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I.V.Barashenkov, N.V.Alexeeva*, E.V.Zemlyanaya

TWO- AND THREE-DIMENSIONAL OSCILLONS IN NONLINEAR FARADAY RESONANCE

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*University of Cape Town, Rondebosch 7701, South Africa

Oscillons are localised two-dimensional oscillating structures which have recently been detected in experiments on vertically vibrated layers of granular material [1], Newtonian fluids and suspensions [2, 3]. Numerical simulations established the existence of stable oscillons in a variety of pattern-forming systems, including the Swift-Hohenberg and Ginsburg-Landau equations, period-doubling maps with continuous spatial coupling, semicontinuum theories and hydrodynamic models [4, 3]. Although these simulations provided a great deal of insight into the phenomenology of the oscillons (in particular, demarcated their existence area on the corresponding phase diagrams), little is known about the mechanism by which they acquire or loose their stability.

In this Letter, we consider a model equation which has *exact* oscillon solutions and allows an accurate characterisation of their existence and stability domains. The main purpose of this work is to understand how the oscillons manage to resist the general tendencies toward nonlinearity-induced blow-up or dispersive decay which are characteristic for localised excitations in two-dimensional media. Our model admits a straightforward generalisation to three dimensions and we use this opportunity to explore the existence of stable oscillons in 3D as well.

The model consists of a D-dimensional lattice of parametrically driven nonlinear oscillators (e.g. pendula) [5] with the nearest-neighbour coupling:

$$\frac{d^2}{d\tau^2}\phi_{\mathbf{k}} + \alpha \frac{d}{d\tau}\phi_{\mathbf{k}} + 2\kappa D\, \phi_{\mathbf{k}} - \kappa \sum_{|\mathbf{m}-\mathbf{k}|=1} \phi_{\mathbf{m}}$$
$$+(1+\rho\cos 2\omega t)\sin\phi_{\mathbf{k}} = 0; \quad \mathbf{k}=(k_1, ..., k_D). \tag{1}$$

Assuming that the coupling is strong: $\kappa = \epsilon^{-1}$; that the damping and driving are weak: $\alpha = \gamma\epsilon^2$, $\rho = 2h\epsilon^2$; and that the driving half-frequency is just below the edge of the linear spectrum gap: $\omega^2 = 1-\epsilon^2$, the oscillators execute small-amplitude librations of the form $\phi_{\mathbf{k}} = 2\epsilon\psi(t, \mathbf{x_k})e^{-i\omega\tau} + c.c. + O(\epsilon^3)$, where $t = \epsilon^2\tau/2$, $\mathbf{x_k} = \epsilon^{3/2}\mathbf{k}$ and the slowly varying amplitude satisfies

$$i\psi_t + \nabla^2\psi + 2|\psi|^2\psi - \psi = h\psi^* - i\gamma\psi, \tag{2}$$

the parametrically driven damped nonlinear Schrödinger (NLS) equation. In 2D, this equation was invoked as a phenomenological model of nonlinear Faraday resonance in water [3]. It also describes an optical resonator with different losses for the two polarisation components of the field [6]. In the absence of the damping and driving, all localised initial conditions in the 2D and 3D NLS equation are known to either disperse or blow-up in finite time [7, 8, 9]. Surprisingly, numerical simulations of (2) with sufficiently large h and γ revealed the occurrence of stable (or possibly long-lived) stationary localised excitations [3]. However no analytic solutions were found, and a possible stabilisation mechanism remained unclear.

In fact there are two exact (though not explicit) stationary radially-symmetric solutions given by

$$\psi^{\pm} = \mathcal{A}_{\pm} e^{-i\theta_{\pm}} \mathcal{R}_0(\mathcal{A}_{\pm} r); \quad (r^2 = x_1^2 + ... + x_D^2), \tag{3}$$

where $\mathcal{A}_{\pm}^2 = 1 \pm \sqrt{h^2 - \gamma^2}$, $\theta_+ = \frac{1}{2}\arcsin(\gamma/h)$, $\theta_- = \frac{\pi}{2} - \theta_+$, and $\mathcal{R}_0(r)$ is the bell-shaped nodeless solution of

$$\nabla_r^2 \mathcal{R} - \mathcal{R} + 2\mathcal{R}^3 = 0; \quad \mathcal{R}_r(0) = \mathcal{R}(\infty) = 0. \tag{4}$$

(Below we simply write $\mathcal{R}$ for $\mathcal{R}_0$.) In (4), $\nabla_r^2 = \partial_r^2 + (D-1)r^{-1}\partial_r$. Solutions of Eq.(4) in $D = 2$ and 3 are well documented in literature. (See e.g. [7] and refs therein.) One advantage of having an explicit dependence on h and γ, is that the existence domain is characterised by an explicit formula. The soliton ψ^+ exists for all $h > \gamma$; the ψ^- exists for $\gamma < h < \sqrt{1+\gamma^2}$. It is pertinent to add here that for $h < \gamma$, *all* initial conditions decay to zero. This follows from the rate equation

$$\partial_t |\psi|^2 = \frac{2}{r}[r(\chi_r |\psi|^2)_r]_r + 2|\psi|^2(h \sin 2\chi - \gamma), \tag{5}$$

where $\psi = |\psi| e^{-i\chi}$. Defining $N = \int |\psi|^2 d\mathbf{x}$, Eq.(5) implies $N_t \leq 2(h-\gamma)N$ whence $N(t) \to 0$ as $t \to \infty$.

We now examine the stability of the two solitons. Linearising Eq.(2) in the small perturbation

$$\delta\psi(\mathbf{x}, t) = e^{(\mu - \Gamma)\tilde{t} - i\theta_{\pm}}[u(\tilde{\mathbf{x}}) + iv(\tilde{\mathbf{x}})], \tag{6}$$

where $\tilde{\mathbf{x}} = \mathcal{A}_{\pm}\mathbf{x}$, $\tilde{t} = \mathcal{A}_{\pm}^2 t$, we get an eigenvalue problem

$$L_1 u = -(\mu + \Gamma)v, \quad (L_0 - \epsilon)v = (\mu - \Gamma)u, \tag{7}$$

where $\Gamma = \gamma/\mathcal{A}_\pm^2$ and the operators

$$L_0 \equiv -\tilde{\nabla}^2 + 1 - 2\mathcal{R}^2(\tilde{r}), \quad L_1 \equiv L_0 - 4\mathcal{R}^2(\tilde{r}), \tag{8}$$

with $\tilde{\nabla}^2 = \sum_{i=1}^D \partial^2/\partial\tilde{x}_i^2$. (We are dropping tildas below.) The quantity ϵ, $\epsilon = \pm 2\sqrt{h^2 - \gamma^2/\mathcal{A}_\pm^2}$, is positive for the ψ^+ soliton and negative for ψ^-. Each ϵ defines a "parabola" on the (h,γ)-plane:

$$h = \sqrt{\epsilon^2/(2-\epsilon)^2 + \gamma^2}. \tag{9}$$

A remarkable property of the model (1)-(2) is that the stability analysis can be reduced to a *one*-parameter eigenvalue problem. Introducing $\lambda^2 = \mu^2 - \Gamma^2$ and changing $v(\mathbf{x}) \to (\mu + \Gamma)\lambda^{-1}v(\mathbf{x})$ [10], Eq.(7) is brought to

$$(L_0 - \epsilon)v = \lambda u, \quad L_1 u = -\lambda v. \tag{10}$$

Since $\mathcal{R}_0(r)$ is nodeless in $0 \le r < \infty$, and $L_0\mathcal{R}_0 = 0$, the operator $L_0 - \epsilon$ is positive definite for $\epsilon < 0$. In this case the eigenvalue can be found as a minimum of the Rayleigh quotient:

$$-\lambda^2 = \min_w \frac{\langle w|L_1|w\rangle}{\langle w|(L_0-\epsilon)^{-1}|w\rangle}. \tag{11}$$

The operator L_1 has D zero eigenvalues associated with the translation eigenfunctions $\partial_i\mathcal{R}(r)$, $i = 1, 2, ...D$; hence it also has a negative eigenvalue with a radial-symmetric eigenfunction $w_0(r)$. Substituting w_0 into the quotient in (11), we get $-\lambda^2 < 0$ whence $\mu > \Gamma$. Thus the soliton ψ^- is unstable (against a nonoscillatory mode) for all D, h and γ, and may be safely disregarded.

Before proceeding to the stability of ψ^+ (for which we have $\epsilon > 0$), we make a remark on the undamped, undriven case ($\epsilon = 0$.) In 3D, the eigenvalue problem (10) has a zero eigenvalue associated with the phase invariance of the unperturbed NLS equation (2) and another one, associated with the scaling symmetry:

$$\begin{pmatrix} L_0 & 0 \\ 0 & L_1 \end{pmatrix} \begin{pmatrix} \mathcal{R} \\ -\frac{1}{2}(r\mathcal{R})_r \end{pmatrix} = \begin{pmatrix} 0 \\ \mathcal{R} \end{pmatrix}. \tag{12}$$

Both the eigenvector $(\mathcal{R}, 0)^T$ and the rank-2 generalised eigenvector $(0, -\frac{1}{2}(r\mathcal{R})_r)^T$ are radially-symmetric. In 2D the number of repeated zero eigenvalues associated with radially-symmetric invariances is four; in addition to those in (12) we have a two-parameter group of the lens transformations [7, 8] giving rise to

$$\begin{pmatrix} L_0 & 0 \\ 0 & L_1 \end{pmatrix} \begin{pmatrix} \frac{1}{8}r^2\mathcal{R} \\ g \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}(r\mathcal{R})_r \\ \frac{1}{8}r^2\mathcal{R} \end{pmatrix}, \tag{13}$$

with some $g(r)$. When $h^2-\gamma^2$ (or, equivalently, ϵ) deviates from zero, all the above invariances break down and the two (respectively, four) eigenvalues move away from the origin on the plane of complex λ. The directions of their motion are crucial for the stability properties.

We can calculate $\lambda(\epsilon)$ perturbatively, assuming

$$\lambda = \lambda_1\epsilon^{\frac{1}{4}} + \lambda_2\epsilon^{\frac{2}{4}} + \lambda_3\epsilon^{\frac{3}{4}} + ..., \tag{14}$$
$$u = u_1\epsilon^{\frac{1}{4}} + u_2\epsilon^{\frac{2}{4}} + ..., \quad v = \mathcal{R} + v_1\epsilon^{\frac{1}{4}} + v_2\epsilon^{\frac{2}{4}} + ...,$$

where $v_i = v_i(r)$, $u_i = u_i(r)$. Substituting into (10), the order $\epsilon^{1/4}$ gives $u_1 = -\lambda_1 L_1^{-1}\mathcal{R}$. Using (12), u_1 is found explicitly: $u_1 = (\lambda_1/2)(r\mathcal{R})_r$. At the order $\epsilon^{2/4}$ we get $u_2 = -\lambda_2 L_1^{-1}\mathcal{R}$ and equation $L_0 v_2 = \lambda_1 u_1$. Since L_0 has a null eigenvector, $\mathcal{R}(r)$, this equation is only solvable if

$$\lambda_1 \int \mathcal{R}(r)u_1(r)d\mathbf{x} = -\lambda_1^2 \tfrac{D-2}{4} \int \mathcal{R}^2(r)d\mathbf{x} = 0. \tag{15}$$

In the two-dimensional case the condition (15) is satisfied for any λ_1 whereas in $D=3$ we have to set $\lambda_1 = 0$. Next, at the orders $\epsilon^{3/4}$ and $\epsilon^{4/4}$ we obtain, respectively,

$$L_0 v_3 = \lambda_2 u_1 + \lambda_1 u_2 = \lambda_1\lambda_2(r\mathcal{R})_r, \tag{16}$$
$$L_0 v_4 = R + \lambda_1 u_3 + \lambda_2 u_2 + \lambda_3 u_1. \tag{17}$$

Eq.(16) is solvable both in 2D and 3D. The solvability condition for (17) reduces to

$$\lambda_1^4 = -\frac{\langle\mathcal{R}|\mathcal{R}\rangle}{\langle\mathcal{R}|L_1^{-1}L_0^{-1}L_1^{-1}|\mathcal{R}\rangle} = -16\frac{\int \mathcal{R}^2 d\mathbf{x}}{\int \mathcal{R}^2 r^2 d\mathbf{x}}, \tag{18}$$
$$\lambda_2^2 = \frac{\langle\mathcal{R}|\mathcal{R}\rangle}{\langle\mathcal{R}|L_1^{-1}|\mathcal{R}\rangle} = 4, \tag{19}$$

in two and three dimensions, respectively.

Thus we arrive at two different bifurcation scenarios. In 3D, where $\lambda_1 = 0$ and λ_2 is real, two imaginary eigenvalues $\pm|\lambda_2|\epsilon^{1/2}$ converge at the origin as $\epsilon \to 0$ from the left. (This does not mean that the ψ^- soliton is stable as there still is a pair of finite real eigenvalues for $\epsilon < 0$.) As ϵ grows to positive values, the imaginary pair $\pm|\lambda_2|\epsilon^{1/2}$ moves onto the real axis. The numerical analysis [11] of the eigenvalue problem (10) shows that when ϵ is further increased, the four real eigenvalues collide, pairwise, and acquire imaginary parts. Importantly, for all $0 < \epsilon < 1$ the imaginary parts remain smaller in magnitude than the real parts. As one can readily check, this means that $\mathrm{Re}\mu$ remains greater than Γ all the time, implying that the three-dimensional ψ^+ soliton is unstable for all h and γ.

The bifurcation occurring in 2D is more unusual. As ϵ approaches zero from the left, *four* eigenvalues converge at the origin, two along the real and two along imaginary axis: $\lambda \approx \pm|\lambda_1|(-\epsilon)^{1/4}, \pm i|\lambda_1|(-\epsilon)^{1/4}$. As ϵ moves to positive, the four eigenvalues start diverging at 45° to the real and imaginary axes. Hence to the leading order, $\mathrm{Im}\lambda \approx \mathrm{Re}\lambda$, and in order to make the conclusion about the stability, we need to calculate the higher-order corrections. The order $\epsilon^{5/4}$ produces a solvability condition

$$\lambda_1^3\lambda_2\langle\mathcal{R}|L_1^{-1}L_0^{-1}L_1^{-1}|\mathcal{R}\rangle = \frac{\lambda_1^3\lambda_2}{16}\int \mathcal{R}^2 r^2 d\mathbf{x} = 0,$$

yielding $\lambda_2 = 0$. (Here we made use of (13).) Finally, the order $\epsilon^{6/4}$ defines λ_3 (where $g(r)$ is as in (13)):

$$\lambda_3 = \frac{1}{\lambda_1} + \frac{\lambda_1^3}{2}\frac{\int g(r)\mathcal{R}(r)r^2 d\mathbf{x}}{\int \mathcal{R}^2(r)r^2 d\mathbf{x}}. \tag{20}$$

Taking λ_1 in the first quadrant, $\lambda_1 = e^{i\pi/4}|\lambda_1|$, and doing the integrals in (18), (20) numerically, we conclude that λ_3 is in the second quadrant, $\lambda_3 = e^{3i\pi/4}|\lambda_3|$, which implies that $|\mathrm{Im}\lambda| > |\mathrm{Re}\lambda|$. In terms of λ, the stability criterion $\mathrm{Re}\mu \le \Gamma$ is written as $\gamma \ge \gamma_c$, where

$$\gamma_c(\epsilon) \equiv \frac{2}{2-\epsilon} \cdot \frac{\mathrm{Re}\lambda(\epsilon)\,\mathrm{Im}\lambda(\epsilon)}{\sqrt{(\mathrm{Im}\lambda)^2 - (\mathrm{Re}\lambda)^2}}. \tag{21}$$

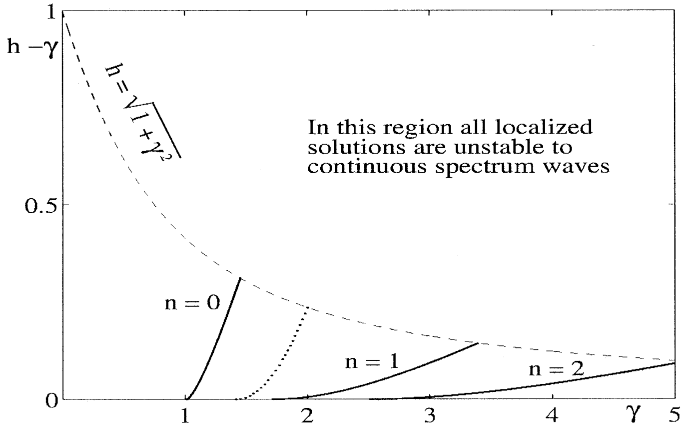

Figure 1. Stability diagram for two-dimensional solitons. The $(\gamma, h-\gamma)$-plane is used for visual clarity. No localised or periodic attractors exist in the region $h<\gamma$ (below the horisontal axis). The region of stability of the soliton ψ_n^+ with $n=0,1,2$, lies to the right of the corresponding solid curve. The dotted curve gives the variational approximation to the stability boundary of the ψ_0^+ soliton: $h=(1+\gamma^4)^{1/2}$, $\gamma \geq \sqrt{2}$.

The smallest γ for which the soliton can be stable, is given by

$$\lim_{\epsilon\to 0}\gamma_c(\epsilon)=\frac{1}{2\sqrt{2}}|\lambda_1|^{3/2}|\lambda_3|^{-1/2}. \tag{22}$$

Substituting for λ_1, λ_3 their numerical values, (22) gives $\gamma_c(0) = 1.00647$. For $\epsilon \neq 0$ we obtained $\lambda(\epsilon)$ by solving the eigenvalue problem (10) directly [11]. Here we have restricted ourselves to radially-symmetric $u(r)$ and $v(r)$. Expressing ϵ via γ_c from (21) and feeding into (9), we get the stability boundary on the (h,γ)-plane (Fig.1).

Asymmetric perturbations do not lead to any instabilities in 2D. To show this, we factorise, in (10), $u(\mathbf{x}) = \tilde{u}(r)e^{im\varphi}$ and $v(\mathbf{x}) = \tilde{v}(r)e^{im\varphi}$, where $\tan\varphi = y/x$ and m is an integer. The eigenproblem (10) remains the same, with only the operators L_0 and L_1 being replaced by

$$L_0^{(m)} \equiv -\nabla_r^2 + m^2/r^2 + 1 - 2\mathcal{R}^2, \quad L_1^{(m)} \equiv L_0^{(m)} - 4\mathcal{R}^2.$$

The crucial observation now is that $L_0^{(m)}$ with $m^2 \geq 1$ does not have *any* (not even positive) discrete eigenvalues. We verified this numerically for $m^2 = 1$; this rules out their appearance for all other m. Therefore the operator $L_0^{(m)} - \epsilon$ with $\epsilon < 1$ is positive definite, and the eigenvalues of the problem (10) can be found from the variational principle (11). The operator $L_1^{(1)}$ has a zero eigenvalue with the eigenfunction $w^{(1)}(r) = \mathcal{R}_r(r)$ which has no nodes for $0 < r < \infty$; hence its all other eigenvalues (if exist) are positive. This also implies that $L_1^{(m)}$ with $m^2 > 1$ are positive definite. Thus the minimum of the Rayleigh quotient (11) is zero for $m^2 = 1$ and positive for $m^2 > 1$.

Besides the nodeless solution $\mathcal{R}_0(r)$, the "master" equation (4) has solutions $\mathcal{R}_n(r)$ with n nodes, $n = 1, 2, \ldots$. These give rise to a sequence of nodal solutions of the damped-driven NLS (2), defined by Eq.(3) with $\mathcal{R}_0 \to \mathcal{R}_n$. It is easy to realise that the solitons ψ_n^- are unstable for all h, γ and D. Indeed, since $L_0 \mathcal{R}_n = 0$, the operator L_0 has $(n-1)$ negative eigenvalues, with eigenfunctions $z_k(r)$. For $\epsilon < 0$ we can search for eigenvalues of (10) as minima of the quotient (11) on the subspace of functions orthogonal to all z_k, $k = 1, \ldots, n-1$. Since L_1 has D null eigenfunctions $\partial_i \mathcal{R}_n$, it also has a negative eigenvalue with a radially-symmetric eigenfunction $w_{n-1}(r)$ having $(n-1)$ nodes, and hence it has $(n-1)$ more negative eigenvalues with eigenfunctions $w_0(r), w_1(r), \ldots, w_{n-2}(r)$. Therefore we can set up a linear combination $C_0 w_0 + \ldots + C_{n-1} w_{n-1}$ such that it belongs to the subspace in question and at the same time renders the quotient in (11) negative.

To examine the stability of the ψ_n^+ soliton, we solved the eigenvalue problem (10) numerically, in the class of radially-symmetric eigenfunctions. In 3D, positive real eigenvalues are present in the spectrum for all ϵ; thus the three-dimensional nodal solitons are always unstable. In 2D, for $\epsilon = 0$ the spectrum consists of n complex quadruplets $\pm\lambda_n, \pm\lambda_n^*$ and four zero eigenvalues. As ϵ grows to positive values, the trajectories of the four eigenvalues diverging from the origin can be described by formulas (14), (18), (20) where now $\mathcal{R}$ stands for $\mathcal{R}_n$. Importantly, the imaginary parts of these eigenvalues — as well as of the other complex quadruplets — are greater in absolute value than their real parts. Therefore the nodal ψ_n^+ solutions are stable for sufficiently large γ. Calculating $\gamma_c(\epsilon)$ for each of the quadruplets from (21), choosing the largest of these $n+1$ values, and substituting into (9), we obtained the stability boundary on the

(h, γ)-plane for the ψ_1^+ and ψ_2^+ solitons (Fig.1).

Lastly, we need to understand the stabilisation mechanism in qualitative terms. To this end, we use the variational approach. The equation (2) is derivable from the stationary action principle with the Lagrangian

$$\mathcal{L} = e^{2\gamma t}\mathrm{Re}\int (i\psi_t\psi^* - |\nabla\psi|^2 - |\psi|^2 + |\psi|^4 - h\psi^2)d\mathbf{x}.$$

Choosing the ansatz $\psi = \sqrt{A}e^{-i\theta-(B+i\sigma)r^2}$ [12, 13] with A, B, θ, σ functions of t, this reduces, in 2D, to

$$\mathcal{L} = e^{2\gamma t}\frac{A}{B}\left[\dot{\theta} - 1 + \frac{\dot{\sigma}}{2B} - \frac{2B}{\cos^2\phi} + \frac{A}{2} - h\cos(\phi + 2\theta)\cos\phi\right]; \quad \tan\phi = \sigma/B. \tag{23}$$

The 4-dimensional dynamical system defined by (23), has two stationary points representing the $\psi^\pm$ solitons. In agreement with the stability properties of the solitons in the full PDE, the ψ^+ stationary point is unstable for small γ but stabilises for larger dampings (Fig.). When γ is large we can expand $A = A_0 + \frac{1}{\gamma}A_1 + ..., B = B_0 + \frac{1}{\gamma}B_1 +..., \theta = \frac{\pi}{4} + \frac{1}{\gamma}\theta_1 + ..., \sigma = \frac{1}{\gamma}\sigma_1 + ...$ Letting $h = \gamma + \frac{c}{2\gamma}$ where $0 \leq c \leq 1$, defining $T = \frac{t}{\gamma}$ and matching coefficients of like powers of $\frac{1}{\gamma}$, yields a 2-dimensional system

$$dA_0/dT = A_0[c + 8\sigma_1 - 4\theta_1^2 + 2(\sigma_1/B_0)^2], \tag{24}$$

$$dB_0/dT = 8\sigma_1 B_0 + 4\sigma_1\theta_1 + 4(\sigma_1^2/B_0), \tag{25}$$

$$\theta_1 = \tfrac{1}{2} + 2B_0 - \tfrac{3}{4}A_0, \quad \sigma_1 = \tfrac{1}{2}A_0B_0 - 2B_0^2. \tag{26}$$

Like their parent system (23), Eqs.(24)-(26) have two fixed points, the saddle at $B_0^- = \frac{1}{2} - \sqrt{c}$, $A_0^- = 4B_0^-$ and a stable focus at $B_0^+ = \frac{1}{2} + \sqrt{c}$, $A_0^+ = 4B_0^+$.

According to (5), the soliton's phase $\chi = \theta + \sigma r^2$ controls the creation and annihilation of the soliton's elementary constituents (whose density is $|\psi|^2$). (If Eq.(2) is used as a model equation for Faraday resonance in granular media or fluids, $\int |\psi|^2\, d\mathbf{x}$ has the meaning of the number of grains or mass of the fluid captured in the oscillon.) Since the creation and annihilation occurs mainly in the core of the soliton, the variable phase component σr^2 plays a marginal role in this process. Instead, the significance of the quantity σ is in that it controls the flux of the constituents between the core and the periphery of the soliton — see, again, equation (5).

If we perturb the stationary point ψ^+ in the 4-dimensional phase space of (23), the variables θ and σ will zap, within a very short time $\Delta t \sim \frac{1}{\gamma}$, onto the 2-dimensional subspace defined by the constraints (26). After this short transient the evolution of θ and σ will be immediately following that of the soliton's amplitude $\sqrt{A}$ and width B. In the case of the ψ^+ soliton, this provides a negative feedback: perturbations in A and B produce only such changes in the phase and flux that the new values of θ and σ stimulate the recovery of the stationary values of A and B. (The phase θ works to restore the number of constituents while σ rearranges them within the soliton.) In the case of the ψ^- the feedback is positive: the perturbation-induced phase and flux (26) strive to amplify the perturbation of the soliton's amplitude and width still further. Finally, for small γ the coupling of θ and σ to A_0 and B_0 is via differential rather than algebraic equations. In this case the dynamics of the phase and flux is inertial and their changes may not catch up with those of the amplitude and width. The feedback loop is destroyed and the soliton destabilises.

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Двух- и трехмерные осциллоны
в нелинейном фарадеевском резонансе

Исследуются двух- и трехмерные локализованные осциллирующие структуры в простой модельной системе, представляющей нелинейный фарадеевский резонанс. Показано, что соответствующее нелинейное уравнение Шредингера для амплитуды имеет последовательность точных солитонных решений. Двухмерные солитоны являются устойчивыми при определенных значениях параметров; следовательно, диссипация и параметрическая накачка могут предотвратить коллапс и дисперсионное затухание солитонов в двухмерном случае. Напротив, трехмерные осциллоны всегда неустойчивы.

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Two- and Three-Dimensional Oscillons
in Nonlinear Faraday Resonance

We study two- and three-dimensional localized oscillating patterns in a simple model system exhibiting nonlinear Faraday resonance. The corresponding nonlinear Schrödinger equation for the amplitude is shown to have sequences of exact soliton solutions. The 2D solitons are found to be stable in certain parameter ranges; hence the damping and parametric driving are capable of preventing the nonlinear blowup and dispersive decay of solitons in two dimensions. On the contrary, the 3D oscillons are shown to be always unstable.

The investigation has been performed at the Laboratory of Information Technologies, JINR.

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