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DETERMINATION OF Z-BOZON SOURCE  
RADIUS PRODUCTION BY IDENTICAL  
PARTICLES INTERFEROMETRY

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### 1-1. Both particles come from the two sources

Bose-Einstein correlations arise from the interference of identical bosons emitted from different points of external sources [1-5].

When the sources are "close", we are unable to distinguish, which particle comes from which source.

The probability of the emission of two identical spinless bosons with 4-momenta  $p_1$  and  $p_2$  from two sources with the average 4-coordinates  $x_1$  and  $x_2$  has the form:

$$W(p_1, p_2) = 2|U(p_1)|^2 |U(p_2)|^2 \{1 + \cos[(p_1 - p_2)(x_1 - x_2)]\},$$

where  $x_1 = (t_1, r_1)$ ,  $x_2 = (t_2, r_2)$ ,

$|U(p)|$  is the one-particle momentum spectrum. We obtain, averaging over the distribution of points  $x_1$  and  $x_2$  :

$$W(p_1, p_2) = 2W_0 (1 + \langle \cos qx \rangle),$$

where  $W_0 = |U(p)|^2$ ,  $q = p_1 - p_2$ ,  $x = x_1 - x_2$ .

In case of independent one-particle sources with the same distribution we can pass directly to the space-time parameter distribution instead of difference one. Then the correlation function looks like:

$$W(p_1, p_2) = W(p_1, p_2) / (2W_0) = 1 + |F(q)|^2 ,$$

where  $F(q) = \int f(z) \exp(-iqz) d^4z$  is a Fourier transformation (a form factor) of the function  $f(x)$ , that describes the space-time distribution of sources.

Experimentally the correlation effect is determined by the expression:

$$S(q_i) = W(p_1, p_2) / W_{bg}(p_1, p_2) = 1 + \lambda H(q_i)$$

where  $W_{bg}$  is a background of two-particle spectrum,  $\lambda$  is an additional parameter. In a case of full interference  $H(q_i)$  has a value 1 at  $p_1 - p_2$  ( $H(0)=1$ ) and tends to 0 as  $p_1 - p_2$  increases.  $MAX(S(q_i)) = 2$ .

Within the framework of the simple model of a radiating spherical surface of radius  $R$  with incoherent point-like oscillators of lifetime  $T$ , Kopylov and Podgoretsky <sup>[2,5]</sup> suggest the following parameterization:

$$S(q_t, q_0) = \gamma \{ (1 + \delta q_t) (1 + 4\lambda J_1^2(\beta q_t) / (\beta q_t)^2) \} (1 + (T q_0)^2) \quad (1)$$

where  $J_1$  is the first-order Bessel function,  $q_0 = E_i - E_j$ ,

$q_t$  is the projection square of the vector  $q = (p_1 - p_2)$  onto the plane perpendicular to the total momentum of the particles pair and represents the transverse momentum difference of the two particles relatively to the direction  $(p_1 + p_2) / Mod(p_1 + p_2)$ ,  $\gamma$  is a normalization factor,  $\delta$  allows small variations of  $R$  at large  $q_{h,t}$ .

The transverse size of the source is related to  $\beta$  by  $R_t = hc / \beta$  where  $T$  is the time of emission. It can be mentioned that any contamination of non-identity particles in the analyzed sample reduces the value of  $\lambda$ .

Goldhaber's et. al.<sup>[1]</sup> parameterization suggests the gaussian shape of the source spatial distribution. This is a relativistic invariant form:

$$S(q_h) = \gamma(1 + \delta q_h)(1 + \lambda \Phi). \quad (2)$$

Here

$$\Phi \sim \exp(-\alpha q_h), \quad (3)$$

where  $q_h = M_{ii}^2 - 4m_i^2$ ,  $M_{ii}$  is an effective mass of two particles,  $\alpha$  is connected with effective dimension of the source as  $R = hc/\sqrt{\alpha}$ .

In the framework of independent one-particle sources model the correlation function  $S(q)$  for identical particles with spin 1/2 without polarization will be equal to<sup>[8]</sup>:

$$S(q) = 1 - 1/2 |F(q)|^2. \quad (4)$$

As follows, the correlation function  $S(q)$  of two identical fermions will approach to 1/2, when  $q \rightarrow 0$  (instead to 2 for identical spinless bosons). Then it should be noted, if two particles with spin 1/2, are polarized totally in the same direction, the correlation function, as well as the cross-section of their generation, tends to zero.

Usually the next kind of correlation function is taken:

$$S(dp) = (N/N_b) dN(dp) / dN_b(dp) \quad (5)$$

where  $dp = \text{abs}(p_1 - p_2)$ ,  $p_1$  and  $p_2$  are particles momenta,  $dN(dp)$  is the number particles pairs,  $dN_b(dp)$  is the number of background pair.

Here we note, that when  $p_1 = p_2$ , then  $dp^2 = q_t^2$ .

## 1-2. One particle from the pair comes from the intervening particle (or resonance)

Let us see now the case with two identical particles, one of them coming directly from the origin source and the other one - a decay's product of the intervening particle (or resonance) (see Fig. 1). The correlation between such particles is determined by the

intervening particle decay range  $L$  as well as by the dimension  $R$  of the producing region <sup>[11]</sup>.

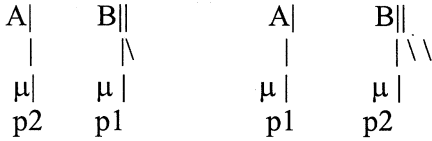


Fig 1.

The correlation function  $S(q)$  here is parameterized as follows:  
 $S(q_t) = \gamma(1 + \delta q_t)(1 - \lambda \exp(-q_t^2 R_t^2 - q_0^2 (T^2 + R_{||}^2 / v_p^2) / (1 + \gamma^2)))$ , (6)

where  $y = q_0 \tau + q_L$ ,

$\tau$ ,  $L$  are the intervening particle decay time and the range,  $T$  is the emitting source,  $v_p$  is the resonance velocity.

Following <sup>[14]</sup>, we shall write:

$$S(q_h) = \gamma(1 + \delta q_h)(1 - \lambda \Phi_{Res}). \quad (7)$$

Here

$$\Phi_{Res} = C \exp(-\alpha q_h) (1/\xi) \arctan \xi \quad (8)$$

and  $\xi = (2p_{c.m.}/\Gamma) \text{SQRT}(q_h/(4\mu^2 + q_h))$ .

## 2-1. The proposed experiment

We propose to measure the  $q_h$  and  $q_t$  spectra of muon pairs within a range  $0 \div 0.4$  (GeV/c)<sup>2</sup> for the following QCD processes:

$$\begin{aligned} q_i q_j &\rightarrow q_i q_j, & q_i (\text{anti } q_i) &\rightarrow q_k (\text{anti } q_k), \\ q_i (\text{anti } q_l) &\rightarrow g g, & q_i g &\rightarrow q_i g, \\ g g &\rightarrow q_k (\text{anti } q_k), & g g &\rightarrow g g. \end{aligned} \quad \{A\}$$

Nor higher-order processes are explicitly included, nor any higher-order loop corrections to the  $2 \Rightarrow 2$  processes.

In addition to above QCD-JETs, the samples of the Z-bosons generations and decays:

$$f+f \rightarrow g+\gamma/Z, \quad f+f \rightarrow Z^0 + Z^0, \quad f+f \rightarrow Z^0 + W$$

were simulated.

Effective mass spectra for Z-bosons are shown in fig. 2.

The samples of muon pairs for 10 JETS energy and 5 samples of Z-decays to  $\mu^+$ ,  $\mu^-$ -mesons were generated with PYTHIA 5-7.

The next selection cuts were used:

1.  $P_t$  of muons:  $[5 \div 20]$  GeV/c;
2.  $\Delta q_h < 0.4$  GeV/c.  $\Delta q_h$  – distribution is shown in Fig. 3.
3. Muons production radius precision must be not more than 10 %.

The  $q_h$  and  $q_t$  errors are nearly the same as ones for  $p_t$ . We note here, that the CMS –detector (matching with a central tracker) will have the following expecting  $p_t$  - resolution <sup>[10]</sup>:

for  $0 < |\eta| < 2.4$  relative errors  $\Delta p_t/p_t$  are estimated as  $[0.5 \div 1.]\%$  for  $p_t < 10$  GeV/c, as  $[1.5 \div 5.]\%$  for  $p_t < 100$  GeV/c and as  $[5. \div 20.]\%$  for  $p_t \sim 1$  TeV/c.

So, the necessary resolution may be reached only after matching with central tracker for the muons with  $p_t < 20$  GeV/c.

## 2.2 Results

### 2.2.1. Errors

From the point of view of the  $Q_h$  -errors for muon pairs from  $E_{JET} = 2000$  GeV nearly 1/3 of all events have the suitable errors. Therefore the factor that will reduce the statistics will be 0.31 for all events from  $[0. \div 0.4]$  (GeV/c)<sup>2</sup> interval. Now we get the estimation for the statistics required for radius measuring with resolution not exceeding 10%, if we take in account the next demands:

$s = n_{eff} / N_{pair} = 10^{-2}$ , the upper limit of the interference effect;

$k=3$  is the number of standard deviations of the effect above the background;

$\delta_q = 0.01$  (GeV/c)<sup>2</sup> is the disposition of maximum;

$q_s = 0.04$  (GeV/c)<sup>2</sup> is a studying interval of  $q$ .

Thus:  $N_{tot} = 3k^2 \delta_q / (q_s s^2) = 10^5$  muon pairs, or  $10^5$  Z-decays.

### 2.2.2. Background problem

In quantitative studies on the interference signal, the question of what is to be taken as a background is of importance. In most publications  $I_{BGD}$  is created from the pairs of unlike charged particles. This background is obviously not free from dynamic correlation. To avoid this, we take advantage of a generated background. 10 000 muon pairs for 10 JETS energy were generated with PYTHIA 5.7 and later used for background investigations.

In the tables 1÷3 are represented the numbers of tried events at  $E_{jet}=50, 500, 2000$  GeV and the numbers of events, received at the end for each of the simulated processes.

Table 1.  $E_{jet}= 50$  GeV

Processes	Cross section (mb)	Generated	Good events
$q_i q_j \rightarrow \dots q_i q_j$	9.278E-04	24649	
$q_i (\text{anti } q_i) \rightarrow q_k (\text{anti } q_k)$	1.256E-05	322	
$q_i (\text{anti } q_i) \rightarrow \dots g g$	1.257E-05	341	
$q_i g \rightarrow \dots q_i g$	8.057E-04	211160	
$g g \rightarrow \dots q_k (\text{anti } q_k)$	6.287E-04	16549	
$g g \rightarrow \dots g g$	1.722E-02	454301	
All incl. subprocesses	2.686E-02	707322	384

Table 2  $E_{jet}= 500$  GeV

Processes	Cross section (mb)	Generated	Good
$q_i q_j \rightarrow \dots q_i q_j$	1.437E-07	55207	
$q_i (\text{anti } q_i) \rightarrow q_k (\text{anti } q_k)$	3.634E-09	1448	
$q_i (\text{anti } q_i) \rightarrow \dots g g$	2.447E-09	952	
$q_i g \rightarrow \dots q_i g$	4.720E-07	181135	
$g g \rightarrow \dots q_k (\text{anti } q_k) q_i$	1.929E-08	7365	
$g g \rightarrow \dots g g$	3.390E-07	129637	
All incl. subprocesses	9.801E-07	375744	209

Table 3.  $E_{\text{jet}} = 2000 \text{ GeV}$

Processes	Cross section (mb)	Generated	Good events
$q_i q_j \rightarrow \dots q_i q_j$	4.168E-11	236989	
$q_i (\text{anti } q_i) \rightarrow q_k (\text{anti } q_k)$	4.471E-13	2546	
$q_i (\text{anti } q_i) \rightarrow \dots g g$	2.458E-13	1403	
$q_i g \rightarrow \dots q_i g$	3.055E-11	173798	
$g g \rightarrow \dots q_k (\text{anti } q_k) q_l$	2.808E-13	1598	
$g g \rightarrow \dots g g$	4.354E-12	24519	
All incl. subprocesses	7.756E-11	440853	169

### 2.2.3 Counting rates for CMS

The collecting rates of the subprocesses  $\{A\}$  with the minimum bias trigger for certain JET's energy are shown in Tabl.4.

Table 4.

$E_{\text{jet}}$ (GeV)	Number of events per one Sec
50	$10^2$
200	$4 \times 10^{-2}$
500	$3 \times 10^{-3}$
1000	$5 \times 10^{-5}$
2000	$1 \times 10^{-7}$

Analysing the proposed Tables, we can see that 1000 Sec are necessary to obtain, for example,  $10^5$  events and  $\approx 100$  Z-decays !

## 2. Source radius for Z – bosons

To reach the producing space dimension of Z-bozons, we



simulated at 1 GeV a sample of events, containing Z-bosons, which further suffer a decay into muons or into muons and other particles. The pairs of muons were constituted by combination from from Z-sample and from the simulated QCD-JET. Simulated  $q_h$  - spectra from the 1 TeV jets for different values of muons pairs are presented in the Fig. 4.

Interference effects, corresponding to different values of source radius, carried in  $q_h$ -spectra by (5). As we see from Fig. 5, for small values of producing radius {0.25, 0.5, or 1.} fm, an essential difference between two corresponding  $q_h$ -distributions can be observed. This result allows us to determine the producing region of Z-bosons until 1fm.

## CONCLUSION

It is shown, that interference effect, coming from Z-bosons, can be scrupulous studied, when Z-bosons are produced within small space regions. The space radius of Z-bozons production may be determined as satisfactory, if Z-bosons are born in the region of  $\sim 1$ fm.

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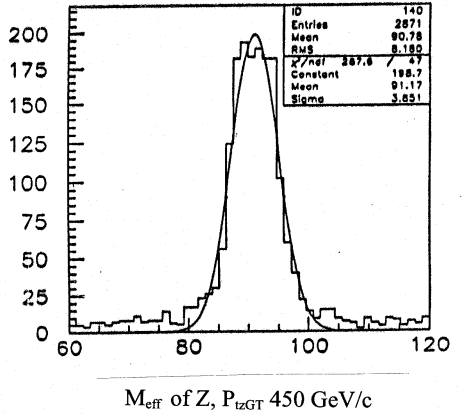
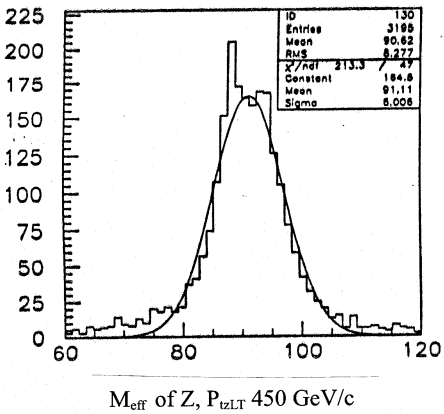
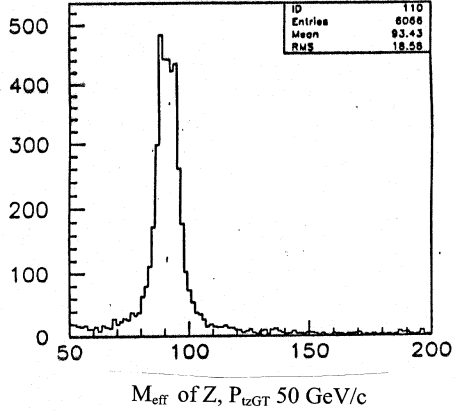
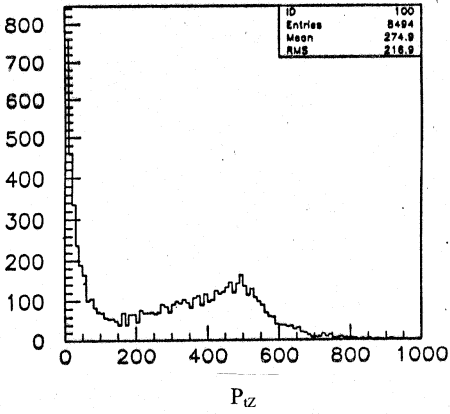


Fig. 2. Effective mass spectrum of simulated Z-bozons.

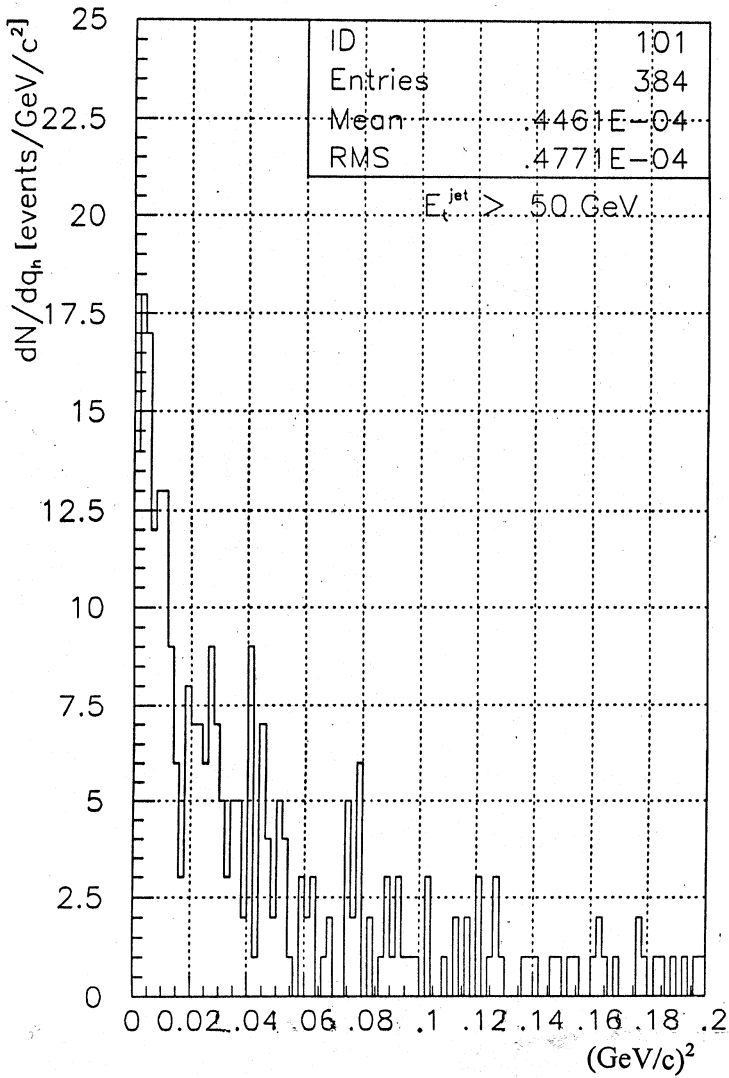


Fig. 3.  $\Delta q_h$  - distribution

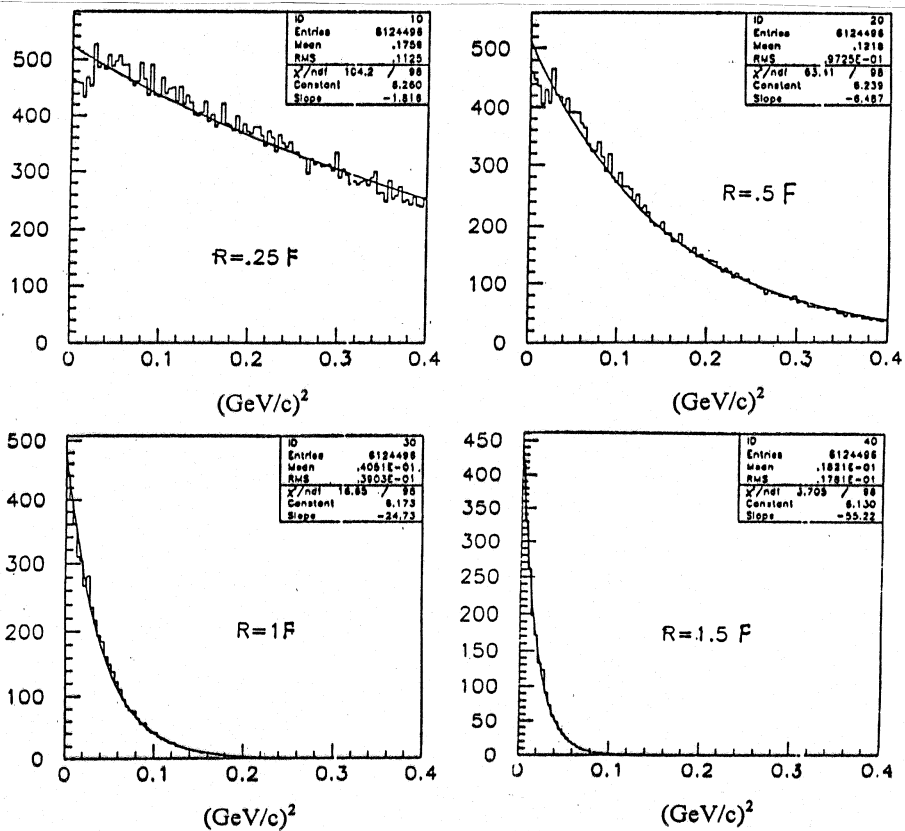


Fig. 4.  $q_h$ -distributions of  $\Phi$ -functions for different values of source dimension  $R$ .

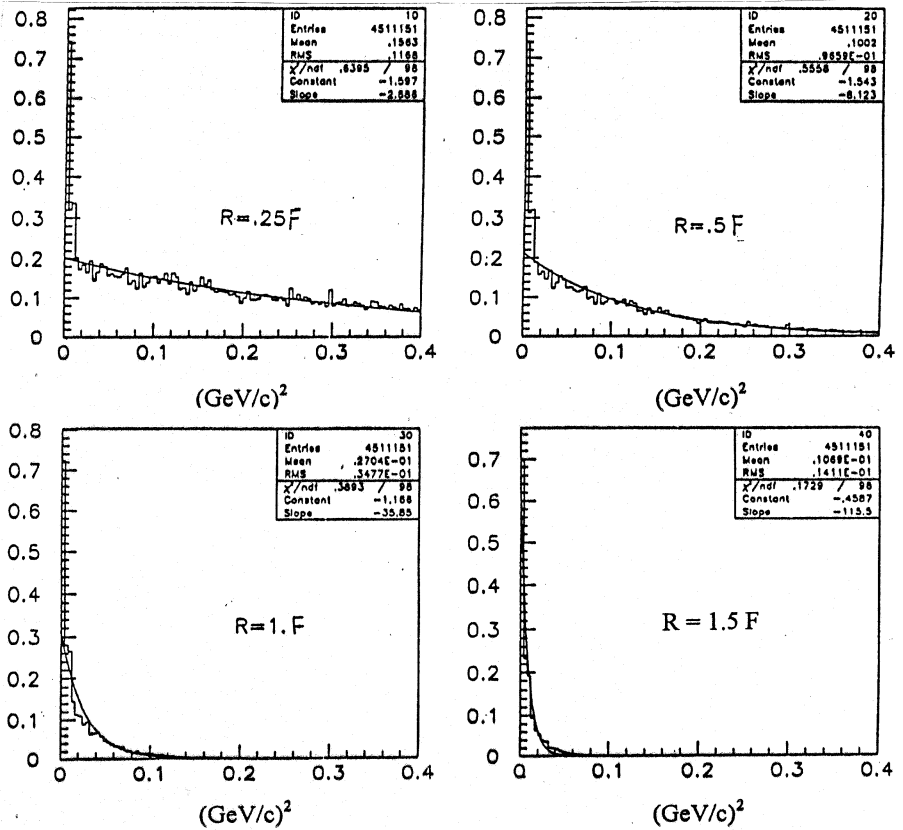


Fig. 5.  $q_h$ —distributions of  $\Phi_{Res}$  - functions for different values of source dimension  $R$ .

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Пенев В.Н., Шкловская А.И.  
Определение радиуса области рождения  $Z$ -бозонов  
методом интерференции тождественных частиц

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Метод определения радиуса области рождения  $Z$ -бозонов основан на интерференционном эффекте двух идентичных мюонов, один из которых происходит от  $Z$ -бозона, а другой — взят из фона. Разница в четырехмерных импульсах двух партнеров пары стремится к нулю. На основе большого числа моделированных пар, проходящих через CMS-детектор, показано, что радиус области рождения  $Z$ -бозонов в пределах 1 фм может быть определен с удовлетворительной точностью.

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Penev V.N., Chklovskaya A.I.  
Determination of  $Z$ -Bozon Source Radius Production  
by Identical Particles Interferometry

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The method of determination of  $Z$ -bozon source radius production is based on interference effect between two identical muons, one of which comes from  $Z$ -bozon, and the other is taken from the background. The difference between the four-momenta of these two partners of the pair tends to zero. It is shown on the basis of simulated samples of muon pairs passing through the CMS-detectors, that the  $Z$ -bozon sources radius of  $\sim 1$  fm can be clearly determined.

The investigation has been performed at the Laboratory of High Energies, JINR.

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