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FIRST RADIAL EXCITATIONS OF SCALAR MESON NONET AND THE GLUEBALL

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1. INTRODUCTION

The description of scalar mesons with masses from 0.4 to 1.7 GeV is an actual and complex problem attracting attention of many physicists for last years [1–4]. The final solution of this problem is not yet found. It is complicated by the fact that, in this interval of masses, there exists a scalar glueball which is noticeably mixed with scalar isoscalar quarkonia. In [5–7], we for the first time suggested to interpret 18 scalar mesons, lying in the mass interval under consideration, as two meson nonets: the ground nonet of scalar quarkonia (with masses below 1 GeV) and the nonet of their first radial excitations (with masses greater than 1 GeV). However, here, an additional scalar meson state [8] has been seen experimentally. The origin of this extra, 19th, state is supposed to be connected with the scalar glueball. Two possible candidates for the glueball are often argued: $f_0(1500)$ and $f_0(1710)$ [1, 2, 4–7, 9–11]. Our present paper is devoted to solving the problem of identification of the true glueball state with one of these states. This is to be done by introducing the scalar glueball into the effective meson Lagrangian investigated in [5–7].

A nonlocal $U(3) \times U(3)$ quark model of the Nambu–Jona-Lasinio (NJL) type was first suggested in [12, 13] to describe the ground and radially excited nonets of pseudoscalar and vector mesons. Next, in [5–7], this model was used to study scalar meson nonets. Its Lagrangian was completed by the 't Hooft interaction to describe the singlet-octet mixing in the scalar and pseudoscalar sectors. For the description of excited states, simple Lorentz-covariant form factors of a polynomial form were used. To investigate the first radial excitations, polynomials of the second order by momentum sufficed. The form factor was chosen in a form which allowed to reproduce all low-energy theorems in the chiral limit and the mechanism of spontaneous chiral symmetry breaking (SCSB) [12]. This model was applied for the description of strong decays of radially excited scalar, pseudoscalar, and vector mesons [5–7, 14].

The chiral symmetry allowed us to use the same form factors both for the pseudoscalar and scalar mesons. As a result, using the masses of excited pseudoscalar meson states, we predicted the mass spectrum of radially excited states of scalar mesons. We also showed that 18 scalar meson states with masses from 0.4 to 1.7 GeV can be considered as two scalar meson nonets: the ground and first radially excited [5–7]. The state $f_0(1710)$ was considered as a quarkonium (the radial excitation of $f_0(980)$). A calculation of widths for the strong decay modes of these mesons and subsequent matching them with experimental data corroborated our conclusions concerning the quark nature of the 18 states. Meanwhile, the state $f_0(1500)$ happened to be beyond our model, and its description required introducing a glueball into the model.

To solve the problem of describing the glueball, simple models that describe the ground scalar quarkonia states only together with the glueball were constructed in [9–11]. There, we introduced a glueball into a $U(3) \times U(3)$ quark Lagrangian with the 't Hooft interaction by means of the dilaton model. The dilaton model has been often used for this purpose by many authors [1,15–17].

The mixing of the glueball with scalar isoscalar quarkonia was described and widths of the main modes of their strong decays were calculated. Our calculations showed that, among the most probable candidates for the glueball, $f_0(1500)$ and $f_0(1710)$, the state $f_0(1500)$ better meets the assumption that it is the glueball than $f_0(1710)$. However, the final decision must be made after including radially excited states and taking account of mixing between five scalar isoscalar states (four quarkonia and a glueball), and describing their decays.

Methods used in [5–7], and in [9,10] are unified in the present paper to construct an extended nonlocal $U(3) \times U(3)$ model with the glueball, allowing to describe all 19 scalar meson states in the interested interval of masses. After calculation of widths of the strong decay modes of scalar mesons, we once more saw that the most probable candidate for the glueball is the state $f_0(1500)$. Meanwhile, the glueball gets noticeably mixed with the states $f_0(400-1200)$ and $f_0(1370)$, mostly composed of u and d quarks, and is almost not represented in $f_0(980)$ and $f_0(1710)$ containing mostly s quarks. Isovector and strange states change little after introducing the glueball. Therefore, the results, obtained here for isovectors and strange mesons, are close to those derived in [5–7], where the glueball was not considered. All changes are connected with new value of the constant K which, unlike papers [5–7], is fixed in our paper not only by masses of η and η' but also by the lower experimental bound for the mass of the lightest scalar isoscalar state $f_0(400-1200)$.

The structure of our paper is following. In section 2, a nonlocal chiral quark model of the NJL type with the six-quark 't Hooft interaction is bosonized to construct an effective meson Lagrangian. In section 3, the meson Lagrangian is extended by introducing a scalar glueball as a dilaton on the base of scale invariance. The gap equations, the divergence of the dilatation current and quadratic terms of the effective meson Lagrangian are derived in sect. 4. There, we also diagonalize quadratic terms. Numerical estimates of the model parameters are given in sect. 5. In section 6, the widths for the main modes of strong decays of scalar mesons are calculated. The discussion over the obtained results is given in sect. 7. A detailed description of how to calculate the quark loop contribution to the width of strong decays of scalar mesons is given Appendix A.

2. $U(3) \times U(3)$ LAGRANGIAN FOR QUARKONIA

We start from an effective $U(3) \times U(3)$ quark Lagrangian of the following form (see [5–7]):

$$L = L_{\text{free}} + L_{\text{NJL}} + L_{\text{tH}}, \tag{1}$$

$$L_{\text{free}} = \bar{q}(i\hat{\partial} - m^0)q \tag{2}$$

$$L_{\rm NJL} = \frac{G}{2}\sum_{n=1}^{N}\sum_{a=1}^{9}[(j^a_{{\rm S},n})^2 + (j^a_{{\rm P},n})^2] \quad (3)$$

$$L_{\rm tH} = -K\left\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\right\}, \quad (4)$$

where $L_{\rm free}$ is the free quark Lagrangian with q and $\bar{q}$ being u, d, or s quark fields; m^0 is a current quark mass matrix with diagonal elements: $m^0_{\rm u}$, m^0_d, $m^0_{\rm s}$ ($m^0_{\rm u} \approx m^0_d$). The term $L_{\rm NJL}$ contains nonlocal four-quark vertices of the Nambu–Jona-Lasinio type which have the current-to-current form. The quark currents are defined in accordance with [5–7, 12, 13]:

$$j^a_{{\rm S(P)},n}(x) = \int d^4x_1 d^4x_2 \bar{q}(x_1) F^a_{{\rm S(P)},n}(x; x_1, x_2) q(x_2), \quad (5)$$

where the subscript S is for scalar and P for the pseudoscalar currents. The term $L_{\rm tH}$ is the six-quark 't Hooft interaction which is supposed to be local, so no form factor is introduced in $L_{\rm tH}$.

For $n > 1$, currents (5) are nonlocal due to form factors $F^a_{{\rm S(P)},n}$. This way of introducing nonlocality allows to consider radially excited meson states, which is impossible in the standard local NJL model. In general, the number of radial excitations N is infinite, but we restrict our-selves with $N = 2$, leaving only the ground and first radially excited states, because extending this model by involving more heavier particles is not valid for this class of models.

Let us define the form factors in the momentum space.

$$\begin{aligned} & F^a_{{\rm S(P)},n}(x; x_1, x_2) = \\ & = \int \frac{d^4P}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} \exp\frac{i}{2}\Big((P+k)(x-x_1) + \\ & +(P-k)(x-x_2)\Big) F^a_{{\rm S(P)},n}(k|P), \end{aligned} \quad (6)$$

where P is the total momentum of a meson and k is the relative momentum of quarks inside the meson. As it was mentioned in the Introduction, here we follow papers [5–7,12,13], where the form factors $F^a_{{\rm S(P)},n}(k|P)$ are chosen in the momentum space as follows:

$$F^a_{{\rm S},n}(k|P) = \tau_a c_a f^a_n(k_\perp), \quad F^a_{{\rm P},n}(k|P) = i\gamma_5\tau_a c_a f^a_n(k_\perp), \quad (7)$$

and the functions f^a_n, ($n = 1, 2$) are

$$f^a_1(k_\perp) = 1, \quad f^a_2(k_\perp) = 1 + d_a|k_\perp|^2. \quad (8)$$

These depend on the transverse relative momentum of the quarks:

$$k_\perp = k - \frac{P\cdot k}{P^2}P. \quad (9)$$

In the rest frame of a meson $P = (M, \mathbf{0})$, the vector $k_\perp$ equals $(0, \mathbf{k})$, thereby the form factors can be considered as functions of 3-dimensional momentum. Further calculations will be carried out in this particular frame. The matrices τ_a are expressed via the Gell-Mann λ_a matrices as follows:

$$\begin{aligned} &\tau_a = \lambda_a \quad (a = 1, ..., 7), \quad \tau_8 = (\sqrt{2}\lambda_0 + \lambda_8)/\sqrt{3}, \\ &\tau_9 = (-\lambda_0 + \sqrt{2}\lambda_8)/\sqrt{3}. \end{aligned} \tag{10}$$

Here $\lambda_0 = \sqrt{2/3}\,\mathbf{1}$, with $\mathbf{1}$ being the unit matrix.

Each form factor function contains a slope parameter d_a which is fixed by special conditions given in sect. 4 (see eq. (55) below). The arbitrary parameter c_a can be absorbed by the four-quark interaction constat G. As a result, we obtain arbitrary constants $\bar{G}_a = c_a^2 G$, where only four constants $\bar{G}_1$, $\bar{G}_4$, $\bar{G}_8$, and $\bar{G}_9$ are free because the following relations take place:

$$\bar{G}_1 = \bar{G}_2 = \bar{G}_3, \quad \bar{G}_4 = \bar{G}_5 = \bar{G}_6 = \bar{G}_7. \tag{11}$$

Thus, the term $L_{\rm NJL}$ (see (3)) çan be rewritten for the ground and first radially excited states in the following form:

$$L_{\rm NJL} = \frac{G}{2}\sum_{a=1}^{9}[(j^a_{\rm S,1})^2 + (j^a_{{\rm P},n})^2] + \frac{1}{2}\sum_{a=1}^{9}\bar{G}_a[(\tilde{j}^a_{\rm S,2})^2 + (\tilde{j}^a_{{\rm P},n})^2], \tag{12}$$

where

$$j^a_{\rm S(P),2} = c_a \tilde{j}^a_{\rm S(P),2}. \tag{13}$$

As it follows from our further calculations (see sect. 4), we have only 3 different form factor functions:

$$f_2^{\rm u} = 1 + d_{\rm u}\mathbf{k}^2, \quad f_2^{\rm s} = 1 + d_{\rm s}\mathbf{k}^2, \quad f_2^{\rm us} = 1 + d_{\rm us}\mathbf{k}^2. \tag{14}$$

The values o constants $d_{\rm u}$, $d_{\rm s}$, and $d_{\rm us}$ are given in sect. 5. As a consequence of such a definition of the form factor functions, all arbitrariness connected with introducing form factors reveals itself only in mass definitions (see (63)), while the interaction of excited mesons is free of arbitrary parameters.

Instead of Lagrangian (1), it is convenient to use its equivalent form containing only four-quark vertices whose interaction constants take account of the 't Hooft interaction. Using the method described in [5–7, 9, 18–20], we obtain

$$\begin{aligned} L = &\bar{q}(i\hat{\partial} - \overline{m}^0)q + \\ &+\frac{1}{2}\sum_{a,b=1}^{9}\left[G^{(-)}_{ab} j^a_{\rm S,1} j^b_{\rm S,1} + G^{(+)}_{ab} j^a_{\rm P,1} j^b_{\rm P,1}\right] + \\ &+\frac{1}{2}\sum_{a=1}^{9}\bar{G}_a\left[j^a_{\rm S,2} j^a_{\rm S,2} + j^a_{\rm P,2} j^a_{\rm P,2}\right], \end{aligned} \tag{15}$$

where

$$
\begin{aligned}
&G_{11}^{(\pm)} = G_{22}^{(\pm)} = G_{33}^{(\pm)} = G \pm 4Km_{\rm s}\mathcal{J}_{0,1}^{\Lambda}[1],\\
&G_{44}^{(\pm)} = G_{55}^{(\pm)} = G_{66}^{(\pm)} = G_{77}^{(\pm)} = G \pm 4Km_{\rm u}\mathcal{J}_{1,0}^{\Lambda}[1],\\
&G_{88}^{(\pm)} = G \mp 4Km_{\rm s}\mathcal{J}_{0,1}^{\Lambda}[1], \quad G_{99}^{(\pm)} = G,\\
&G_{89}^{(\pm)} = G_{98}^{(\pm)} = \pm 4\sqrt{2}Km_{\rm u}\mathcal{J}_{1,0}^{\Lambda}[1],\\
&G_{ab}^{(\pm)} = 0 \quad (a \neq b; \quad a,b = 1,\dots,7),\\
&G_{a8}^{(\pm)} = G_{a9}^{(\pm)} = G_{8a}^{(\pm)} = G_{9a}^{(\pm)} = 0 \quad (a = 1,\dots,7),
\end{aligned}
\tag{16}
$$

and $\overline{m}^0$ is a diagonal matrix composed of modified current quark masses:

$$\overline{m}_{\rm u}^0 = m_{\rm u}^0 - 32Km_{\rm u}m_{\rm s}\mathcal{J}_{1,0}^{\Lambda}[1]\mathcal{J}_{0,1}^{\Lambda}[1], \tag{17}$$

$$\overline{m}_{\rm s}^0 = m_{\rm s}^0 - 32Km_{\rm u}^2\mathcal{J}_{1,0}^{\Lambda}[1]^2, \tag{18}$$

introduced here to avoid double counting of the 't Hooft interaction in gap equations (see [9,20]). Here $m_{\rm u}$ and $m_{\rm s}$ are constituent quark masses, and $I_1^{\Lambda}(m_a)$ stands for a regularized integral over the momentum space. It is convenient to define all integrals that will appear further in the paper via the functional $\mathcal{J}$:

$$\mathcal{J}_{l,n}^{\Lambda}[f] = -i\frac{N_c}{(2\pi)^4}\int d^4k \frac{f(\mathbf{k})\theta(\Lambda^2 - \mathbf{k}^2)}{(m_{\rm u}^2 - k^2)^l(m_{\rm s}^2 - k^2)^n}, \tag{19}$$

where f is a product of form factor functions, and $N_c = 3$ is the number of colors. Since the integral is divergent for some values of l and n, it is regularized by a 3-dimensional cutoff Λ.

After bosonization of Lagrangian (15) we obtain:

$$
\begin{aligned}
\tilde{\mathcal{L}}(\bar{\sigma},\phi) = \tilde{L}_{\rm G}(\bar{\sigma},\phi) - i\mathrm{Tr}\ln\Big\{ i\hat{\partial} - \overline{m}^0 + \\
+ \sum_{n=1}^{2}\sum_{a=1}^{9}\tau_a g_{a,n}(\bar{\sigma}_{a,n} + i\gamma_5\sqrt{Z}\phi_{a,n})f_n^a\Big\},
\end{aligned}
\tag{20}
$$

where

$$
\begin{aligned}
&\tilde{L}_{\rm G}(\bar{\sigma},\phi) = \\
&= -\frac{1}{2}\sum_{a,b=1}^{9} g_{a,1}\bar{\sigma}_{a,1}\left(G^{(-)}\right)^{-1}_{ab} g_{b,1}\bar{\sigma}_{b,1} - \frac{Z}{2}\sum_{a,b=1}^{9} g_{a,1}\phi_{a,1}\left(G^{(+)}\right)^{-1}_{ab} g_{b,1}\phi_{b,1} - \\
&- \sum_{a=1}^{9}\frac{g_{a,2}^2}{2\bar{G}_a}(\sigma_{a,2}^2 + \phi_{a,2}^2)
\end{aligned}
\tag{21}
$$

As it follows from our further calculations of quark loop diagrams, the vacuum expectation values (VEV) of the fields $\bar{\sigma}_{8,1}$ and $\bar{\sigma}_{9,1}$ are not equal to zero, while

$\langle\bar{\sigma}_{a,1}\rangle_0 = 0$, $(a = 1,\dots,7)$. This is connected with the existence of tadpole diagrams (Fig. 1(a)) for the ground meson states. Therefore, it is necessary to introduce new fields $\sigma_{a,n}$ with zero VEV $\langle\sigma_{8,n}\rangle_0 = \langle\sigma_{9,n}\rangle_0 = 0$, using the following relations:

$$\begin{aligned} g_{8,1}\bar{\sigma}_{8,1} - \overline{m}^0_{\mathrm{u}} &= g_{8,1}\sigma_{8,1} - m_{\mathrm{u}}, \\ g_{9,1}\bar{\sigma}_{9,1} + \frac{\overline{m}^0_{\mathrm{s}}}{\sqrt{2}} &= g_{9,1}\sigma_{9,1} + \frac{m_{\mathrm{s}}}{\sqrt{2}}. \end{aligned} \tag{22}$$

VEV taken from (22) give gap equations connecting current and constituent quark masses (see (53) and (54) in sect. 4). This is a consequence of spontaneous breaking of chiral symmetry (SBCS). As a result, we obtain [9,18]:

$$\begin{aligned} &\mathcal{L}(\sigma,\phi) = L_{\mathrm{G}}(\sigma,\phi) - \\ &-i\,\mathrm{Tr}\ln\left\{i\hat{\partial} - m + \sum_{n=1}^{2}\sum_{a=1}^{9}\tau_a g_{a,n}(\sigma_{a,n} + i\gamma_5\sqrt{Z}\phi_{a,n})f^a_n\right\} = \\ &= L_{\mathrm{kin}}(\sigma,\phi) + L_{\mathrm{G}}(\sigma,\phi) + L_{\mathrm{loop}}(\sigma,\phi). \end{aligned} \tag{23}$$

The term $L_{\mathrm{G}}(\sigma,\phi)$ is

$$\begin{aligned} &L_{\mathrm{G}}(\sigma,\phi) = \\ &= -\frac{1}{2}\sum_{a,b=1}^{9}(g_{a,1}\sigma_{a,1} - \mu_a + \bar{\mu}^0_a)\left(G^{(-)}\right)^{-1}_{ab}(g_{b,1}\sigma_{b,1} - \mu_b + \bar{\mu}^0_b) \\ &-\frac{Z}{2}\sum_{a,b=1}^{9} g_{a,1}\phi_{a,1}\left(G^{(+)}\right)^{-1}_{ab} g_{b,1}\phi_{b,1} - \sum_{a=1}^{9}\frac{g^2_{a,2}}{2\bar{G}_a}(\sigma^2_{a,2} + \phi^2_{a,2}). \end{aligned} \tag{24}$$

Here we introduced, for convenience, the constants μ_a and $\bar{\mu}^0_a$ defined as follows: $\mu_a = 0$, $(a = 1,\dots,7)$, $\mu_8 = m_{\mathrm{u}}$, $\mu_9 = -m_{\mathrm{s}}/\sqrt{2}$ and $\bar{\mu}^0_a = 0$, $(a = 1,\dots,7)$, $\bar{\mu}^0_8 = \overline{m}^0_{\mathrm{u}}$, $\bar{\mu}^0_9 = -\overline{m}^0_{\mathrm{s}}/\sqrt{2}$.

The term $L_{\mathrm{kin}}(\sigma,\phi)$ contains the kinetic terms and, in the momentum space, has the following form:

$$L_{\mathrm{kin}}(\sigma,\phi) = \frac{P^2}{2}\sum_{n,j=1}^{2}\sum_{a=1}^{9}\left(\sigma_{a,n}\Gamma^a_{\mathrm{S},nj}\sigma_{a,j} + \phi_{a,n}\Gamma^a_{\mathrm{P},nj}\phi_{a,j}\right), \tag{25}$$

where

$$\begin{aligned} \Gamma^a_{\mathrm{S(P)},11} &= \Gamma^a_{\mathrm{S(P)},22} = 1, \\ \Gamma^a_{\mathrm{S(P)},12} &= \Gamma^a_{\mathrm{S(P)},21} = \gamma^a_{\mathrm{S(P)}}, \end{aligned} \tag{26}$$

$$\gamma^a_{\mathrm{S}} = \begin{cases} \dfrac{\mathcal{J}^{\Lambda}_{2,0}[f^{\mathrm{u}}_2]}{\sqrt{\mathcal{J}^{\Lambda}_{2,0}[1]\mathcal{J}^{\Lambda}_{2,0}[f^{\mathrm{u}}_2 f^{\mathrm{u}}_2]}} & (a = 1,2,3,8), \\ \dfrac{\mathcal{J}^{\Lambda}_{1,1}[f^{\mathrm{us}}_2]}{\sqrt{\mathcal{J}^{\Lambda}_{1,1}[1]\mathcal{J}^{\Lambda}_{1,1}[f^{\mathrm{us}}_2 f^{\mathrm{us}}_2]}} & (a = 4,5,6,7), \\ \dfrac{\mathcal{J}^{\Lambda}_{0,2}[f^{\mathrm{s}}_2]}{\sqrt{\mathcal{J}^{\Lambda}_{0,2}[1]\mathcal{J}^{\Lambda}_{0,2}[f^{\mathrm{s}}_2 f^{\mathrm{s}}_2]}} & (a = 9), \end{cases} \tag{27}$$

$$\gamma_{\rm P}^a = \gamma_{\rm S}^a \sqrt{Z}. \tag{28}$$

The term $L_{\rm loop}(\sigma,\phi)$ is a sum of one-loop (see Fig.1) quark contributions[1)], from which the kinetic term was subtracted:

$$L_{\rm loop}(\sigma,\phi) = L^{(1)}_{\rm loop}(\sigma) + L^{(2)}_{\rm loop}(\sigma,\phi) + L^{(3)}_{\rm loop}(\sigma,\phi) + L^{(4)}_{\rm loop}(\sigma,\phi), \tag{29}$$

where the superscript in brackets stands for the degree of fields. Thus, $L^{(1)}_{\rm loop}$ (Fig. 1(a)) contains the terms linear in the field σ; $L^{(2)}_{\rm loop}$ (Fig. 1(b)), the quadratic ones, and so on. For example[2)],

$$L^{(1)}_{\rm loop}(\sigma,\phi) = 8m_{\rm u} g_{8,1} \mathcal{J}^\Lambda_{1,0}[1]\sigma_{8,1} - 4\sqrt{2} m_{\rm s} g_{9,1} \mathcal{J}^\Lambda_{0,1}[1]\sigma_{9,1}, \tag{30}$$

$$\begin{aligned}
L^{(2)}_{\rm loop}(\sigma,\phi) &= 4\sum_{a=1}^{3} g^2_{a,1}\mathcal{J}^\Lambda_{1,0}[1](\sigma^2_{a,1} + Z\phi^2_{a,1}) + \\
&+2\sum_{a=4}^{7} g^2_{a,1}(\mathcal{J}^\Lambda_{1,0}[1] + \mathcal{J}^\Lambda_{0,1}[1])(\sigma^2_{a,1} + Z\phi^2_{a,1}) + \\
&+4g^2_{8,1}\mathcal{J}^\Lambda_{1,0}[1](\sigma^2_{8,1} + Z\phi^2_{8,1}) + \\
&+4g^2_{9,1}\mathcal{J}^\Lambda_{0,1}[1](\sigma^2_{9,1} + Z\phi^2_{9,1}) + \\
&+4\sum_{a=1}^{3} g^2_{a,2}\mathcal{J}^\Lambda_{1,0}[f^a_2 f^a_2](\sigma^2_{a,2} + \phi^2_{a,2}) + \\
&+2\sum_{a=4}^{7} g^2_{a,2}(\mathcal{J}^\Lambda_{1,0}[f^a_2 f^a_2] + \mathcal{J}^\Lambda_{0,1}[f^a_2 f^a_2])(\sigma^2_{a,2} + \phi^2_{a,2}) + \\
&+4g^2_{8,2}\mathcal{J}^\Lambda_{1,0}[f^8_2 f^8_2](\sigma^2_{8,2} + \phi^2_{8,2}) + \\
&+4g^2_{9,2}\mathcal{J}^\Lambda_{0,1}[f^9_2 f^9_2](\sigma^2_{9,2} + \phi^2_{9,2}) - \\
&-2\sum_{i,j=1}^{2}\Bigg[m^2_{\rm u}\sum_{a=1}^{3}\sigma_{a,i}\Gamma^a_{{\rm S},ij}\sigma_{a,j} + \\
&+\left(\frac{m_{\rm u}+m_{\rm s}}{2}\right)^2\sum_{a=4}^{7}\sigma_{a,i}\Gamma^a_{{\rm S},ij}\sigma_{a,j} + \\
&+m^2_{\rm u}\sigma_{8,i}\Gamma^8_{{\rm S},ij}\sigma_{8,j} + m^2_{\rm s}\sigma_{9,i}\Gamma^9_{{\rm S},ij}\sigma_{9,j}\Bigg].
\end{aligned} \tag{31}$$

The total expressions for $L^{(3)}_{\rm loop}$ and $L^{(4)}_{\rm loop}$ are too lengthy, therefore, we do not show them here. Instead we will extract parts from them when they are needed (see *e.g.* Appendix A).

[1)] Here we keep only the terms of an order of fields not higher than 4 (corresponding diagrams are shown in Fig. 1).

[2)] Here, the expressions (32) and (33) for Yukawa coupling constants were used.

The Yukawa coupling constants $g_{a,i}$ describing the interaction of quarks and mesons appear as a result of renormalization of meson fields (see [5–7,12,13,21] for details):

$$
\begin{aligned}
g_{a,1}^2 &= [4\mathcal{J}_{2,0}^{\Lambda}[1]]^{-1}, \quad (a=1,2,3,8),\\
g_{a,1}^2 &= [4\mathcal{J}_{1,1}^{\Lambda}[1]]^{-1}, \quad (a=4,5,6,7),\\
g_{9,1}^2 &= [4\mathcal{J}_{0,2}^{\Lambda}[1]]^{-1}.
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
g_{a,2}^2 &= [4\mathcal{J}_{2,0}^{\Lambda}[f_2^{\mathrm{u}} f_2^{\mathrm{u}}]]^{-1}, \quad (a=1,2,3,8),\\
g_{a,2}^2 &= [4\mathcal{J}_{1,1}^{\Lambda}[f_2^{\mathrm{us}} f_2^{\mathrm{us}}]]^{-1}, \quad (a=4,5,6,7),\\
g_{9,2}^2 &= [4\mathcal{J}_{0,2}^{\Lambda}[f_2^{\mathrm{s}} f_2^{\mathrm{s}}]]^{-1}.
\end{aligned} \tag{33}
$$

For the pseudoscalar meson fields, π-A_1-transitions lead to the factor Z, describing an additional renormalization of pseudoscalar meson fields, with M_{A_1} being the mass of the axial-vector meson (see [13,21]):

$$
Z = \left(1 - \frac{6m_{\mathrm{u}}}{M_{A_1}^2}\right)^{-1} \approx 1.46. \tag{34}
$$

For the radially excited pseudoscalar states a similar renormalization also takes place, but in this case the renormalization factor turns out to be approximately equal to unit, so it is omitted in our calculations (see [13]).

3. INTRODUCING THE DILATON

According to the prescription described in [9,10], we introduce the dilaton field into Lagrangian (23) as follows: the dimensional model parameters G, Λ, m_a, and K are replaced by the following rule: $G \to G(\chi_c/\chi)^2$, $\Lambda \to \Lambda(\chi/\chi_c)$, $m_a \to m_a(\chi/\chi_c)$, $K \to K(\chi_c/\chi)^5$, where χ is the dilaton field with VEV χ_c. We also define the field χ' as the difference $\chi' = \chi - \chi_c$ that has zero VEV. Below the effective meson Lagrangian is expanded in terms of χ' when calculating the mass terms and vertices describing the interaction of mesons.

The current quark masses break scale invariance and, therefore, should not be multiplied by the dilaton field. The modified current quark masses $\overline{m}_a^0$ are also not multiplied by the dilaton field. Finally, we come to the Lagrangian:

$$
\begin{aligned}
\bar{\mathcal{L}}(\sigma,\phi,\chi) &= L_{\mathrm{kin}}(\sigma,\phi) + \bar{L}_{\mathrm{G}}(\sigma,\phi,\chi) + \bar{L}_{\mathrm{loop}}(\sigma,\phi,\chi) +\\
&\quad + \mathcal{L}(\chi) + \Delta L_{\mathrm{an}}(\sigma,\phi,\chi).
\end{aligned} \tag{35}
$$

The term $L_{\mathrm{kin}}(\sigma,\phi)$ remains unchanged, as it is already scale-invariant.

Here, the term $\bar{L}_{\rm G}(\sigma, \phi, \chi)$ is

$$\bar{L}_{\rm G}(\sigma, \phi, \chi) =$$

$$= -\frac{1}{2}\left(\frac{\chi}{\chi_c}\right)^2 \sum_{a,b=1}^{9} \left(g_{a,1}\sigma_{a,1} - \mu_a \frac{\chi}{\chi_c} + \bar{\mu}_a^0\right) \left(G^{(-)}\right)^{-1}_{ab} \times$$

$$\times \left(g_{b,1}\sigma_{b,1} - \mu_b \frac{\chi}{\chi_c} + \bar{\mu}_b^0\right) -$$

$$-\frac{Z}{2}\left(\frac{\chi}{\chi_c}\right)^2 \sum_{a,b=1}^{9} g_{a,1}\phi_{a,1} \left(G^{(+)}\right)^{-1}_{ab} g_{b,1}\phi_{b,1} -$$

$$-\left(\frac{\chi}{\chi_c}\right)^2 \sum_{a=1}^{9} \frac{g_{a,2}^2}{2\bar{G}_a} (\sigma_{a,2}^2 + \phi_{a,2}^2) \tag{36}$$

Expanding (36) in a power series of χ, we can extract a term that is of order χ^4. It can be absorbed by the term in the pure dilaton potential (see (39) below) which has the same degree of χ. This does not bring essential changes, because such terms are scale-invariant and therefore do not contribute to the divergence of the dilatation current (see (59) below). This would lead only to a redefinition of the constants χ_0 and B of the potential (39).

The term $\bar{L}_{\rm loop}(\sigma, \phi, \chi)$ after introducing dilaton fields takes the form:

$$\bar{L}_{\rm loop}(\sigma, \phi, \chi) = L^{(1)}_{\rm loop}(\sigma)\left(\frac{\chi}{\chi_c}\right)^3 + L^{(2)}_{\rm loop}(\sigma, \phi)\left(\frac{\chi}{\chi_c}\right)^2 +$$

$$+L^{(3)}_{\rm loop}(\sigma, \phi)\frac{\chi}{\chi_c} + L^{(4)}_{\rm loop}(\sigma, \phi). \tag{37}$$

Here $\mathcal{L}(\chi)$ is the pure dilaton Lagrangian

$$\mathcal{L}(\chi) = \frac{P^2}{2}\chi^2 - V(\chi) \tag{38}$$

with the potential

$$V(\chi) = B\left(\frac{\chi}{\chi_0}\right)^4 \left[\ln\left(\frac{\chi}{\chi_0}\right)^4 - 1\right] \tag{39}$$

that has a minimum at $\chi = \chi_0$, and the parameter B represents the vacuum energy when there are no quarks. The kinetic term is given in the momentum space, P being the momentum of the dilaton.

Note that Lagrangian (23) implicitly contains the term $L_{\rm an}$ that is induced by gluon anomalies:

$$L_{\rm an}(\bar{\sigma}, \phi) = -h_\phi \phi_0^2 + h_\sigma \bar{\sigma}_0^2, \tag{40}$$

where ϕ_0 and $\bar{\sigma}_0$ $(\langle\sigma_0\rangle_0 \neq 0)$ are pseudoscalar and scalar meson isosinglets, respectively; and h_ϕ, h_σ are constants; $\phi_0 = \sqrt{2/3}\phi_{8,1} - \sqrt{1/3}\phi_{9,1}$, $\bar{\sigma}_0 = \sqrt{2/3}\bar{\sigma}_{8,1} -$

$\sqrt{1/3}\bar{\sigma}_{9,1}$, where $\phi_{8,1}$ and $\bar{\sigma}_{8,1}$ ($\langle\bar{\sigma}_{8,1}\rangle_0 \neq 0$) consist of u-quarks; and $\phi_{9,1}$, $\bar{\sigma}_{9,1}$ ($\langle\bar{\sigma}_{9,1}\rangle_0 \neq 0$), of s-quarks. In our model, the 't Hooft interaction is responsible for the appearance of these terms.

The term $L_{\rm an}$ breaks scale invariance. However, when the procedure of the scale invariance restoration is applied to Lagrangian (23), the term $L_{\rm an}$ also becomes scale-invariant. To avoid this, one should subtract this part in the scale-invariant form and add it in a scale-breaking (SB) form. This is achieved by including the term $\Delta L_{\rm an}$:

$$\Delta L_{\rm an}(\sigma,\phi,\chi) = -L_{\rm an}(\bar{\sigma},\phi)\left(\frac{\chi}{\chi_c}\right)^2 + L_{\rm an}^{\rm SB}(\sigma,\phi,\chi). \quad (41)$$

Let us define the scale-breaking term $L_{\rm an}^{\rm SB}$. The coefficients h_σ and h_ϕ in (40) can be determined by comparing them with the terms in (24) that describe the singlet-octet mixing[3]. We obtain

$$h_\phi = -\frac{3}{2\sqrt{2}} g_{8,1} g_{9,1} Z \left(G^{(+)}\right)^{-1}_{89}, \quad (42)$$

$$h_\sigma = \frac{3}{2\sqrt{2}} g_{8,1} g_{9,1} \left(G^{(-)}\right)^{-1}_{89}. \quad (43)$$

If these terms were to be made scale-invariant, one should insert $(\chi/\chi_c)^2$ into them (see last term in (41)). However, as the gluon anomalies break scale invariance, we introduce the dilaton field into these terms in a more complicated way. The inverse matrix elements $\left(G^{(+)}\right)^{-1}_{ab}$ and $\left(G^{(-)}\right)^{-1}_{ab}$,

$$\left(G^{(+)}\right)^{-1}_{89} = \frac{-4\sqrt{2} m_{\rm u} K \mathcal{J}^{\Lambda}_{1,0}[1]}{G^{(+)}_{88} G^{(+)}_{99} - \left(G^{(+)}_{89}\right)^2}, \quad (44)$$

$$\left(G^{(-)}\right)^{-1}_{89} = \frac{4\sqrt{2} m_{\rm u} K \mathcal{J}^{\Lambda}_{1,0}[1]}{G^{(-)}_{88} G^{(-)}_{99} - \left(G^{(-)}_{89}\right)^2}, \quad (45)$$

are determined by two different interactions. The numerators are fully defined by the 't Hooft interaction that leads to anomalous terms (40) breaking scale invariance, therefore, we do not introduce here dilaton fields. The denominators are determined by the constant G describing the NJL four-quark interaction, and the dilaton field is inserted into it, according to the prescription given above. Finally, we come to the following form of $L_{\rm an}^{\rm SB}$:

$$L_{\rm an}^{\rm SB}(\sigma,\phi,\chi) = \left[-h_\phi \phi_0^2 + h_\sigma\left(\sigma_0 - F_0\frac{\chi}{\chi_c} + F_0^0\right)^2\right]\left(\frac{\chi}{\chi_c}\right)^4, \quad (46)$$

$$F_0 = \frac{\sqrt{2} m_{\rm u}}{\sqrt{3} g_{8,1}} + \frac{m_{\rm s}}{\sqrt{6} g_{9,1}}, \quad F_0^0 = \frac{\sqrt{2}\overline{m}_{\rm u}^0}{\sqrt{3} g_{8,1}} + \frac{\overline{m}_{\rm s}^0}{\sqrt{6} g_{9,1}}. \quad (47)$$

[3] The singlet-octet mixing is fully determined by the 't Hooft interaction.

From it, we immediately obtain the term $\Delta L_{\rm an}$:

$$\Delta L_{\rm an}(\sigma,\phi,\chi) = \left[h_\phi\phi_0^2 - h_\sigma\left(\sigma_0 - F_0\frac{\chi}{\chi_c} + F_0^0\right)^2\right]\left(\frac{\chi}{\chi_c}\right)^2\left[1-\left(\frac{\chi}{\chi_c}\right)^2\right] \quad (48)$$

4. EQUATIONS

One of the principal requirements for an effective meson Lagrangian with a glueball is that the terms linear in σ and χ' should be absent in the effective Lagrangian. This leads to the equations:

$$\left.\frac{\delta\bar{\mathcal{L}}}{\delta\sigma_{8,1}}\right|_{\substack{\phi=0\\ \sigma=0\\ \chi=\chi_c}} = \left.\frac{\delta\bar{\mathcal{L}}}{\delta\sigma_{9,1}}\right|_{\substack{\phi=0\\ \sigma=0\\ \chi=\chi_c}} = \left.\frac{\delta\bar{\mathcal{L}}}{\delta\chi}\right|_{\substack{\phi=0\\ \sigma=0\\ \chi=\chi_c}} = $$
$$= \left.\frac{\delta\bar{\mathcal{L}}}{\delta\sigma_{8,2}}\right|_{\substack{\phi=0\\ \sigma=0\\ \chi=\chi_c}} = \left.\frac{\delta\bar{\mathcal{L}}}{\delta\sigma_{9,2}}\right|_{\substack{\phi=0\\ \sigma=0\\ \chi=\chi_c}} = 0. \quad (49)$$

Gap equations follow from them. For the ground states of quarkonia ($\sigma_{a,1}$) and the dilaton field χ', we obtain:

$$(m_{\rm u} - \bar{m}_{\rm u}^0)\left(G^{(-)}\right)^{-1}_{88} - \frac{m_{\rm u} - \bar{m}_{\rm u}^0}{\sqrt{2}}\left(G^{(-)}\right)^{-1}_{89} - 8m_{\rm u}\mathcal{J}_{1,0}^\Lambda[1] = 0, \quad (50)$$

$$(m_{\rm s} - \bar{m}_{\rm s}^0)\left(G^{(-)}\right)^{-1}_{99} - \sqrt{2}(m_{\rm s} - \bar{m}_{\rm s}^0)\left(G^{(-)}\right)^{-1}_{98} - 8m_{\rm s}\mathcal{J}_{0,1}^\Lambda[1] = 0, \quad (51)$$

$$4B\left(\frac{\chi_c}{\chi_0}\right)^3\frac{1}{\chi_0}\ln\left(\frac{\chi_c}{\chi_0}\right)^4 + \frac{1}{\chi_c}\sum_{a,b=8}^{9}\bar{\mu}_a^0\left(G^{(-)}\right)^{-1}_{ab}(\bar{\mu}_b^0 - 3\mu_b)$$
$$-\frac{2h_\sigma}{\chi_c}\left(F_0 - F_0^0\right)^2 = 0. \quad (52)$$

Using (17) and (18), one can rewrite equations (50) and (51) in the well-known form [20] (see Fig. 2):

$$m_{\rm u}^0 = m_{\rm u} - 8Gm_{\rm u}\mathcal{J}_{1,0}^\Lambda[1] - 32Km_{\rm u}m_{\rm s}\mathcal{J}_{1,0}^\Lambda[1]\mathcal{J}_{0,1}^\Lambda[1], \quad (53)$$
$$m_{\rm s}^0 = m_{\rm s} - 8Gm_{\rm s}\mathcal{J}_{0,1}^\Lambda[1] - 32K(m_{\rm u}\mathcal{J}_{1,0}^\Lambda[1])^2. \quad (54)$$

For the excited states ($\sigma_{a,2}$), we require that the corresponding gap equations have the trivial solution, $\langle\sigma_{a,2}\rangle_0 = 0$. This is one of the possible particular solutions of equations (49). An advantage of such a solution is that in this case the quark condensates and constituent quark masses remain unchanged after introducing radially excited states. This solution surely exists if the tadpole diagram (Fig. 1(a)) for the excited scalar is equal to zero (see [7,12]). This leads to the condition:

$$\mathcal{J}_{1,0}^\Lambda[f_2^{\rm u}] = \mathcal{J}_{0,1}^\Lambda[f_2^{\rm s}] = 0. \quad (55)$$

In addition to these conditions, we impose the third one:

$$\mathcal{J}^{\Lambda}_{1,0}[1+d_{\mathrm{us}}\mathbf{k}^2]+\mathcal{J}^{\Lambda}_{0,1}[1+d_{\mathrm{us}}\mathbf{k}^2]=0, \tag{56}$$

which fixes the constant d_{us}. The calculation of the second variation of the effective potential will ensure us that the solution that we have chosen give the minimum of the potential.

In addition to the gap equations, an Ward identity for the divergence of dilation current, coming from QCD should be taken into account. The identity reads:

$$\langle \partial_\mu S^\mu \rangle = \mathcal{C}_{\mathrm{g}} - \sum_{q=\mathrm{u,d,s}} m_q^0 \langle \bar{q}q \rangle_0 , \tag{57}$$

where

$$\mathcal{C}_{\mathrm{g}} = \left(\frac{11N_c}{24}-\frac{N_{\mathrm{f}}}{12}\right)\left\langle \frac{\alpha}{\pi}\left(G^a_{\mu\nu}\right)^2\right\rangle_0 , \tag{58}$$

and N_{f} is the number of flavours, $\left\langle \frac{\alpha}{\pi}(G^a_{\mu\nu})^2\right\rangle_0$ and $\langle \bar{q}q\rangle_0$ are the gluon and quark condensates with α being the QCD constant of strong interaction.

Let us now consider VEV of the divergence of the dilatation current S^μ [9,15] calculated from the potential of Lagrangian (35):

$$\langle \partial_\mu S^\mu \rangle = \left[\sum_{i=1}^{2}\sum_{a=1}^{9}\left(\sigma_{a,n}\frac{\partial V}{\partial \sigma_{a,n}}+\phi_{a,n}\frac{\partial V}{\partial \phi_{a,n}}\right)+\chi\frac{\partial V}{\partial \chi}-4V\right]\Bigg|_{\substack{\chi=\chi_c\\ \sigma=0\\ \phi=0}} =$$

$$= 4B\left(\frac{\chi_c}{\chi_0}\right)^4 - 2h_\sigma\left(F_0-F_0^0\right)^2 - \sum_{q=\mathrm{u,d,s}} \overline{m}_q^0 \langle qq\rangle_0 . \tag{59}$$

Here $V=V(\chi)+\bar{V}(\sigma,\phi,\chi)$, and $\bar{V}(\sigma,\phi,\chi)$ is the potential part of Lagrangian $\bar{\mathcal{L}}(\sigma,\phi,\chi)$ (see (35)) that does not contain the pure dilaton potential (39). In the expression given in (59), the following relation of the quark condensates to integrals $\mathcal{J}^{\Lambda}_{1,0}[1]$ and $\mathcal{J}^{\Lambda}_{0,1}[1]$ was used:

$$4m_{\mathrm{u}}\mathcal{J}^{\Lambda}_{1,0}[1] = -\langle \bar{u}u\rangle_0 = -\left\langle \bar{d}d\right\rangle_0 , \quad 4m_{\mathrm{s}}\mathcal{J}^{\Lambda}_{0,1}[1] = -\langle \bar{s}s\rangle_0 , \tag{60}$$

and that these integrals are connected with constants $G^{(-)}_{ab}$ through gap equations, as it will be shown below (see (50) and (51)). Comparing (59) with the QCD expression (57), one can see that the term $\sum m_q^0\langle\bar{q}q\rangle$ on the right-hand side of (57) is canceled by the corresponding contribution from current quark masses on the right-hand side of (59). Equating the right-hand sides of (59) and (57),

$$\mathcal{C}_{\mathrm{g}} - \sum_{q=\mathrm{u,d,s}} m_q^0 \langle \bar{q}q\rangle_0 =$$

$$= 4B\left(\frac{\chi_c}{\chi_0}\right)^4 - 2h_\sigma\left(F_0-F_0^0\right)^2 - \sum_{q=\mathrm{u,d,s}} \overline{m}_q^0 \langle \bar{q}q\rangle_0 , \tag{61}$$

we obtain the correspondence

$$\mathcal{C}_{\mathrm{g}} = 4B\left(\frac{\chi_c}{\chi_0}\right)^4 + \sum_{a,b=8}^{9}(\bar{\mu}_a^0 - \mu_a^0)\left(G^{(-)}\right)^{-1}_{ab}(\mu_b - \bar{\mu}_b^0) -$$
$$-2h_\sigma\left(F_0 - F_0^0\right)^2, \tag{62}$$

where $\mu_a^0 = 0 \quad (a = 1, \dots 7)$, $\mu_8^0 = m_{\mathrm{u}}^0$, and $\mu_9^0 = -m_{\mathrm{s}}^0/\sqrt{2}$. This equation relates the gluon condensate, whose value is taken from other sources (see, *e.g.*, [22]), to the model parameter B. The next step is to investigate gap equations.

To determine the masses of quarkonia and of the glueball, let us consider the part of Lagrangian (35) which is quadratic in fields σ and χ' together with kinetic terms and which is denoted by $L^{(2)}$:

$$L^{(2)}(\sigma,\phi,\chi') = \frac{1}{2}\sum_{n,j=1}^{2}\left[\sum_{a=1}^{3}(P^2 - 4m_{\mathrm{u}}^2)\sigma_{a,n}\Gamma^a_{\mathrm{S},nj}\sigma_{a,j} + \right.$$
$$+\sum_{a=4}^{7}(P^2 - (m_{\mathrm{u}} + m_{\mathrm{s}})^2)\sigma_{a,n}\Gamma^a_{\mathrm{S},nj}\sigma_{a,j} +$$
$$\left. +(P^2 - 4m_{\mathrm{u}}^2)\sigma_{8,n}\Gamma^8_{\mathrm{S},nj}\sigma_{8,j} + (P^2 - 4m_{\mathrm{s}}^2)\sigma_{9,n}\Gamma^9_{\mathrm{S},nj}\sigma_{9,j}\right] -$$
$$-\frac{1}{2}g_{8,1}^2\left[\left(G^{(-)}\right)^{-1}_{88} - 8\mathcal{J}^{\Lambda}_{1,0}[1]\right]\sigma_{8,1}^2 -$$
$$-\frac{1}{2}g_{9,1}^2\left[\left(G^{(-)}\right)^{-1}_{99} - 8\mathcal{J}^{\Lambda}_{0,1}[1]\right]\sigma_{9,1}^2 -$$
$$-\frac{1}{2}g_{8,2}^2\left[1/\bar{G}_8 - 8\mathcal{J}^{\Lambda}_{1,0}[f_2^{\mathrm{u}}f_2^{\mathrm{u}}]\right]\sigma_{8,2}^2 -$$
$$-\frac{1}{2}g_{9,2}^2\left[1/\bar{G}_9 - 8\mathcal{J}^{\Lambda}_{0,1}[f_2^{\mathrm{s}}f_2^{\mathrm{s}}]\right]\sigma_{9,2}^2 -$$
$$-g_{8,1}g_{9,1}\left(G^{(-)}\right)^{-1}_{89}\sigma_{8,1}\sigma_{9,1} - \frac{M_{\mathrm{g}}^2\chi'^2}{2} +$$
$$+\sum_{a,b=8}^{9}\frac{\bar{\mu}_a^0}{\chi_c}(G^{(-)})^{-1}_{ab}g_{b,1}\sigma_{b,1}\chi' +$$
$$+\frac{4h_\sigma(F_0 - F_0^0)}{\chi_c\sqrt{3}}\left(\sigma_{9,1} - \sigma_{8,1}\sqrt{2}\right)\chi', \tag{63}$$

where

$$M_{\mathrm{g}}^2 = \frac{1}{\chi_c^2}\left(4\mathcal{C}_{\mathrm{g}} + \sum_{a,b=8}^{9}\bar{\mu}_a^0\left(G^{(-)}\right)^{-1}_{ab}(2\bar{\mu}_b^0 - \mu_b) + \right.$$
$$\left. + \sum_{a,b=8}^{9}4\mu_a^0\left(G^{(-)}\right)^{-1}_{ab}(\mu_b - \bar{\mu}_b^0) - 4h_\sigma F_0^2 + 4h_\sigma(F_0^0)^2\right) \tag{64}$$

is the glueball mass before taking account of mixing effects. Here, the gap equations and equation (62) are taken into account.

From this Lagrangian, after diagonalization, we obtain the masses of five scalar isoscalar meson states: $\sigma_{\rm I}$, $\sigma_{\rm II}$, $\sigma_{\rm III}$, $\sigma_{\rm IV}$, and $\sigma_{\rm V}$ and a matrix of mixing coefficients b that connects the nondiagonalized fields $\sigma_{8,1}, \sigma_{9,1}, \sigma_{8,2}, \sigma_{9,2}, \chi'$ with the physical ones $\sigma_{\rm I}, \sigma_{\rm II}, \sigma_{\rm III}, \sigma_{\rm IV}, \sigma_{\rm V}$:

$$\begin{pmatrix} \sigma_{8,1} \\ \sigma_{9,1} \\ \sigma_{8,2} \\ \sigma_{9,2} \\ \chi' \end{pmatrix} = \begin{pmatrix} b_{\sigma_{8,1}\sigma_{\rm I}} & b_{\sigma_{8,1}\sigma_{\rm II}} & b_{\sigma_{8,1}\sigma_{\rm III}} & b_{\sigma_{8,1}\sigma_{\rm IV}} & b_{\sigma_{8,1}\sigma_{\rm V}} \\ b_{\sigma_{9,1}\sigma_{\rm I}} & b_{\sigma_{9,1}\sigma_{\rm II}} & b_{\sigma_{9,1}\sigma_{\rm III}} & b_{\sigma_{9,1}\sigma_{\rm IV}} & b_{\sigma_{9,1}\sigma_{\rm V}} \\ b_{\sigma_{8,2}\sigma_{\rm I}} & b_{\sigma_{8,2}\sigma_{\rm II}} & b_{\sigma_{8,2}\sigma_{\rm III}} & b_{\sigma_{8,2}\sigma_{\rm IV}} & b_{\sigma_{8,2}\sigma_{\rm V}} \\ b_{\sigma_{9,2}\sigma_{\rm I}} & b_{\sigma_{9,2}\sigma_{\rm II}} & b_{\sigma_{9,2}\sigma_{\rm III}} & b_{\sigma_{9,2}\sigma_{\rm IV}} & b_{\sigma_{9,2}\sigma_{\rm V}} \\ b_{\chi'\sigma_{\rm I}} & b_{\chi'\sigma_{\rm II}} & b_{\chi'\sigma_{\rm III}} & b_{\chi'\sigma_{\rm IV}} & b_{\chi'\sigma_{\rm V}} \end{pmatrix} \begin{pmatrix} \sigma_{\rm I} \\ \sigma_{\rm II} \\ \sigma_{\rm III} \\ \sigma_{\rm IV} \\ \sigma_{\rm V} \end{pmatrix}. \qquad (65)$$

The values of matrix elements are given in Table 1.

For the isovector states we introduce physical states: the ground a_0 and radially excited $\hat{a}_0$. The corresponding mixing coefficients are represented as elements of a matrix connecting the physical fields a_0, $\hat{a}_0$ with the fields a_{01}, a_{02} before mixing:

$$\begin{pmatrix} a_{01} \\ a_{02} \end{pmatrix} = \begin{pmatrix} b_{a_{01}a_0} & b_{a_{01}\hat{a}_0} \\ b_{a_{02}a_0} & b_{a_{02}\hat{a}_0} \end{pmatrix} \begin{pmatrix} a_0 \\ \hat{a}_0 \end{pmatrix}. \qquad (66)$$

Their values are $b_{a_{01}a_0} = 0.918$, $b_{a_{02}a_0} = 0.138$, $b_{a_{01}\hat{a}_0} = 0.761$, $b_{a_{02}\hat{a}_0} = -1.18$. The mixing of strange scalar meson states is described as follows:

$$\begin{pmatrix} K^*_{01} \\ K^*_{02} \end{pmatrix} = \begin{pmatrix} b_{K^*_{01}K^*_0} & b_{K^*_{01}\hat{K}^*_0} \\ b_{K^*_{02}K^*_0} & b_{K^*_{02}\hat{K}^*_0} \end{pmatrix} \begin{pmatrix} K^*_0 \\ \hat{K}^*_0 \end{pmatrix}. \qquad (67)$$

The values of matrix elements are: $b_{K^*_{01}K^*_0} = 0.866$, $b_{K^*_{02}K^*_0} = 0.232$, $b_{K^*_{01}\hat{K}^*_0} = 0.750$, $b_{K^*_{02}\hat{K}^*_0} = -1.12$. Here, the physical states are K^*_0 and its radial excitation $\hat{K}^*_0$. The states K^*_{01} and K^*_{02} correspond to the nondiagonalized Lagrangian.

After the diagonalization, we obtain the kinetic and mass terms of the effective Lagrangian in a diagonal form. Expressions for the quadratic terms in the case of isovector and strange mesons are given in [5–7].

5. MODEL PARAMETERS AND NUMERICAL ESTIMATES

The basic parameters of our model are G, Λ, $m_{\rm u}$, and $m_{\rm s}$. They are fixed by the pion weak decay constant $F_\pi = 93$ MeV, the ρ meson decay constant $g_\rho \approx 6.14$, and the masses of pion and kaon [21,23,24]. To fix Λ and $m_{\rm u}$, the Goldberger-Treiman relation $g_{\rm u} F_\pi \sqrt{Z} = m_{\rm u}$ and the equation $g_\rho = \sqrt{6} g_{\rm u}$ are used. The parameter G is determined by the pion mass; and $m_{\rm s}$, by the kaon mass. Their values do not change both after the radially excited states [5–7, 12, 13] and the dilaton fields are introduced [9, 10]:

$$\begin{aligned} &m_{\rm u} = 280 \text{ MeV}, \ m_{\rm s} = 417 \text{ MeV}, \ \Lambda = 1.03 \text{ GeV}, \\ &G = 3.202 \text{ GeV}^{-2}. \end{aligned} \qquad (68)$$

To have a correct description of η and η', one should fix the 't Hooft interaction constant by the masses of η and η'. The lower bound for the lightest scalar isoscalar meson mass is also taken into account here. As a result, for the model masses we obtain: $M_\eta \approx 500$ MeV, $M_{\eta'} \approx 870$ MeV, and for K:

$$K = 4.4 \text{ GeV}^{-5}. \tag{69}$$

After introducing the radially excited states, there appear new parameters: the slope parameters d_a and the arbitrary parameters c_a. The constants d_a are not arbitrary and are fixed by conditions (55) and (56):

$$d_{\rm u} = -1.77 \text{ GeV}^{-2}, \quad d_{\rm s} = -1.72 \text{ GeV}^{-2}, d_{\rm us} = -1.75 \text{ GeV}^{-2}. \tag{70}$$

The parameters c_a are absorbed by the four quark interaction constants $\bar{G}_a$ and influence only masses of mesons. They are fixed from experiment by masses of excited pseudoscalar meson states. As a result, we obtain:

$$\begin{aligned} \bar{G}_1 = 4.45 \text{ GeV}^{-2}, \bar{G}_4 = 5.12 \text{ GeV}^{-2}, \\ \bar{G}_8 = 4.64 \text{ GeV}^{-2}, \bar{G}_9 = 5.09 \text{ GeV}^{-2}. \end{aligned} \tag{71}$$

Due to the chiral symmetry of Lagrangian (3), the same values of the form factor parameters are used both for the scalar and pseudoscalar mesons, which allows us to predict masses of excited scalar states[4].

After the dilaton is introduced, new three parameters χ_0, χ_c, and B appear. To fix the new parameters, one should use equations (62), (52), and the physical glueball mass. As a result, we obtain for χ_0 and B:

$$\begin{aligned} \chi_0 = \chi_c \exp\Bigg\{ -\Bigg[\sum_{a,b=8}^{9} \bar{\mu}_a^0 \left(G^{(-)}\right)^{-1}_{ab} (3\mu_b - \bar{\mu}_b^0) + +2h_\sigma \left(F_0 - F_0^0\right)^2 \Bigg] \Big/ \\ \Big/ 4\Bigg[\mathcal{C}_{\rm g} - \sum_{a,b=8}^{9} (\bar{\mu}_a^0 - \mu_a^0) \left(G^{(-)}\right)^{-1}_{ab} (\mu_b - \bar{\mu}_b^0) + 2h_\sigma \left(F_0 - F_0^0\right)^2 \Bigg] \Bigg\}, \end{aligned} \tag{72}$$

$$B = \frac{1}{4} \left(\mathcal{C}_{\rm g} - \sum_{a,b=8}^{9} (\bar{\mu}_a^0 - \mu_a^0) \left(G^{(-)}\right)^{-1}_{ab} (\mu_b - \bar{\mu}_b^0) + 2h_\sigma \left(F_0 - F_0^0\right)^2 \right) \left(\frac{\chi_0}{\chi_c}\right)^4. \tag{73}$$

We adjust the parameter χ_c, so that the mass of the scalar meson state $\sigma_{\rm IV}$ would be close to 1500 MeV ($\chi_c = 219$ MeV)[5]. For the constants χ_0 and B we have: $\chi_0 = 203$ MeV, $B = 0.007$ GeV^4. We found that, if the state $f_0(1710)$ is supposed to be the glueball, the result turns out to be in worse agreement with

[4] The excited meson K' is an exception. Insofar as the experimental status of the excited K' meson is unclear, we use the experimental value of the mass of $K_0^*(1430)$ to determine $\bar{G}_4$ and predict the mass of K'.

[5] To reach more close agreement with experimental data in the description of strong decays of $\sigma_{\rm IV}$, we chose the model value of $M_{\sigma_{\rm IV}} = 1550$ MeV (mass + half-width)

experiment (see Conclusion). The masses of scalar mesons calculated in our model together with their experimental values are given in Table 2.

6. STRONG DECAYS OF SCALAR ISOSCALAR MESONS

Once all parameters are fixed, we can estimate the decay widths for the main modes of strong decays of scalar mesons: $\sigma_l \to \pi\pi, KK$, $\eta\eta$, $\eta\eta'$, and 4π $(2\sigma, \sigma 2\pi \to 4\pi)$, where $l = \mathrm{I, II, III, IV}$, and V; decays of excited isovectors: $\bar{a}_0 \to \eta\pi$, $\bar{a}_0 \to \eta\pi$, $\bar{a}_0 \to KK$; and of strange mesons: $K\pi$.

Note that, in the energy region under consideration (up to 1.7 GeV), we work on the brim of the validity of exploiting the chiral symmetry and scale invariance that were used to construct our effective Lagrangian. Thus, our results should be considered rather as qualitative.

Let us start with the decay of a scalar isoscalar meson into a pair of pions. The corresponding amplitude has the form:

$$A_{\sigma_l \to \pi\pi} = A^{(1)}_{\sigma_l \to \pi\pi} + A^{(2)}_{\sigma_l \to \pi\pi}, \tag{74}$$

where the first part comes from contact terms of Lagrangian (35) that describe the decay of the glueball into pions. These terms come from $\bar{L}_G(\sigma, \phi, \chi)$ and $(\chi/\chi_c)^2 L^{(2)}_{\mathrm{loop}}(\sigma, \phi)$ (see (36) and (37)). They turn into the pion mass term if $\chi = \chi_c$. Expanding around $\chi = \chi_c$ in terms of χ' and choosing the term linear in χ', we obtain, after the mixing effects are taken into account, the following:

$$A^{(1)}_{\sigma_l \to \pi\pi} = -\frac{M_\pi^2}{\chi_c} b_{\chi' \sigma_l}, \tag{75}$$

where M_π is the pion mass, and $b_{\chi'\sigma_l}$ is a mixing coefficient (see (65) and Table 1). The second contribution $A^{(2)}_{\sigma_l \to \pi\pi}$ describes the decay of the quarkonium part of σ_l and is determined by triangle quark loop diagrams (see Figs. 1(c) and 3). For details of their calculation see Appendix A. As a result, we obtain the following widths for decays of scalar isoscalar mesons into two pions:

$$\begin{aligned}
\Gamma_{\sigma_{\mathrm{I}} \to \pi\pi} &\approx 600 \text{ MeV}, \\
\Gamma_{\sigma_{\mathrm{II}} \to \pi\pi} &\approx 36 \text{ MeV} (20 \text{ MeV}), \\
\Gamma_{\sigma_{\mathrm{III}} \to \pi\pi} &\approx 680 \text{ MeV} (480 \text{ MeV}), \\
\Gamma_{\sigma_{\mathrm{IV}} \to \pi\pi} &\approx 100 \text{ MeV}, \\
\Gamma_{\sigma_{\mathrm{V}} \to \pi\pi} &\approx 0.3 \text{ MeV}.
\end{aligned} \tag{76}$$

To calculate decay widths, we used the model masses of scalar mesons. For the state σ_{II} hereafter we give in brackets the values obtained for its experimental mass. Concerning the state σ_{III}, the values in brackets correspond to calculations performed for the lowest experimental limit for its mass (1200 MeV). Note that in the last two cases the widths are noticeably smaller than those derived for the model masses.

Decays of scalar isoscalar mesons into kaons are described by the amplitude:

$$A_{\sigma_l \to KK} = A^{(1)}_{\sigma_l \to KK} + A^{(2)}_{\sigma_l \to KK}, \tag{77}$$

where $A^{(1)}_{\sigma_l \to KK}$ originates from the same source as $A^{(1)}_{\sigma_l \to \pi\pi}$ and is determined by the kaon mass:

$$A^{(1)}_{\sigma_l \to KK} = -\frac{2M_K^2}{\chi_c} b_{\chi' \sigma_l}, \tag{78}$$

while the other part $A^{(2)}_{\sigma_l \to KK}$ again comes from quark loop diagrams (see Appendix A). The decay widths thereby are[6)]

$$\begin{aligned} \Gamma_{\sigma_{\rm III} \to KK} &\approx 260\ \text{MeV}(125\ \text{MeV}), \\ \Gamma_{\sigma_{\rm IV} \to KK} &\approx 28\ \text{MeV}, \\ \Gamma_{\sigma_{\rm V} \to KK} &\approx 250\ \text{MeV}. \end{aligned} \tag{79}$$

The state $\sigma_{\rm I}$ cannot decay into kaons, as it is below the threshold.

The amplitude describing decays of scalar isoscalar mesons into $\eta\eta$ has a more complicated form, because it contains a contribution from $\Delta L_{\rm an}$. The singlet-octet mixing between pseudoscalar isoscalar states should also be taken into account here. Using the expression for the fields $\phi_{8,1}$ and $\phi_{9,1}$ through the physical ones η and η':

$$\phi_{8,1} = b_{\phi_{8,1}\eta}\eta + b_{\phi_{8,1}\eta'}\eta' + \ldots, \tag{80}$$

$$\phi_{9,1} = b_{\phi_{9,1}\eta}\eta + b_{\phi_{9,1}\eta'}\eta' + \ldots, \tag{81}$$

where ... stand for the excited η and η' that we do not need here and therefore omit them. The mixing coefficients for the scalar pseudoscalar meson states were calculated in [5–7]. In the current calculation their values changed little because the parameter K has changed, thus, $b_{\phi_{8,1}\eta} = 0.777$, $b_{\phi_{8,1}\eta'} = -0.359$, $b_{\phi_{9,1}\eta} = 0.546$, $b_{\phi_{9,1}\eta'} = 0.701$. Thus, we obtain for the amplitude:

$$A_{\sigma_l \to \eta\eta} = A^{(1)}_{\sigma_l \to \eta\eta} + A^{(2)}_{\sigma_l \to \eta\eta} + A^{(3)}_{\sigma_l \to \eta\eta}. \tag{82}$$

Here the contact term $A^{(1)}_{\sigma_l \to \eta\eta}$ has the form:

$$A^{(1)}_{\sigma_l \to \eta\eta} = -\frac{M_\eta^2}{\chi_c} b_{\chi' \sigma_l}. \tag{83}$$

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6) The decay of $\sigma_{\rm II}$ into kaons occurs almost at the threshold, therefore, we cannot give a reliable estimate for this process.

The second term $A^{(2)}_{\sigma_l\to\eta\eta}$ comes from a quark loop calculation (see Appendix A), and the third term $A^{(3)}_{\sigma_l\to\eta\eta}$ originates from $\Delta L_{\rm an}$ (see (48)):

$$A^{(3)}_{\sigma_l\to\eta\eta} = \frac{2h_\phi}{3\chi_c}\left(\sqrt{2}b_{\phi_{8,1}\eta} - b_{\phi_{9,1}\eta}\right)^2. \tag{84}$$

As result, we obtain the following decay widths:

$$\begin{aligned}\Gamma_{\sigma_{\rm III}\to\eta\eta} &\approx 62\ {\rm MeV}(26\ {\rm MeV}),\\ \Gamma_{\sigma_{\rm IV}\to\eta\eta} &\approx 4\ {\rm MeV},\\ \Gamma_{\sigma_{\rm V}\to\eta\eta} &\approx 23\ {\rm MeV}.\end{aligned} \tag{85}$$

The state $\sigma_{\rm V}$ can also decay into $\eta\eta'$. The corresponding amplitude is

$$A_{\sigma_l\to\eta\eta'} = A^{(2)}_{\sigma_l\to\eta\eta'} + A^{(3)}_{\sigma_l\to\eta\eta'}. \tag{86}$$

The contact term $A^{(1)}_{\sigma_l\to\eta\eta'}$ is absent here. The term $A^{(2)}_{\sigma_l\to\eta\eta'}$ comes from quark loop diagrams, as usual, and the last term has the form:

$$A^{(3)}_{\sigma_l\to\eta\eta'} = \frac{4h_\phi}{3\chi_c}\left(\sqrt{2}b_{\phi_{8,1}\eta} - b_{\phi_{9,1}\eta}\right)\left(\sqrt{2}b_{\phi_{8,1}\eta'} - b_{\phi_{9,1}\eta'}\right). \tag{87}$$

The decay width is approximately equal to 100 MeV.

The scalar meson states $\sigma_{\rm III}$, $\sigma_{\rm IV}$, and $\sigma_{\rm V}$ can decay into four pions. This decay can occur via intermediate scalar mesons. Similar calculations for $f_0(1500)$ were done in our previous works [9, 10]. Insofar as our calculations are qualitative, we consider here, instead of the direct processes that involve $\sigma_{\rm I}$-resonances, simpler decays: into $2\sigma_{\rm I}$ and $\sigma_{\rm I}2\pi$ as final states. Our investigation have shown that the result thus obtained can be a good estimate for the decay into 4π.

Let us consider decays into $2\sigma_{\rm I}$. Its amplitude can be divided into two parts:

$$A_{\sigma_l\to\sigma_{\rm I}\sigma_{\rm I}} = A^{(1)}_{\sigma_l\to\sigma_{\rm I}\sigma_{\rm I}} + A^{(2)}_{\sigma_l\to\sigma_{\rm I}\sigma_{\rm I}}. \tag{88}$$

To calculate the first term $A^{(1)}_{\sigma_l\to\sigma_{\rm I}\sigma_{\rm I}}$, one should first take, from the effective meson Lagrangian, the terms that contains only scalar meson fields in the third degree before taking account of mixing effects. The corresponding vertices have the form:

$$a_1\chi'^3 + a_2\chi'^2\sigma_{8,1} + a_3\chi'\sigma_{8,1}^2 + a_4\chi'\sigma_{8,2}^2, \tag{89}$$

where the coefficients a_k are given in Appendix A (see (A.120)–(A.123)). These vertices come from $\bar{L}_{\rm G}$, $\mathcal{L}(\chi)$, and $\Delta L_{\rm an}$ (see eqs. (36), (39)), and (48)). We neglected here the terms with $\sigma_{9,i}$ fields which represent quarkonia made of s-quarks, because we are interested in decays into pions that do not contain s-quarks.

Up to this moment, the contribution $A^{(1)}_{\sigma_l\to\sigma_{\rm I}\sigma_{\rm I}}$ was considered. As to the term $A^{(2)}_{\sigma_l\to\sigma_{\rm I}\sigma_{\rm I}}$ in (88) connected with quark loops, its calculation is given in Appendix. As a result, we obtain the following decay widths:

$$\begin{aligned}\Gamma_{\sigma_{\rm III}\to\sigma_{\rm I}\sigma_{\rm I}} &\approx 40\ {\rm MeV},\\ \Gamma_{\sigma_{\rm IV}\to\sigma_{\rm I}\sigma_{\rm I}} &\approx 200\ {\rm MeV},\\ \Gamma_{\sigma_{\rm V}\to\sigma_{\rm I}\sigma_{\rm I}} &\approx 1\ {\rm MeV}.\end{aligned} \tag{90}$$

Four pions in the final state can be produced also through the process with one $\sigma_{\rm I}$-resonance ($\sigma_l \to \sigma_{\rm I}2\pi \to 4\pi$). To estimate this process, we calculate the decay into $\sigma 2\pi$ as a final state. The amplitude again can be divided into two parts:

$$A_{\sigma_l\to\sigma_{\rm I}2\pi} = A^{(1)}_{\sigma_l\to\sigma_{\rm I}2\pi} + A^{(2)}_{\sigma_l\to\sigma_{\rm I}2\pi}. \tag{91}$$

The first term has the form:

$$\begin{aligned}A^{(1)}_{\sigma_l\to\sigma_{\rm I}2\pi} &= -\frac{M_\pi^2}{\chi_c^2} b_{\chi'\sigma_l} b_{\chi'\sigma_{\rm I}} + \frac{8m_{\rm u}}{\chi_c} b_{\chi'\sigma_l} \mathcal{J}^\Lambda_{2,0}[\bar{f}_{\sigma_{\rm I}}\bar{f}_\pi\bar{f}_\pi]\\ &+\frac{8m_{\rm u}}{\chi_c} b_{\chi'\sigma_{\rm I}} \mathcal{J}^\Lambda_{2,0}[\bar{f}_{\sigma_l}\bar{f}_\pi\bar{f}_\pi],\end{aligned} \tag{92}$$

where $\bar{f}_a$ are "physical" form factor functions defined in Appendix A. The pure quark contribution is calculated as described in Appendix A. The result is

$$A^{(2)}_{\sigma_l\to\sigma_{\rm I}2\pi} = -4\mathcal{J}^\Lambda_{2,0}[\bar{f}_{\sigma_l}\bar{f}_{\sigma_{\rm I}}\bar{f}_\pi\bar{f}_\pi]. \tag{93}$$

The corresponding decay widths are negligibly small

$$\begin{aligned}\Gamma_{\sigma_{\rm III}\to\sigma 2\pi} &\approx 1\ {\rm MeV},\\ \Gamma_{\sigma_{\rm IV}\to\sigma 2\pi} &\approx 2\ {\rm MeV},\\ \Gamma_{\sigma_{\rm V}\to\sigma 2\pi} &\approx 0.6\ {\rm MeV}.\end{aligned} \tag{94}$$

Comparing the obtained results with experimental data (see Table 3), one can see that the decays $\sigma_{\rm I} \to \pi\pi$ and $\sigma_{\rm II} \to \pi\pi$ are in satisfactory agreement with experiment. For the states $\sigma_{\rm III}$, $\sigma_{\rm IV}$, and $\sigma_{\rm V}$, we have reliable values only for their total widths measured experimentally. Our results allow us to obtain just the order of magnitude for the decay widths, exceeding the experimental values by a factor of $2.0 \div 3.0$.

Concerning partial decay modes, the state $f_0(1500)$ decays mostly into 4π and 2π. According to the experimental data analysis given in [25], the ratio $\Gamma_{4\pi}/\Gamma_{2\pi} \approx 1.34$. We obtain $\Gamma_{4\pi}/\Gamma_{2\pi} \approx 2$, which is in qualitative agreement with [25]. The decays into 4π and 2π are suppressed for the state $f_0(1710)$. Its main decay mode is into kaons. This agrees with the analysis of experimental data given in [25] and corroborates our assumption that $f_0(1500)$ is rather a glueball.

The amplitudes describing decays of excited state $\hat{a}_0$ into $\eta\pi$, $\eta'\pi$, and KK are calculated through triangle quark loop diagrams and look as follows:

$$A_{\hat{a}_0\to\eta\pi} = 16m_u(\mathcal{J}_{2,0}^{\Lambda}[\bar{f}_{\hat{a}_0}\bar{f}_{\eta}^{\mathrm{u}}\bar{f}_{\pi}] - P_1\cdot P_2\mathcal{J}_{3,0}^{\Lambda}[\bar{f}_{\hat{a}_0}\bar{f}_{\eta}^{\mathrm{u}}\bar{f}_{\pi}]), \tag{95}$$

$$A_{\hat{a}_0\to\eta'\pi} = 16m_u(\mathcal{J}_{2,0}^{\Lambda}[\bar{f}_{\hat{a}_0}\bar{f}_{\eta'}^{\mathrm{u}}\bar{f}_{\pi}] - P_1\cdot P_2\mathcal{J}_{3,0}^{\Lambda}[\bar{f}_{\hat{a}_0}\bar{f}_{\eta'}^{\mathrm{u}}\bar{f}_{\pi}]), \tag{96}$$

$$A_{\hat{a}_0\to KK} = 8m_u(C_{\mathrm{uu}}\mathcal{J}_{2,0}^{\Lambda}[\bar{f}_{\hat{a}_0}\bar{f}_K\bar{f}_K] + C_{\mathrm{us}}\mathcal{J}_{1,1}^{\Lambda}[\bar{f}_{\hat{a}_0}\bar{f}_K\bar{f}_K]) - \\ -P_1\cdot P_2 8m_s\mathcal{J}_{2,1}^{\Lambda}[\bar{f}_{\hat{a}_0}\bar{f}_K\bar{f}_K], \tag{97}$$

where the constants C are defined in Appendix (see (A.118)). The momenta P_1 and P_2 are those of the secondary particles. Their product is expressed via masses of mesons (see (A.102) in Appendix) As a result we obtain:

$$\begin{aligned} \Gamma_{\hat{a}_0\to\eta\pi} &= 250\ \mathrm{MeV}, \\ \Gamma_{\hat{a}_0\to\eta'\pi} &= 36\ \mathrm{MeV}, \\ \Gamma_{\hat{a}_0\to KK} &= 160\ \mathrm{MeV}. \end{aligned} \tag{98}$$

The total width is thereby 446 MeV.

The decay amplitude of $K_0^*(1430)$ into $K\pi$ has the form:

$$A_{\hat{K}_0^*\to K\pi} = 8m_s\mathcal{J}_{1,1}^{\Lambda}[\bar{f}_{\hat{K}_0^*}\bar{f}_K^{\mathrm{u}}\bar{f}_{\pi}] - 8m_sP_1\cdot P_2\mathcal{J}_{2,1}^{\Lambda}[\bar{f}_{\hat{K}_0^*}\bar{f}_K^{\mathrm{u}}\bar{f}_{\pi}]. \tag{99}$$

The decay width is

$$\Gamma_{\hat{K}_0^*\to K\pi} = 200\ \mathrm{MeV}. \tag{100}$$

There is also possibility of the state $\hat{K}_0^*$ to decay into $K3\pi$ via the processes $\hat{K}_0^* \to K_0^*\pi\pi \to K3\pi$, $\hat{K}_0^* \to K_0^*\sigma_{\mathrm{I}} \to K3\pi$ and $\hat{K}_0^* \to K\pi\sigma_{\mathrm{I}} \to K3\pi$. A rough estimate of the corresponding decay widths shows that it can add ~50 MeV to the total width of $\hat{K}_0^*$.

7. CONCLUSION AND DISCUSSION

In papers [9, 10] we suggested a chiral quark model with the scalar glueball. However, in these papers, only ground states of scalar quarkonia were considered. To describe the whole spectrum of scalar meson in the mass interval from 0.4 to 1.7 GeV, one needs to introduce radially excited meson states. This has already been done, however without the glueball, in papers [5–7]. The radially excited quarkonia were described by means of form factors. Each of these form factors was a polynomial in the momentum space and had two parameters: the external c_a and the slope parameter d_a. In general, the external parameters c_a can always be absorbed by the four-quark interaction constant G, giving rise to four different interaction constants $\bar{G}_a$, connected with excited meson states. The constants determine only masses of excited mesons and do not affect their interaction of mesons. Only the slope parameters d_a influence decay amplitudes. And they are fixed by conditions (55) and (56) and are not arbitrary.

In papers [5–7], we have shown for the first time that 18 scalar meson states with masses lying between 0.4 GeV and 1.7 GeV can be considered as two nonets of scalar quarkonia. In the present work, we introduced a glueball into the Lagrangian investigated in [5–7] and described mixing of five scalar isoscalar meson states: $\sigma_{\rm I}$, $\sigma_{\rm II}$, $\sigma_{\rm III}$, $\sigma_{\rm IV}$, and $\sigma_{\rm V}$ with the masses: 400, 1070, 1320, 1550, and 1670 MeV, respectively. We showed that $f_0(1500)$ is rather a glueball. This conclusion has followed the analysis of strong decays of the meson state $f_0(1500)$. Indeed, according to our calculations, the state $f_0(1500)$ decays mostly into 4π and 2π, the decay into 4π being more probable. This is in agreement with experiment [8, 25]. Meanwhile, the decays of $f_0(1710)$ into 4π and 2π are suppressed as compared with those into kaons and η mesons (see [8, 25]). On the other hand, if the model parameters were fixed from the supposition that $f_0(1710)$ was the glueball, the main decay mode of $f_0(1710)$ would be 4π ($\Gamma_{4\pi}$ =150 MeV), the remaining partial widths would be too small: $\Gamma_{\pi\pi}$ =3 MeV, Γ_{KK} =5 MeV, $\Gamma_{\eta\eta}$ =2 MeV, $\Gamma_{\eta\eta'}$ =2 MeV. For the state $f_0(1500)$ in this case, the main decay would be into kaons ($\Gamma_{KK} = 250$ MeV), the other modes would give: $\Gamma_{\pi\pi}$=10 MeV, $\Gamma_{\eta\eta} = 34$ MeV, $\Gamma_{4\pi}$ =90 MeV. This crucially disagrees with experiment [25].

Note that, after the glueball is introduced into the effective meson Lagrangian, the mass of $\sigma_{\rm I}$ noticeably decreased as compared with the result from [5–7]. This is a consequence of the noticeable mixing between the glueball and the ground and radially excited $\bar{u}u$ ($\bar{d}d$) quarkonia, $f_0(400-1200)$ and $f_0(1370)$. The obtained mass and decay width of $\sigma_{\rm I}$ are in satisfactory agreement with recent experimental data [8, 26, 27]. On the other hand, the $\bar{s}s$ quarkonia mix with the glueball at a small proportion (see Table 1). Therefore, after introducing the glueball (see [5–7]), the masses of $\sigma_{\rm II}$ and $\sigma_{\rm V}$ change less than the mass of $\sigma_{\rm I}$. However, here we obtain better agreement with experiment for the mass of $\sigma_{\rm V}$ than in [5–7]. For $\sigma_{\rm IV}$, we obtain that the state contains 67% of the glueball, which is in agreement with [2]. After this analysis, we identify the five scalar isoscalar states $\sigma_{\rm I}$, $\sigma_{\rm II}$, $\sigma_{\rm III}$, $\sigma_{\rm IV}$, and $\sigma_{\rm V}$ with physically observed meson states in the following sequence: $f_0(400-1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ (see Table 2). We also have excited isovectors: $\hat{a}_0$ with the mass 1530 MeV and strange scalar meson $\hat{K}_0^*$ with the mass 1430 MeV, respectively.

The chiral symmetry has played a crucial role in calculations. It allowed us to predict masses of scalar mesons, using masses of pseudoscalars.

Let us remind that our model is based on the $U(3) \times U(3)$ chiral symmetry and scale invariance of an effective meson Lagrangian. Both symmetries are very approximate for the energies under consideration. Therefore, our results are rather qualitative. As a result, the obtained decay width agrees with experiment only in the order of magnitude. Nevertheless, we hope that the model gives, on the whole, a correct description of scalar meson properties.

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A. CALCULATION OF THE QUARK LOOP CONTRIBUTION INTO THE STRONG DECAY AMPLITUDES

In the calculation of the quark loop contributions to decay amplitudes, we follow our papers [5–7], where the external momentum dependence of decay amplitudes was taken into account.

It is convenient to take account of the mixing effects before integration. To demonstrate how to do this, let us first calculate the decay of the state $\sigma_{\rm I}$ into pions. As one can see, eight[7] diagrams (Fig. 3) contribute to this process. The expression for the amplitude is as follows (see (8) for the definition of form factor functions):

$$\begin{aligned} A^{(2)}_{\sigma_{\rm I}\to\pi\pi} &= 8m_{\rm u}[g_{8,1}b_{\sigma_{8,1}\sigma_{\rm I}}(g^2_{1,1}Zb^2_{\pi_1\pi}\mathcal{J}^{\Lambda}_{2,0}[1] + \\ &+2g_{1,1}g_{1,2}\sqrt{Z}b_{\pi_1\pi}b_{\pi_2\pi}\mathcal{J}^{\Lambda}_{2,0}[f^{\rm u}_2] + g^2_{1,2}b^2_{\pi_2\pi}\mathcal{J}^{\Lambda}_{2,0}[f^{\rm u}_2 f^{\rm u}_2]) + \\ &+g_{8,2}b_{\sigma_{8,2}\sigma_{\rm I}}(g^2_{1,1}Zb^2_{\pi_1\pi}\mathcal{J}^{\Lambda}_{2,0}[f^{\rm u}_2] + \\ &+2g_{1,1}g_{1,2}\sqrt{Z}b_{\pi_1\pi}b_{\pi_2\pi}\mathcal{J}^{\Lambda}_{2,0}[f^{\rm u}_2 f^{\rm u}_2] + \\ &+g^2_{1,2}b^2_{\pi_2\pi}\mathcal{J}^{\Lambda}_{2,0}[f^{\rm u}_2 f^{\rm u}_2 f^{\rm u}_2]) - \\ &-P_1\cdot P_2\,(g_{8,1}b_{\sigma_{8,1}\sigma_{\rm I}}(g^2_{1,1}Zb^2_{\pi_1\pi}\mathcal{J}^{\Lambda}_{3,0}[1] + \\ &+2g_{1,1}g_{1,2}\sqrt{Z}b_{\pi_1\pi}b_{\pi_2\pi}\mathcal{J}^{\Lambda}_{3,0}[f^{\rm u}_2] + g^2_{1,2}b^2_{\pi_2\pi}\mathcal{J}^{\Lambda}_{3,0}[f^{\rm u}_2 f^{\rm u}_2]) + \\ &+g_{8,2}b_{\sigma_{8,2}\sigma_{\rm I}}(g^2_{1,1}Zb^2_{\pi_1\pi}\mathcal{J}^{\Lambda}_{3,0}[f^{\rm u}_2] + \\ &+2g_{1,1}g_{1,2}\sqrt{Z}b_{\pi_1\pi}b_{\pi_2\pi}\mathcal{J}^{\Lambda}_{3,0}[f^{\rm u}_2 f^{\rm u}_2] + \\ &+g^2_{1,2}b^2_{\pi_2\pi}\mathcal{J}^{\Lambda}_{3,0}[f^{\rm u}_2 f^{\rm u}_2 f^{\rm u}_2]))], \end{aligned} \tag{A.101}$$

The product of the momenta of secondary particles can be expressed through masses of mesons:

$$P_1\cdot P_2 = \frac{1}{2}(M^2 - M^2_1 - M^2_2), \tag{A.102}$$

where M is the mass of the decaying meson, and M_1 and M_2 are the masses of secondary particles ($M = M_{\sigma_{\rm I}}$, $M_1 = M_2 = M_\pi$ in this case). Let us continue (A.101) and calculate the sum before integration. The resulting expression becomes short:

$$A^{(2)}_{\sigma_{\rm I}\to\pi\pi} = 8m_{\rm u}(\mathcal{J}^{\Lambda}_{2,0}[\bar f_{\sigma_{\rm I}}\bar f_\pi \bar f_\pi] - P_1\cdot P_2\mathcal{J}^{\Lambda}_{3,0}[\bar f_{\sigma_{\rm I}}\bar f_\pi \bar f_\pi]), \tag{A.103}$$

[7] Two of them are identical, which leads to the symmetry factor of 2.

where $\bar{f}_a$ are form factor functions for the physical meson states, defined as follows:

$$\bar{f}_{\sigma_{\rm I}} = g_{8,1} b_{\sigma_{8,1}\sigma_{\rm I}} + g_{8,2} b_{\sigma_{8,2}\sigma_{\rm I}} f_2^{\rm u}, \tag{A.104}$$

$$\bar{f}_\pi = g_{1,1} b_{\pi_1\pi} \sqrt{Z} + g_{1,2} b_{\pi_2\pi} f_2^{\rm u}. \tag{A.105}$$

The coefficients $b_{\pi_1\pi}$ appear because of the mixing between the ground and excited pion states. Their values are: $b_{\pi_1\pi} \approx 0.997$, $b_{\pi_2\pi} \approx 0.007$. Concerning the decays into the other pairs of pseudoscalars, the calculation of the corresponding contribution is carried out in the same manner. We will discriminate these form factor functions by the superscripts u and s, respectively. Below we give the physical form factors that were used in the calculation:

$$\bar{f}^{\rm u}_{\sigma_l} = g_{8,1} b_{\sigma_{8,1}\sigma_l} + g_{8,2}(1 + d_{\rm u}\mathbf{k}^2) b_{\sigma_{8,2}\sigma_l}, \tag{A.106}$$
$$\bar{f}^{\rm s}_{\sigma_l} = g_{9,1} b_{\sigma_{9,1}\sigma_l} + g_{9,2}(1 + d_{\rm s}\mathbf{k}^2) b_{\sigma_{9,2}\sigma_l}, \tag{A.107}$$
$$\bar{f}_\pi = g_{1,1} b_{\pi_1\pi}\sqrt{Z} + g_{1,2}(1 + d_{\rm u}\mathbf{k}^2) b_{\pi_2\pi}, \tag{A.108}$$
$$\bar{f}_K = g_{4,1} b_{K_1K}\sqrt{Z} + g_{4,2}(1 + d_{\rm us}\mathbf{k}^2) b_{K_2K}, \tag{A.109}$$
$$\bar{f}_{\hat{a}_0} = g_{1,1} b_{a_{0\,1}\hat{a}_0} + g_{1,2}(1 + d_{\rm u}\mathbf{k}^2) b_{a_{0\,2}\hat{a}_0}, \tag{A.110}$$
$$\bar{f}_{K_0^*} = g_{4,1} b_{K^*_{0\,1}\hat{K}^*_0} + g_{4,2}(1 + d_{\rm us}\mathbf{k}^2) b_{K^*_{0\,2}\hat{K}^*_0}, \tag{A.111}$$
$$\bar{f}^{\rm u}_\eta = g_{8,1} b_{\phi_{8,1}\eta}\sqrt{Z} + g_{8,2}(1 + d_{\rm u}\mathbf{k}^2) b_{\phi_{8,2}\eta}, \tag{A.112}$$
$$\bar{f}^{\rm u}_{\eta'} = g_{8,1} b_{\phi_{8,1}\eta'}\sqrt{Z} + g_{8,2}(1 + d_{\rm u}\mathbf{k}^2) b_{\phi_{8,2}\eta'}, \tag{A.113}$$
$$\bar{f}^{\rm s}_\eta = g_{9,1} b_{\phi_{9,1}\eta}\sqrt{Z} + g_{9,2}(1 + d_{\rm s}\mathbf{k}^2) b_{\phi_{9,2}\eta}, \tag{A.114}$$
$$\bar{f}^{\rm s}_{\eta'} = g_{9,1} b_{\phi_{9,1}\eta'}\sqrt{Z} + g_{9,2}(1 + d_{\rm s}\mathbf{k}^2) b_{\phi_{9,2}\eta'}. \tag{A.115}$$

Let us write the quark-loop contribution to the vertices of the effective meson Lagrangian in terms of physical meson states. Only the vertices describing the processes, which we are interested in, are given below. For l =I,II,III,IV, V, we have

$$\begin{aligned} &A^{(2)}_{\sigma_l\to\pi\pi}\sigma_l(2\pi^+\pi^- + \pi^0\pi^0) + A^{(2)}_{\sigma_l\to KK}\sigma_l(K^+K^- + K^0\bar{K}^0) + \\ &+A^{(2)}_{\sigma_l\to\eta\eta}\sigma_l\eta\eta + A^{(2)}_{\sigma_l\to\eta\eta'}\sigma_l\eta\eta'. \end{aligned} \tag{A.116}$$

$$\begin{aligned} A^{(2)}_{\sigma_l\to\pi\pi} &= 8m_{\rm u}(\mathcal{J}^\Lambda_{2,0}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}_\pi\bar{f}_\pi] - P_1\cdot P_2\mathcal{J}^\Lambda_{3,0}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}_\pi\bar{f}_\pi]), \\ A^{(2)}_{\sigma_l\to KK} &= 8m_{\rm u}(C_{\rm uu}\mathcal{J}^\Lambda_{2,0}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}_K\bar{f}_K] + C_{\rm us}\mathcal{J}^\Lambda_{1,1}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}_K\bar{f}_K]) - \\ &-8\sqrt{2}m_{\rm s}(C_{\rm ss}\mathcal{J}^\Lambda_{0,2}[\bar{f}^{\rm s}_{\sigma_l}\bar{f}_K\bar{f}_K] + C_{\rm su}\mathcal{J}^\Lambda_{1,1}[\bar{f}^{\rm s}_{\sigma_l}\bar{f}_K\bar{f}_K]) - \\ &-P_1\cdot P_2(8m_{\rm s}\mathcal{J}^\Lambda_{2,1}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}_K\bar{f}_K] - 8\sqrt{2}m_{\rm u}\mathcal{J}^\Lambda_{1,2}[\bar{f}^{\rm s}_{\sigma_l}\bar{f}_K\bar{f}_K]), \\ A^{(2)}_{\sigma_l\to\eta\eta} &= 8m_{\rm u}\mathcal{J}^\Lambda_{2,0}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}^{\rm u}_\eta\bar{f}^{\rm u}_\eta] - 8\sqrt{2}m_{\rm s}\mathcal{J}^\Lambda_{0,2}[\bar{f}^{\rm s}_{\sigma_l}\bar{f}^{\rm s}_\eta\bar{f}^{\rm s}_\eta] - \\ &-P_1\cdot P_2(8m_{\rm u}\mathcal{J}^\Lambda_{3,0}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}^{\rm u}_\eta\bar{f}^{\rm u}_\eta] - 8\sqrt{2}m_{\rm s}\mathcal{J}^\Lambda_{0,3}[\bar{f}^{\rm s}_{\sigma_l}\bar{f}^{\rm s}_\eta\bar{f}^{\rm s}_\eta]), \end{aligned}$$

$$A^{(2)}_{\sigma_l\to\eta\eta'} = 16m_{\rm u}\mathcal{J}^{\Lambda}_{2,0}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}^{\rm u}_{\eta}\bar{f}^{\rm u}_{\eta'}] - 16\sqrt{2}m_{\rm s}\mathcal{J}^{\Lambda}_{0,2}[\bar{f}^{\rm s}_{\sigma_l}\bar{f}^{\rm s}_{\eta}\bar{f}^{\rm s}_{\eta'}] - \\ -P_1\cdot P_2(16m_{\rm u}\mathcal{J}^{\Lambda}_{3,0}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}^{\rm u}_{\eta}\bar{f}^{\rm u}_{\eta'}] - 16\sqrt{2}m_{\rm s}\mathcal{J}^{\Lambda}_{0,3}[\bar{f}^{\rm s}_{\sigma_l}\bar{f}^{\rm s}_{\eta}\bar{f}^{\rm s}_{\eta'}]), \tag{A.117}$$

where

$$C_{\rm uu} = \frac{2m_{\rm u}}{m_{\rm u}+m_{\rm s}}, \quad C_{\rm us} = \frac{m_{\rm s}(m_{\rm u}-m_{\rm s})}{m_{\rm u}(m_{\rm u}+m_{\rm s})}, \\ C_{\rm ss} = \frac{2m_{\rm s}}{m_{\rm u}+m_{\rm s}}, \quad C_{\rm su} = \frac{m_{\rm u}(m_{\rm s}-m_{\rm u})}{m_{\rm s}(m_{\rm u}+m_{\rm s})}. \tag{A.118}$$

Now we consider the decays of a scalar isoscalar meson into a pair of $\sigma_{\rm I}$. To calculate the quark loop contribution to the corresponding decay amplitudes, one should follow the method described above for the pseudoscalar mesons. The quark loop contribution can be represented as a set of diagrams that results in a sum of integrals which then can be converted into a single integral over the physical form factors for scalar isoscalar mesons. Thus, one obtains:

$$A^{(2)}_{\sigma_l\to\sigma_{\rm I}\sigma_{\rm I}} \approx 8m_{\rm u}\mathcal{J}^{\Lambda}_{2,0}[\bar{f}^{\rm u}_{\sigma_l}\bar{f}^{\rm u}_{\sigma_{\rm I}}\bar{f}^{\rm u}_{\sigma_{\rm I}}] \tag{A.119}$$

for l =III, IV, V. In conclusion, we display the coefficients a_k that determine contact terms (89):

$$a_1 = -\frac{1}{\chi_c^3}\Big[\frac{10}{3}C_{\rm g} + \sum_{a,b=8}^{9}\Big(-\frac{4}{3}\bar{\mu}^0_a(G^{(-)})^{-1}_{ab}\mu_b + \\ +\frac{7}{3}\bar{\mu}^0_a\left(G^{(-)}\right)^{-1}_{ab}\bar{\mu}^0_b + \frac{1}{6}\mu^0_a\left(G^{(-)}\right)^{-1}_{ab}(\mu_b - \bar{\mu}^0_b)\Big) + \\ +h_\sigma(16F_0^2 - 18F_0F_0^0 + 4(F_0^0)^2)\Big], \tag{A.120}$$

$$a_2 = -\frac{\sqrt{2}h_\sigma}{\sqrt{3}\chi_c^2}(14F_0 - 10F_0^0) - \frac{1}{\chi_c^2}\sum_{a=8}^{9} g_{8,1}\left(G^{(-)}\right)^{-1}_{8a}\bar{\mu}^0_a, \tag{A.121}$$

$$a_3 = \frac{4h_\sigma}{3\chi_c} - \frac{1}{\chi_c}\left(g^2_{8,1}\left((G^{(-)})^{-1}_{88} - 8\mathcal{J}^{\Lambda}_{1,0}[1]\right) + 4m^2_{\rm u}\right), \tag{A.122}$$

$$a_4 = \frac{1}{\chi_c}\left(g^2_{8,2}\left(1/\bar{G}_8 - 8\mathcal{J}^{\Lambda}_{2,0}[f^{\rm u}_2 f^{\rm u}_2]\right) + 4m^2_{\rm u}\right). \tag{A.123}$$

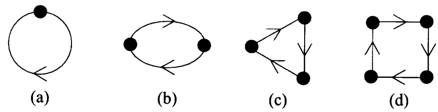

Figure 1 The set of diagrams contributing to the effective meson Lagrangian. Tadpoles (a) determine to gap equations, polarization diagrams (b) give kinetic and mass terms, triangle diagrams (c) and boxes (d) describe the interaction of mesons.

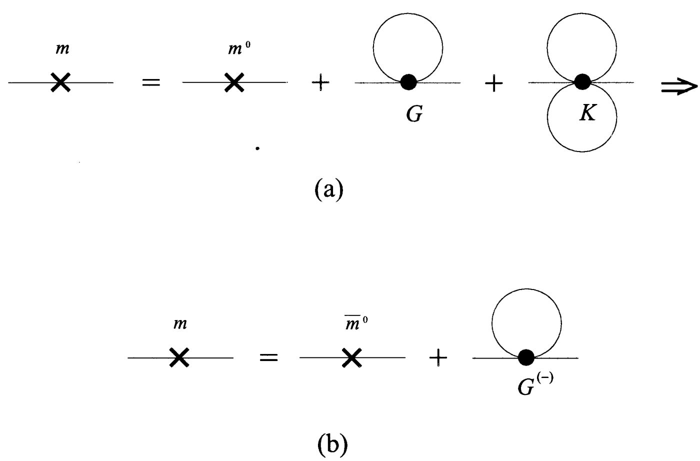

Figure 2 The set of diagrams defining gap equations. In the case (b) the six-quark interaction is taken into account in the matrix of constants $G^{(-)}$. In order to avoid the double counting of the contribution of 't Hooft interaction, the current quark masses m^0 is replaced by the modified masses $\overline{m}^0$.

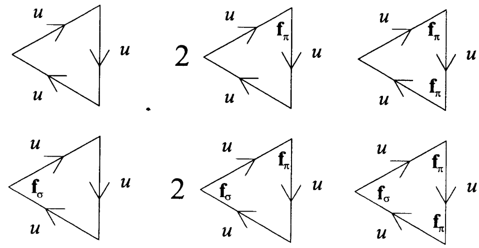

Figure 3 The set of diagrams describing the decay of a scalar meson into a pair of pions. The vertices where a form factor occurs are marked by **f**. In this set of diagram, the quark-meson vertices correspond to meson fields before taking into account mixing effects.

Table 1. Elements of the matrix b, describing mixing in the scalar isoscalar sector. The singlet-octet and quarkonia-glueball mixing effects are taken into account.

	$\sigma_{\rm I}$	$\sigma_{\rm II}$	$\sigma_{\rm III}$	$\sigma_{\rm IV}$	$\sigma_{\rm V}$
$\sigma_{8,1}$	0.973	0.137	0.393	0.548	0.048
$\sigma_{8,2}$	−0.064	0.204	−0.978	−0.647	−0.047
$\sigma_{9,1}$	−0.225	0.876	0.160	0.011	0.628
$\sigma_{9,2}$	0.025	0.146	0.136	−0.082	−1.09
χ'	−0.266	0.095	−0.495	0.813	−0.116

Table 2. The model and experimental masses of scalar isoscalar meson states.

	Theor.	Exp. [8]
$\sigma_{\rm I}$	400	408 [26], 387 [27]
$\sigma_{\rm II}$	1070	980±10
$\sigma_{\rm III}$	1320	1200–1500
$\sigma_{\rm IV}$	1550	1500±10
$\sigma_{\rm V}$	1670	1712±5
$\hat{a}_0$	1530	1474±19
K_0^*	1430	1429±6

Table 3. The partial and total decay widths (in MeV) of scalar isoscalar meson states. (*)For the meson state $\sigma_{\rm II}$, there is possible a decay into kaons, which we did not calculate here, because its mass is at the threshold. We show only the lowest limit for its total decay width allowing for the decay into kaons that can increase the total decay width. The value is given for the model mass 1070 MeV. Next, in brackets, we also give the decay width calculated for the experimental mass 980 MeV. In the case of $\sigma_{\rm III}$, two values are given for its model mass and (in brackets) for the lowest bound for its experimental mass (1200 MeV).

Decays	$f_0(400-1200)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$a_0(1250)$	$K_0^*(1430)$
$\pi\pi$	600	36 (20)	680 (480)	100	0.3	–	–
$K\bar{K}$	–	–	260 (125)	28	250	160	–
$\eta\eta$	–	–	62 (26)	4	20	–	–
$\eta\eta'$	–	–	–	–	100	–	–
$4\pi(2\sigma_{\rm I})$	–	–	40	200	1	–	–
$\pi\eta$	–	–	–	–	–	250	–
$\pi\eta'$	–	–	–	–	–	36	–
$K\pi$	–	–	–	–	–	–	200
$K3\pi$ $(K_0^*\pi\pi$, $K_0^*\sigma_{\rm I}$, $K\pi\sigma_{\rm I})$	–	–	–	–	–	–	~50
$\Gamma_{\rm tot}$	600	$> 40(> 20)(*)$	1040 (670)	330	370	450	250
$\Gamma_{\rm tot}^{\rm exp}$	600–1200	40–100	200–500	112±10	133±14	265±13	287±23

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Волков М.К., Юдичев В.Л. E2-2001-43
Первые радиальные возбуждения скалярного мезонного нонета и глюбол

В нелокальной $U(3)\times U(3)$ киральной кварковой модели с локальным взаимодействием 't Хофта описаны первые радиальные возбуждения скалярных мезонов. Для описания радиально-возбужденных состояний использованы простые лоренц-ковариантные формфакторы полиномиального вида в импульсном пространстве. Благодаря киральной симметрии формфакторы для скалярных состояний совпадают с формфакторами для псевдоскаляров. В результате, используя экспериментальные значения масс псевдоскалярных мезонов, мы предсказываем спектр масс основных и радиально-возбужденных скалярных мезонных состояний. Скалярный глюбол введен в эффективный мезонный лагранжиан при помощи дилатонной модели. Показано, что 19 скалярных состояний с массами от 0,4 до 1,7 ГэВ могут быть интерпретированы как два скалярных нонета и глюбол. Вычислены ширины сильных распадов скалярных мезонов. Показано, что состояние f_0 (1500) является наиболее вероятным кандидатом на скалярный глюбол.

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Volkov M.K., Yudichev V.L. E2-2001-43
First Radial Excitations of Scalar Meson Nonet and the Glueball

In a nonlocal $U(3)\times U(3)$ chiral quark model with the local 't Hooft interaction, the first radial excitations of scalar mesons are described. Simple Lorentz-covariant form factors, having a polynomial form in the momentum space, are used for the description of radially excited states. Due to the chiral symmetry, the form factors for scalar states coincide with those for pseudoscalars. As a result, using the experimental values for the masses of pseudoscalar mesons, we predict the mass spectrum of the ground and radially excited scalar meson states. The scalar glueball is introduced into the effective meson Lagrangian by means of the dilaton model. It is shown that 19 scalar states with masses from 0.4 to 1.7 GeV can be interpreted as two scalar nonets and a glueball. Strong decay widths of scalar mesons are calculated. The state f_0 (1500) is shown to be the most probable candidate for the scalar glueball.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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