Re}\frac{32(4\pi\alpha)^4\mathbf{k}_1^2 F(k^2)\mathrm{e}^{\mathrm{i}\delta_1}}{(k_1^2k_2k)^2}\frac{1}{4}\mathrm{Tr}[\hat{p}_1'\hat{B}_\perp^\sigma\hat{p}_1\hat{\tilde{R}}_{\sigma\perp}],
\end{aligned} \tag{23}
$$

[2]We are grateful to professors V. Serbo and I. Ginzburg for discussion on this point.

where

$$s_1 = s\alpha = \frac{\mathbf{p}_1'^2}{x} + \frac{\mathbf{q}_1^2 + m^2}{x_1} + \frac{\mathbf{q}_2^2 + m^2}{x_2}$$

is a jet invariant mass squared, and

$$k_2 = p_1 - p_1', \qquad k = q_1 + q_2, \qquad k_1^2 = -\left[\mathbf{k}_1^2 + M^2\left(\frac{s_1}{s}\right)^2\right],$$

$$\hat{B}_\sigma = \frac{1}{s}(\hat{Q}\hat{p}_2\gamma_\sigma + \frac{1}{x}\gamma_\sigma\hat{p}_2\hat{Q}) + \frac{2}{s_1 x}p'_{1\sigma}\hat{Q}, \tag{24}$$

and $\hat{R}_\sigma = \mathcal{M}_{\sigma\mu}\gamma^\mu$ (see (6)). Proton spin correlations are absent here since the exchange photon in the WW (see Page 5) approximation cannot carry spin information.

Taking into account the strong phases in the pion amplitudes, the above formula is modified as follows:

$$\begin{aligned} \mathrm{d}\sigma^{\mathrm{odd}}_{\mathrm{jet-1}} &= -\frac{\alpha^4 L_p}{\pi^3 s_1^2 k_2^2 k^2}\left[I\,\mathrm{Re}\left\{F(k^2)e^{\mathrm{i}(\delta_1-\delta_2)}\right\}\right. \\ &\left. + \tilde{I}\,\mathrm{Re}\left\{F(k^2)\left(e^{\mathrm{i}(\delta_1-\delta_0)} - e^{\mathrm{i}(\delta_1-\delta_2)}\right)\right\}\right]\frac{\mathrm{d}^2q_1\mathrm{d}^2q_2\mathrm{d}x_1\mathrm{d}x_2}{xx_1x_2}. \end{aligned} \tag{25}$$

For the kinematics with $|\mathbf{p}_1'| \to 0$ the charge-odd part of the cross section takes the form:

$$\begin{aligned} \mathrm{d}\sigma^{\mathrm{odd}}_{\mathrm{jet-1}}\Big|^{(\pi\pi)}_{\mathbf{q}_2=-\mathbf{q}_1} &= \frac{\alpha^4 L_p}{\pi^3}\frac{\mathbf{q}_1^2 - \mathbf{q}_2^2}{(\mathbf{q}_1+\mathbf{q}_2)^2}\frac{(x_1x_2)^2}{(\mathbf{q}_1^2+m^2)^3 x(1-x)^5} \\ &\times \left\{-2x\mathrm{Re}\left[F(k^2)\left(\mathrm{e}^{\mathrm{i}(\delta_1-\delta_0)} - \mathrm{e}^{\mathrm{i}(\delta_1-\delta_2)}\right)\right]\right. \\ &+ \left.\mathrm{Re}\left[F(k^2)\mathrm{e}^{\mathrm{i}(\delta_1-\delta_2)}\right]\left(1 + x^2 - \frac{m^2}{\mathbf{q}_1^2+m^2}(1+x)^2\right)\right\} \\ &\times \mathrm{d}^2q_1\mathrm{d}^2q_2\mathrm{d}x_1\mathrm{d}x_2. \end{aligned} \tag{26}$$

In the case $\mathrm{e}^{-\mathrm{i}\delta_i} = 1$ this is in agreement with the expression obtained in [2, 7]. We also include here a similar expression for muon-pair production in this limit:

$$\begin{aligned} \mathrm{d}\sigma^{\mathrm{odd}}_{\mathrm{jet-1}}\Big|^{(\mu\mu)}_{\mathbf{q}_2=-\mathbf{q}_1} &= \frac{\alpha^4 L_p}{\pi^3}\frac{\mathbf{q}_1^2 - \mathbf{q}_2^2}{(\mathbf{q}_1+\mathbf{q}_2)^2} \\ &\times \frac{((1-x)^2(\mathbf{q}_1^2+m^2) - 2\mathbf{q}_1^2x_1x_2)(1+x^2) + 4xx_1x_2m^2}{x(1-x)^5} \\ &\times \frac{x_1x_2\mathrm{d}^2q_1\mathrm{d}^2q_2\mathrm{d}x_1\mathrm{d}x_2}{(\mathbf{q}_1^2+m^2)^4}, \end{aligned} \tag{27}$$

Neglecting the effect of the phases, the spin-dependent part for a jet-2 kinematics takes the following form:

$$\begin{aligned} d\sigma^{\rm spin}_{\rm jet-2} &= \frac{\alpha^4 L_e}{\pi^3 s_2^3}\frac{{\rm Im}F^*(k^2)M}{k_2^2 k^2} \\ &\times \left[A(\mathbf{q}_1, x_1; \mathbf{q}_2, x_2)(\mathbf{q}_1 \wedge \mathbf{a})_z - A(\mathbf{q}_2, x_2; \mathbf{q}_1, x_1)(\mathbf{q}_2 \wedge \mathbf{a})_z\right] \\ &\times \frac{d^2 q_1 d^2 q_2 dx_1 dx_2}{x_1 x_2 x}. \end{aligned} \tag{28}$$

The expression for A is given in the Appendix.

For the kinematics when $\mathbf{p}_1'^2 = (\mathbf{q}_1 + \mathbf{q}_2)^2 \ll \mathbf{q}_1^2$ we have (using the δ-function approximation for ${\rm Im}F^*$):

$$\begin{aligned} d\sigma^{\rm spin}_{\rm jet-2}|^{(\pi\pi)}_{\mathbf{q}_2=-\mathbf{q}_1} &= \frac{2\alpha^4 L_e}{\pi^2} M(\mathbf{q}_1 \wedge \mathbf{a})_z\, \delta\left(1 - \frac{v(1-x)^2}{x_1 x_2 m_\rho^2}\right) \\ &\times \frac{(x x_1 x_2)^2}{v(M^2(1-x)^2 + \mathbf{p}_2'^2)(1-x)^3(vx + M^2 x_1 x_2)^3} \\ &\times \left[v - m^2(1+x) - \frac{M^2 x_1 x_2}{x} - \frac{v}{4}\frac{(1-x)^3}{x_1 x_2}\right] \\ &\times d^2 q_1 d^2 q_2 dx_1 dx_2, \qquad v = \mathbf{q}_1^2 + m^2. \end{aligned} \tag{29}$$

Now let us discuss the accuracy of formulæ presented. It is, of course, determined by the omitted terms; they are of order

$$\frac{m^2}{s}, \quad \frac{s_i}{s}, \quad \text{and} \quad \frac{1}{L}, \tag{30}$$

as compared to unity. For the DESY experimental conditions the accuracy is less than 10%. For the inclusive set-up one must consider the 3-pion production process. Compared with 2-pion production, this has a phase-volume suppression factor

$$\int \frac{d^2 q}{\pi M^2} \sim \left(\frac{m}{M}\right)^2 \sim 10^{-2}.$$

For intermediate energies, such as at VEPP-2M and DAΦNE, despite the rather small suppression factor $m^2/s \sim 0.01$ for the theoretical background of the remaining Feynman diagrams (there are present six gauge-invariant sets, of which we have used just two), the WW enhancement factor (a large logarithm) and the specific choice of kinematics $|\mathbf{q}_1 + \mathbf{q}_2| \sim 0$ nevertheless provides an accuracy of the same order for jet-1 kinematics as obtained for jet-2 type.

2 Conclusion

The charge-odd contribution to the spectrum in a jet-1 kinematics for the case $|\mathbf{p}_1'| \ll |\mathbf{q}_1|$ may be put in the following form:

$$m_\rho^2 \frac{d\sigma_{\text{jet}-1}^{\text{odd}(\pi^+\pi^-)}}{d^2\mathbf{q}_1 dx_1 dx_2} = 0.7 \cdot 10^{-2} \text{ nb} \cdot \frac{\mathbf{q}_1^2 - \mathbf{q}_2^2}{(\mathbf{q}_1 + \mathbf{q}_2)^2} \times \left[f_1 \sin(\delta_0 - \delta_1) + f_2 \sin(\delta_2 - \delta_1) \right], \quad (31)$$

with f_1, f_2 being smooth functions of the order of unity,

$$f_1 = -\frac{2}{x_1 + x_2}, \qquad f_2 = \frac{(1+x)^2}{x(x_1+x_2)}. \quad (32)$$

The contribution to spin-dependent part in the same kinematics can be put in a form

$$\frac{d\sigma_{\text{jet}-2}^{\text{spin}(\pi^+\pi^-)}}{d\phi dx_1 dx_2} = 1.8 \cdot 10^{-2} \text{ nb } \cdot |a| f_3 \frac{d\mathbf{p}_1'^2}{M^2(1-x)^2 + \mathbf{p}_1'^2} \cdot \sin\phi,$$
$$f_3 = \frac{1.22x}{\sqrt{x_1 x_2}} \frac{x_1 x_2 x - 1.49(x_1+x_2)^2 x_1 x_2 - 0.25x(x_1+x_2)^3}{(x + 1.49(x_1+x_2)^2)^3}. \quad (33)$$

Here $|a|$ is a degree of the target transverse polarisation; ϕ — the azimuthal angle between directions of target maximum polarisation and transverse momentum of a negative pion $\mathbf{q}_1$.

Typical values of the functions f_i for a set of x_1, x_2 values are given in the Table 1 for HERMES (DESY) conditions ($s = 60\ GeV^2$ and $|\mathbf{q}_1| \approx |\mathbf{q}_2| \sim 0.5 \div 1.5\ GeV$).

x_1	x_2	f_1	f_2	f_3	f_4
0.2	0.2	−5.000	10.667	0.030	0.000
0.2	0.4	−3.333	8.167	−0.068	−0.173
0.2	0.6	−2.500	9.000	−0.053	−0.141
0.3	0.4	−2.857	8.048	−0.075	−0.065

Table 1: *The values of functions f_1, f_2 [Eq. (32)], f_3 [Eq. (33)] and f_4 [Eq. (1)] for typical HERMES conditions* $|\mathbf{p}_1'| \ll |\mathbf{q}_1|$, 0.2 GeV $< |\mathbf{q}_1| <$ 1.2 GeV, $0.2 \le x_i \le 0.6$.

The consideration given above has been carried out within a framework of QED. In the case of a jet-1 kinematics it is reasonable to replace a photon exchange between pion and nucleon by a P*omeron* one in double-photon amplitudes which results in enhancement factor α_s/α: $\sigma_0 \to (\alpha_s/\alpha)\sigma_0$.

A visible effect of asymmetry in pion pair production was measured at Cornell[3] where a number of most energetic π^- moving along e^- direction exceeds that of most energetic π^+ in the same direction.

The formulæ given above may be applied to pion pair production at e^+e^- collider, besides the error caused by contributions of annihilation-type Feynman diagrams falls down with growth of the total center-of-mass energy $\sqrt{s}$ being the quantity of the order of $m^2/s < 3\%$ for J/ψ and B factories.

Nevertheless the charge-odd effects in $\pi^+\pi^-$ production may definitely be measured at Φ-factories taking advantage of kinematics discussed above.

Charge-odd effects in K_S, K_L pair production may clearly be seen at J/ψ- and B-factories.

Appendix

The general expression in a jet-1 kinematics for the charge-odd contribution to a pion pair production cross section in the WW approximation is given in Eq. (25) with the quantities $I, \tilde{I}$

$$\begin{aligned} I &= -\mathbf{Q}\mathbf{p}_1' + \frac{(1+x)(x_1-x_2)}{x} p_1 p_1' + \frac{1+x}{2x_1x_2}\mathbf{b}\mathbf{Q} - \frac{x_1-x_2}{2x_1x_2}\mathbf{b}\mathbf{p}_1' \\ &- \frac{\mathbf{p}_1'\mathbf{a}}{x_1x_2x}\frac{(2p_1Q2p_1'Q - Q^2 2p_1p_1')}{s_1^2} + \frac{2(-\mathbf{p}_1'^2 Qp_1 + \mathbf{Q}\mathbf{p}_1' p_1p_1')}{xs_1} \\ &- \frac{\mathbf{Q}\mathbf{a}}{x_1x_2s_1}(p_1Q + p_1'Q) + \frac{\mathbf{p}_1'\mathbf{b}}{x_1x_2s_1}(xQp_1 + Qp_1' - (x_1-x_2)p_1p_1') \\ &- \frac{(1-x)Q^2}{2s_1}\frac{\mathbf{p}_1'\mathbf{a}}{x_1x_2x} + \frac{x_1-x_2}{xx_1x_2s_1}Qp_1\mathbf{p}_1'\mathbf{a}, \end{aligned} \tag{34}$$

where

$$\mathbf{b} = x_2\mathbf{q_1} + x_1\mathbf{q_2}, \qquad \mathbf{a} = x_2\mathbf{q_1} - x_1\mathbf{q_2},$$

$$k_2^2 = -2p_1p_1' = -\frac{\mathbf{p}_1^{2\prime}}{x}, \qquad 2p_1'Q = x2p_1Q - 2\mathbf{Q}\mathbf{p}_1',$$

$$2p_1Q = \frac{1}{x_1}(\mathbf{q}_1^2 + m^2) - \frac{1}{x_2}(\mathbf{q}_2^2 + m^2)$$

$$Q^2 = 4m^2 - k^2, \qquad k^2 = \frac{1}{x_1x_2}\left(m^2(1-x)^2 + \mathbf{a}^2\right), \tag{35}$$

[3]V.G. Serbo, private communication.

and

$$
\begin{aligned}
\tilde{I} &= -\frac{2}{1-x}\mathbf{p}_1'\mathbf{Q} + \mathbf{p}_1'^2\left[\frac{(1-x+x^2)(x_1-x_2)}{x^2(1-x)} - \frac{4p_1Q}{s_1x(1-x)}\right] \\
&+\frac{1+x}{x^2(1-x)}\frac{\mathbf{p}_1'^2\,\mathbf{p}_1'\mathbf{Q}}{s_1} + \frac{x_1-x_2}{x^2(1-x)}\frac{(\mathbf{p}_1'^2)^2}{s_1}. \qquad (36)
\end{aligned}
$$

The quantity A relevant for the spin asymmetry looks

$$
\begin{aligned}
\mathcal{A}(\mathbf{q}_1,x_1;\mathbf{q}_2,x_2) &= -s_2\frac{1-x}{x} + \frac{Q^2(1-x)^2}{2xx_1} + \frac{x_1-x_2}{2xx_1}(-2p_2'Qx+2p_2Q) \\
&+\frac{2}{xx_1}[\mathbf{q}_1^2x_2+\mathbf{q}_2^2x_1-(1-x)\mathbf{q}_1\mathbf{q}_2] - \frac{2x_2(\mathbf{q}_1^2-\mathbf{q}_2^2)}{xx_1} \\
&+\left[\frac{x_1}{x_2}(\mathbf{q}_2^2+m^2) - \frac{x_2}{x_1}(\mathbf{q}_1^2+m^2)\right] \\
&\times\left[\frac{x_1-x_2}{xx_1} + \frac{2(\mathbf{q}_2^2-\mathbf{q}_1^2)}{s_2xx_1}\right]. \qquad (37)
\end{aligned}
$$

References

[1] A. Belkov and S. Bunyatov, *Phys. of El. Part. and At.* **13** Part 1 (1982); A. Belkov, S. Bunyatov, K. Mukhin and O. Patarakin, *Pion-pion interaction*, "Energoatomizdat" (1985) (In Russian); L. Nemenov, *Sov. J. Nucl. Phys.* **41** (1985) 629.

[2] V. Serbo and V. Chernyak, *Yad. Fiz.* **19** (1974) 123.

[3] A. Ahmedov, I.V. Akushevich, É.A. Kuraev and P.G. Ratcliffe, e-print hep-ph/9902418, *Eur. Phys. J.* **C11** (1999) 703-708.

[4] A. Arbuzov *et al.*, *Phys. Atom. Nucl.* **57** (1994) 1004.

[5] É.A. Kuraev, L.N. Lipatov, N. Merenkov and M. Strikman, *Yad. Fiz.* **23** (1976) 163.

[6] V. Baier, V. Fadin, V. Khoze and É.A. Kuraev, *Phys. Rep.* **78** (1981) 293.

[7] V. Budnev, I. Ginzburg, G. Meledin and V. Serbo, *Phys. Rep.* **15** (1975) 181.

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Зарядово-нечетные и односпиновые эффекты
в электрон-протонных столкновениях с образованием
двух пионов

Рассмотрены тормозной и двухфотонный механизмы образования двух заряженных пионов в электрон-протонных столкновениях при высоких энергиях. Интерференция между соответствующими амплитудами генерирует зарядово-нечетный вклад в дифференциальное сечение данного процесса. В кинематике, когда струя движется вдоль электрона, спин-независимая часть может быть использована для определения разностей фаз пион-пионного рассеяния в состояниях с орбитальными моментами 0 или 2 и 1, в то время как в кинематике со струей, движущейся вдоль протона, спин-зависимая часть может оказаться полезной при интерпретации экспериментальных данных по односпиновым корреляциям в образовании отрицательно заряженных пионов. Также обсуждается фон и дается оценка точности полученных результатов (<10%). Помимо общих формул приводятся упрощенные выражения для специфической кинематики, когда суммарный поперечный импульс пионов мал.

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Galynsky M.V. et al. E2-2000-42

Charge-Odd and Single-Spin Effects
in Two Pion Production in $e\vec{p}$ Collisions

We consider two-photon and bremsstrahlung mechanisms for the production of two charged pions in high-energy electron (proton) scattering off a transversely polarised proton. Interference between the relevant amplitudes generates a charge-odd contribution to the cross section for the process. In a kinematics with a jet moving along electron spin-independent part may be used for determination of phase differences for pion-pion scattering in the states with orbital momentum 0 or 2 and 1 whereas in a kinematics with a jet moving along proton spin-dependent part may be used to explain the experimental data for single-spin correlations in the production of negatively charged pions. We also discuss the backgrounds and estimate the accuracy of the results at less than 10% level. In addition simplified formulae derived for specific kinematics, with small total transverse pion momentum, are given.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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