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THE PION-NUCLEON Σ-TERM IN A CHIRAL QUARK MODEL

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In last ten years the hypothesis of the content of strange quarks in the valence structure of a nucleon [1, 2] was many times discussed on the basis of the analysis of the pion-nucleon Σ term, . This was caused by the fact that the theoretical estimates of the Σ term obtained in the cited papers without using this hypothesis were substantially smaller than experimental data. Let us notice that till now there are no reliable experimental data for this quantity. The most probable values are:

$$\Sigma_{exp} = 64 \pm 8 \text{ MeV} \quad [3], \tag{1}$$
$$= 92 \pm 6 \text{ MeV} \quad [4]. \tag{2}$$

The last value was obtained quite recently within a new method of analyzing experimental data. In the papers [1, 2] significantly smaller values of the Σ-term were obtained when one does not use the above mentioned hypothesis. Even considering the content of strange quarks in a nucleon, they obtained the estimates lower than 45 MeV.

In our article [5] we showed that in the framework of a linear σ model one can quite satisfactorily explain these experimental data without using strange quarks, considering, together with the diagram describing the scalar form factor of the pion, also the diagram with the intermediate σ-meson (see Fig. 1).

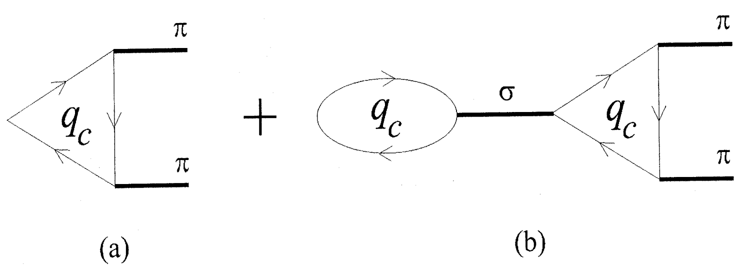

Figure 1: The quark diagrams describing the matrix element $\langle \pi^+(p_1)|\bar{u}u+\bar{d}d|\pi^+(p_2)\rangle$.

Indeed, it was shown that the latter diagram (fig. 1b) basically determines a value of the Σ-term, and it completely cancels the contribution of the scalar form factor of a pion described by the quark triangle diagram (Fig. 1a). The remaining part allows us to describe, in a quite satisfactory way, the experimental data not using the hypothesis of the content of strange quarks in a nucleon.

It is useful to notice that the similar situation takes place in the description of the $\pi\pi$ scattering in the linear σ model where the contribution of the contact π^4 term is completely cancelled by the diagram with the intermediate σ meson (see Fig. 2), and the remaining part of the pole diagram gives the Weinberg formula for the amplitude of the $\pi\pi$ scattering.

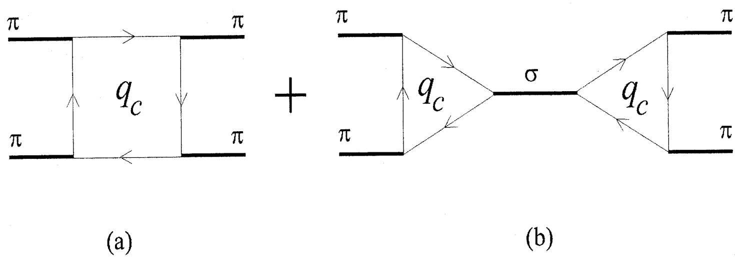

Figure 2: The quark diagrams describing $\pi\pi$-scattering.

$$A_{\pi\pi} \simeq \frac{s}{F_\pi^2}, \tag{3}$$

where $s = (p_1 + p_2)^2$, p_1 and p_2 are momenta of incoming pions and $F_\pi = 93$ MeV is the pion decay constant [6].

To evaluate the Σ-term, in the paper [5] the scalar form factor of the pion was calculated in the chiral symmetric limit when $M_\sigma = 2m$ and $p_1^2 = p_2^2 = 0$ where $m = 280$ MeV is the mass of the constituent u-quark, p_1 and p_2 are pion momenta (Fig. 1)[1].

In the present paper the more realistic value for the σ-meson mass than in [5] is used, namely,

$$m_\sigma = 400 \text{ MeV}, \tag{4}$$

which was obtained in our recent works [8, 9] after taking account of the singlet-octet mixing of scalar isoscalar mesons one with other and with the glueball. Let us note that in our case the σ-meson is identically equal to the experimental scalar state $f_0(400-1200)$ [7]. The singlet-octet mixing is defined by the 't Hooft interaction which appears in the $U(3)\times U(3)$ quark model because of the interaction of instantons [9, 10, 11, 12]. The theoretical value (4) is in a good agreement with the last experimental data [13, 14, 15].

Here we take into account also the dependence of the scalar pion form factor (fig. 1) on external pion momenta. This dependence arises from two sources. The first, because of taking account of the $\pi - a_1$ transitions at external pion legs, the second, because of the dependence of the quark triangle diagram (fig. 1a) on external pion momenta. The dependence of the first kind was considered in the paper [5] and the dependence of the second kind is considered here for the first time.

[1] In this approximation all of the lengths of the $\pi\pi$ scattering are equal to zero (see formula (3)). In order to obtain nontrivial values of the $\pi\pi$ scattering lengths it is necessary to use the real value for the mass of the $f_0(400-1200)$ meson.

In our work [5] we obtained the value of the Σ-term equal to 55 MeV and now we have got the noticeably higher value for the pion-nucleon Σ-term, namely, $\Sigma \simeq 81$ MeV that is closer to the results of the analysis of experimental data obtained in the paper [4] (see eq. (2)).

Let us very briefly define the pion-nucleon Σ-term. The chiral symmetry allows us to connect this term with the even (respecting to isotopic transformations) amplitude of the πN scattering, $D^{(+)}(\nu, t)$ evaluated at the Cheng-Dashen point [16]: $\nu = (s-u)/4M_p = 0$ and $t = 2M_\pi^2$[2]:

$$\Sigma = F_\pi^2 \bar{D}^{(+)}(0, 2M_\pi^2) = \sigma + \Delta. \tag{5}$$

The first term on the RHS of (5) can be considered as the contribution of the amplitude of the scattering of the massless pion on the physical nucleon, whereas the second term, Δ, represents the correction due to nonvanishing pion mass. The pion-nucleon σ-term is defined by the following matrix element:

$$\sigma = \frac{m_{0u} + m_{0d}}{4M_p} \langle P(p)|\bar{u}u + \bar{d}d|P(p)\rangle, \tag{6}$$

where $|P(p)\rangle$ is the one-proton physical state, $M_p = 938$ MeV is the proton mass, and m_{0q} is the mass of the current q-quark ($q = u, d$).

Let us remind the value of the first term on the RHS of (5) obtained by estimating the matrix element $\langle \pi^+(p_1)|\bar{u}u + \bar{d}d|\pi^+(p_2)\rangle$ on the base of the Gell-Mann–Oakes–Renner low-energy theorem [17] obtained in the current algebra and the PCAC approach:

$$\langle \pi^+(p_1)|\bar{u}u + \bar{d}d|\pi^+(p_2)\rangle \simeq \langle \pi^+(0)|\bar{u}u + \bar{d}d|\pi^+(0)\rangle = -\frac{1}{F_\pi^2}\langle 0|\bar{u}u + \bar{d}d|0\rangle = \frac{4v}{F_\pi^2}, \tag{7}$$

where $2v = -\langle 0|\bar{u}u|0\rangle = -\langle 0|\bar{d}d|0\rangle$ is the quark condensate.

Now let us show how one can get this formula in a chiral quark model of Nambu–Jona–Lasinio (NJL) type calculating the quark-loop diagrams depicted in Fig. 1. For this purpose, it is necessary to use the lagrangian L_{int} of the quark-meson interaction obtained in our model [6]

$$L_{int} = \bar{q}(g_\sigma \sigma + i\gamma^5 g_\pi \vec{\pi}.\vec{\tau})q + \frac{i}{F_\pi}(1 - 1/Z)\bar{q}\frac{1}{2}\gamma^\mu\gamma^5 \partial_\mu \vec{\pi}.\vec{\tau} q, \tag{8}$$

where $q = (u, d)$ denotes the field operators of the constituent u and d quarks, $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the isospin Pauli matrices. In this model we have the following relations between the meson-quark coupling constants

$$g_\sigma = \frac{g_\rho}{\sqrt{6}}, \qquad g_\pi = \frac{m}{F_\pi}, \tag{9}$$

[2]Here s, u and t denote the kinematic invariants of the πN scattering: $s + u + t = 2M_\pi^2 + 2M_p^2$.

where $g_\rho = 6.14$ is the $\rho \to 2\pi$ decay constant and $m = m_u = m_d$ is the mass of a constituent quark. The first relation in (9) was obtained for the first time in [6, 18], and the second one is the Goldberger-Treiman relation. On the other hand, the constants g_σ and g_π are linked by the relation

$$g_\pi = \sqrt{Z} g_\sigma, \tag{10}$$

where

$$Z = \left(1 - \frac{6m^2}{M_{a_1}^2}\right)^{-1} \tag{11}$$

is the additional renormalization of pion fields appearing after taking into account the $\pi - a_1$ transitions, $M_{a_1} = 1.26 \pm 0.03$ GeV [7] is the mass of the axial a_1-meson. The relation (10) we can consider as the equation defining the mass of a constituent quark m. This allows us to express the mass of the constituent u-quark through the mass of the a_1-meson M_{a_1}, the pion decay constant F_π, and the coupling constant g_ρ, namely

$$m^2 = \frac{M_{a_1}^2}{12}\left[1 - \sqrt{1 - \frac{4g_\rho^2 F_\pi^2}{M_{a_1}^2}}\right] \tag{12}$$

and we obtain $m = 280$ MeV [6].[3]

The second part of L_{int} (8) appears after redefinition of axial-vector fields by taking into account the $\pi - a_1$ transitions [6, 19]. This part of the lagrangian is necessary to define the momentum dependence of the scalar pion form factor. To reproduce the GMOR result (7) [5], it is enough to use the first part of L_{int} (8), because in the low-energy approximation, where $p_i^2 \to 0$, the second part of L_{int} (8) vanishes.

Let us write the result of the calculation of the diagrams depicted in Fig. 1 [5]:

$$\langle \pi^+(0)|\bar{u}u + \bar{d}d|\pi^+(0)\rangle = 4mZ\{1 + [I_1(m) - 2m^2 I_2(m)]\frac{8g_\sigma^2}{M_\sigma^2}\}, \tag{13}$$

where the integrals $I_n(m)$ $(n = 1, 2)$ in the Euclidean space have the following form [6]

$$I_n(m_u) = \frac{3}{(2\pi)^4}\int \frac{d_E^4 k}{(m^2 + k^2)^n}\theta(\Lambda^2 - k^2), \tag{14}$$

where Λ is the cut-off parameter identified in our chiral quark model with the scale of spontaneous breaking of the chiral symmetry (SBCS). Using the expressions $g_\sigma^2 = (4I_2(m))^{-1} = g_\rho^2/6$ we obtain the value $\Lambda = 1.25$ GeV [6].

[3]The equality $Z = 1$ is formally possible at $M_{a_1} \to \infty$, and it corresponds to the absence of the a_1-mesons in the intermediate state.

In the chiral limit where $M_\sigma^2 = 4m^2$, first and third terms on the RHS in the formula (13) cancell in pairs. Then using the relation for the quark condensate $2v = 4mI_1(m)$ [6] we obtain the result that coincides with the GMOR result (7). This fact was demonstrated in our paper [5]. In this approximation the following value of the σ-term was obtained [5]

$$\sigma = 50 \text{ MeV}. \tag{15}$$

This value is close to the result given in the papers [1, 2] $\sigma = 43$ MeV where the hypothesis of the strange quarks content in a nucleon was used.

The last analysis of experimental data gives a noticeably larger value for the pion-nucleon σ-term [4]. We show here how one can obtain this result in our model using the more realistic value of the mass of the σ-meson ($f_0(400-1200)$), $M_\sigma = 400$ MeV. This value was obtained in [8, 9], where the singlet-octet mixing of scalar isoscalar mesons and influence of the scalar glueball had been taken into account.

To estimate the σ-term corresponding to $M_\sigma = 400$ MeV we can use the formula (13) where it is necessary to introduce into the second term on the RHS, containing the mass of the σ-meson, the coefficient C_σ^2, describing the content of u- and d-quark components in the scalar state $f_0(400-1200)$. In the paper [8] the value $C_\sigma = 0.94$ was obtained . As a result we have the following formula for the matrix element, defining the σ-term (see also formula(6)) [5]

$$\langle P(p)|\bar{u}u + \bar{d}d|P(p)\rangle = \frac{g_{\sigma pp}}{g_\sigma}\bar{u}(p)u(p)\langle 0|\bar{u}u + \bar{d}d|0\rangle =$$
$$= \frac{g_{\sigma pp}}{g_\sigma}\left(1 + [I_1(m) - 2m^2 I_2(m)]\frac{8C_\sigma^2 g_\sigma^2}{M_\sigma^2}\right)\bar{u}(p)u(p), \tag{16}$$

where $u(p)$ is the bispinor, normalized by the condition $\bar{u}(p)u(p) = 2M_p$, $g_{\sigma pp} = M_p/F_\pi$ denotes the coupling constant of the σpp interaction on the mass shell of the proton ($p^2 = M_p^2$). Using (16) and the GMOR relation

$$m_\pi^2 = \frac{2v}{F_\pi^2}(m_{0u} + m_{0d}) \tag{17}$$

we obtain in our chiral model[4]

$$\sigma = 75 \text{ MeV}. \tag{18}$$

Now let us calculate the dependence of the scalar pion form factor on external momenta p_1 and p_2. The first source of this dependence appears after taking into account the possibility of $\pi - a_1$ transitions at the external pion legs. These transitions are described by the last term of the lagrangian L_{int} (8). The contributions were

[4]We have used the following value for the integral $I_1(m_u) = \frac{3}{(4\pi)^2}\left[\Lambda^2 - m_u^2 \ln\left(\frac{\Lambda^2}{m_u^2} + 1\right)\right]$.

estimated in our work [5] and, as a result, the factor $A(M_\pi^2)$ appears on the RHS of the eqs. (13) and (16)[5]:

$$A(M_\pi^2) = 1 + \left(1 - \frac{1}{Z}\right)\frac{M_\pi^2}{2m^2 Z} = 1.026, \tag{19}$$

where we used $p_1^2 = p_2^2 = M_\pi^2$. The second source of the momentum dependence of the scalar pion form factor is related to the momentum dependence of the triangle quark diagram (see fig. 1). This dependence was neglected in our previous calculations. After taking it into account we obtain the following expression

$$\begin{aligned} I^{(\Delta)} &= -i\frac{3}{(2\pi)^4}\int^\Lambda d^4k Tr\left\{\frac{1}{m-\hat{k}-\hat{p}_1}\gamma_5\frac{1}{m-\hat{k}}\gamma_5\frac{1}{m-\hat{k}-\hat{p}_2}\right\} = \\ &= 4m[I_2(m) + p_1p_2I_3(m)] = 4mI_2(m)\left(1+\frac{3M_\pi^2}{8\pi^2F_\pi^2Z}\right). \end{aligned} \tag{20}$$

Where in the last expression we put $p_1p_2 = M_\pi^2$, $4I_2(m) = g_\sigma^{-2}$, $m^2/g_\sigma^2 = F_\pi^2 Z$ and $I_3(m) = 3(32\pi^2m^2)^{-1}$. The integral (20) was taken in Minkowski space. Now we can extract the second additional factor on the RHS in eqs. (13) and (16) describing the p^2-dependence

$$B(M_\pi^2) = 1 + \frac{3M_\pi^2}{8\pi^2F_\pi^2Z} = 1.06 \tag{21}$$

Finally, for the pion-nucleon Σ-term we have

$$\Sigma = \sigma A(M_\pi^2)B(M_\pi^2) = \sigma + \Delta = 81 \text{ MeV} \tag{22}$$

As a consequence, taking into account the p^2-dependence leads to the following value

$$\Delta = \Sigma(m_\pi^2) - \sigma|_{p^2=0} = 6 \text{ MeV} \tag{23}$$

Now let us shortly analyze the obtained results. In this paper we have shown, similarly as in the article [5], that after taking into account the diagrams with the intermediate σ-meson the agreement with experimental data is essentially improved. Therefore, in this model there is no necessity to use the hypothesis of the content of strange quarks in a nucleon.

On the other hand, we note that in both experimental data for Σ-term and for the mass of the scalar σ-meson $f_0(400-1200)$ there is a large uncertainty. Because our model is very sensitive to the dependence of the pion-nucleon Σ-term on the mass of the scalar meson M_σ more precise measurements of the mass of the scalar meson $f_0(400-1200)$ can give us an important information concerning the Σ-term.

[5] We note that in our previous paper [5] there is a mistake in the coefficient at p^2-term. Indeed, this coefficient must be two times larger.

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References

[1] J. Gasser, M. E. Sainio: Sigma-term physics, hep-ph/0002283; J. Gasser, H. Leutwyler, M. P. Locher and M. E. Sainio: Phys. Lett. B **213** (1988) 85

[2] P. Büttiker, Ulf-G. Meissner: Pion-nucleon scattering inside the Mandelstam triangle, hep-ph/0001066

[3] R. Koch: Z. Phys. C **15** (1982) 161

[4] E. Kuraev, V. Samoilov, O. Shevchenko and A. Sissakian: Extraction of σ-term from scattering and annihilation channels data. JINR Communications E4-2001-32, Dubna 2001

[5] A. N. Ivanov, M. Nagy, N. I. Troitskaya and M. K. Volkov: Phys. Lett. B **235** (1990) 331

[6] M. K. Volkov: Part. Nucl. **17** (1986) 433

[7] Review of Particle Physics – Eur. Phys. J. C **15** (2000) 1

[8] M. K. Volkov and V. L. Yudichev: Eur. Phys. J. A **10** (2001) (in press), hep-ph/0011326

[9] M. K. Volkov and V. L. Yudichev: Eur. Phys. J. A **10** (2001) (in press), hep-ph/0103003

[10] M. K. Volkov, M. Nagy and V. L. Yudichev: Nuovo Cimento A **112** (1999) 225

[11] D. Ebert, M. Nagy, M. K. Volkov and V. L. Yudichev: Eur. Phys. J. A **8** (2000) 567

[12] A. E. Dorokhov and N. I. Kochelev: Z. Phys. C **37** (1988) 377; A. E. Dorokhov and N. I. Kochelev: Sov. J. Nucl. Phys. **52** (1990) 135; A. E. Dorokhov and N. I. Kochelev: Sov. J. Part. Nucl. **23** (1992) 522

[13] C. Janssen, B. C. Pearce, K. Holinde and I. Speth: Phys. Rev. D **52** (1995) 2690

[14] B. S. Zou and D. V. Bugg: Phys. Rev. D **50** (1994) 591; Long Li, Bing-song Zou and Guang-lie Li: Phys. Rev. D **63** (2001) 074003

[15] Yu. S. Surovtsev, D. Krupa and M. Nagy: Phys. Rev. D **63** (2001) 054024

[16] T. P. Cheng and R. Dashen: Phys. Rev. Lett. **26** (1971) 594; L. S. Brown, W. J. Pardee and R. D. Peccei: Phys. Rev. D **4** (1971) 2801

[17] M. Gell-Mann, R. J. Oakes and B. Renner: Phys. Rev. **175** (1968) 2195

[18] K. Kikkawa: Progr. Theor. Phys. **56** (1976) 947

[19] A. N. Ivanov, M. Nagy, N. I. Troitskaya and M. K. Volkov: Int. J. Mod. Phys. A **4** (1989) 3193

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Надь М., Русакович Н.Л., Волков М.К. E2-2001-84
Пион-нуклонный Σ-член в киральной кварковой модели

В рамках линейной σ-модели, основанной на $U(3)\times U(3)$ кварковом эффективном лагранжиане, вычислен пион-нуклонный Σ-член. Показана важность учета полюсной диаграммы с промежуточным скалярным мезоном f_0 (400–1200). Для массы мезона было выбрано значение 400 МэВ, соответствующее, с одной стороны, теоретическим предсказаниям, учитывающим синглет-октетное смешивание скалярных и изоскалярных мезонов и глюбола, с другой стороны, недавним экспериментальным данным. Полученное значение $\sigma = 75$ МэВ согласуется с последним анализом экспериментальных данных по π-N-рассеянию. Показано, что для достижения согласия с экспериментальными данными нет необходимости использовать гипотезу о присутствии странных кварков в структуре нуклона.

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Nagy M., Rusakovich N.L., Volkov M.K. E2-2001-84
The Pion-Nucleon Σ-Term in a Chiral Quark Model

The pion-nucleon Σ-term is calculated in a linear σ-model based on the $U(3)\times U(3)$ quark effective lagrangian. The importance of the pole diagram with the scalar meson f_0 (400–1200) is demonstrated. For the mass of this meson the value 400 MeV was chosen, which corresponds, on the one hand, to the theoretical predictions taking into account singlet-octet mixing of scalar and isoscalar mesons and glueball, and on the other hand, to recent experimental data. The obtained value $\sigma = 75$ MeV is in agreement with the latest analysis of experimental data on the π-N-scattering. It is shown that to reach an agreement with experimental data, the hypothesis of the content of strange quarks in the valence structure of a nucleon is not necessary.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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