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OPTIMUM TARGET THICKNESS FOR POLARIMETERS

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1 Introduction

Large acceptance polarimeters with 2π geometry [1] are instruments to study the polarization of secondary protons produced in primary reactions (double scattering experiments). Common target materials in such polarimeters are carbon[2, 3] or CH_2[4, 5], and the measured reaction is

$$\vec{p} + T \rightarrow \textit{one charged particle} + X. \quad (1)$$

Decreasing analyzing power (A_y) with increasing beam momentum [6] impose severe conditions for the polarimeter performance in the region of 5 – 6 GeV/c, as indicated in the proposal of Jlab experiment[7]; these can be achieved only with an optimum choice of thickness and material of a target. Meanwhile, the optimum target thickness is not the subject of consensus yet. For example, the target thickness, used in the experiments [2, 3], was close to the nuclear collision length while it was almost twice higher in the experiments [4, 5]. The measurements of A_y on a carbon target at several beam momenta and target thicknesses [8] have not provided a conclusion about the optimum target thickness.

An attempt to calculate the most common features related to the optimum target thickness in the few GeV region is made in this paper. These results were reported at the SPIN-01 workshop in Dubna [9] for the first time.

2 Equations

The absorption of a primary beam is described by the differential equation

$$\frac{dN_0}{dz} = -\lambda_T^{-1} N_0,$$

where N_0 is the number of unscattered particles, z the target thickness, λ_T the nuclear collision length. The solution of this equation is the well-known expression

$$\varepsilon_0(z) = \frac{N_0(z)}{N_0(0)} = e^{-z/\lambda_T}. \quad (2)$$

The production and absorption of i-th scattered particles is described by the differential equation

$$\frac{dN_i}{dz} = \lambda_p^{-1} N_{i-1} - \lambda_T^{-1} N_i,$$

where λ_p is the collision length for the measured process ($\lambda_p > \lambda_T$). Step by step, the solutions of this equation for each i ($y' + ay = f(x)$) give

$$\varepsilon_i(z) = \frac{N_i(z)}{N_0(0)} = \frac{(z/\lambda_p)^i}{i!} e^{-z/\lambda_T}. \tag{3}$$

The maximum output of particles of an i-th generation takes place at

$$z = i\lambda_T, \tag{4}$$

and does not depend on λ_p.

The total number of interactions is then

$$\varepsilon = \sum_{i=1}^{\infty} \varepsilon_i = (e^{z/\lambda_p} - 1)e^{-z/\lambda_T}, \tag{5}$$

which has a maximum at

$$z_m = \frac{\lambda_T}{k_p} \ln \frac{1}{1 - k_p}, \tag{6}$$

where $k_p = \lambda_T/\lambda_p$. Having an experimental value of ε at an arbitrary value of z, one can find k_p inverting (5):

$$k_p = \frac{\ln\left(1 + \varepsilon(z)e^{z/\lambda_T}\right)}{z/\lambda_T}. \tag{7}$$

Eq.(5) is valid, when the scattered particle is identical to the incident one.

To investigate the angular distributions, let us assume for simplicity that the first scattering is described by

$$\frac{d\varepsilon_1}{dt} = C\ e^{bt}, \tag{8}$$

where t is the squared 4-momentum transfer. As a function of θ, it takes the Gaussian form:

$$\frac{d\varepsilon_1}{dt} = C\ e^{-\theta^2/2\sigma^2},$$

where $\sigma^2 = 1/(2bp^2)$, p - beam momentum. The convolution of n Gaussians produces a new Gaussian with

$$\sigma_n^2 = n\sigma^2.$$

So, the distribution of i-th scattered particles is proportional to $e^{bt/i}$. Finding C from the equation

$$\int_0^\infty Ce^{bt/i}dt = \varepsilon_i(z),$$

we have

$$\frac{d\varepsilon_i}{dt} = \frac{b}{i}\frac{(z/\lambda_p)^i}{i!}e^{-z/\lambda_T}e^{bt/i}. \tag{9}$$

For a proton beam, polarized along the vertical axis, the analyzing power of the process A_y is defined by the following expression:

$$\left(\frac{d\sigma}{dt}\right)_\pm (t,\varphi) = \frac{d\sigma}{dt}(t)[1 + P_\pm A_y(t)\cos\varphi]. \tag{10}$$

Here, the sign $\pm$ refers to spin orientation of incident particles, $P_\pm$ is the value of polarization, φ is the angle between the normal to the scattering plane and the vertical axis, and $d\sigma/dt$ is the differential cross section for the unpolarized beam. After replacing $d\sigma/dt \to d\varepsilon/dt$, Eq. (10) is approximately valid when single scattering events predominate.

The polarimeter performance is expressed in terms of the figure of merit:

$$\mathcal{F}^2(z) = \int \frac{d\varepsilon}{dt} A_y^2(t,z)\, dt = \varepsilon(z)\langle A_y^2\rangle(z), \tag{11}$$

where $\langle A_y^2\rangle$ is the weighted value of A_y^2.

3 Numerical evaluations

The contributions to process (1) are the following:

a. elastic pC and (for CH_2) pp and quasi-elastic pN scattering;

b. reactions with target excitation;

c. reactions with projectile excitation.

In the few GeV region, all these processes contribute with similar probabilities. Reactions (b) practically do not contribute second particles to the polarimeter acceptance while reactions (c), producing many particles in the forward direction, are rejected by the measurement conditions. The only non-negligible subprocess in (c), which produce one charged particle into the polarimeter acceptance, is the reaction

$$p + T \rightarrow n + \pi^+(+\pi^0) + T'.$$

So, one charged particle in the polarimeter acceptance is mostly the proton produced in the reactions (a) and (b), and (5) is rather a good approximation for process (1). Assuming $\sigma_{(b)} \simeq \sigma_{(a)}$ and taking into account data on the total and total elastic cross sections[10], the expected value of k_p in the few GeV region is 0.55–0.65.

It is evident that a sizable contribution to A_y is connected only with single protons in the forward direction. From this point of view, the pion admixture in reaction(1) leads to a certain overestimation of k_p, and so, z_m. For example, if the experimental value obtained for k_p is 0.6 ($z_m = 1.53\lambda_T$) and the estimation of pion admixture is about 10%, one can expect the maximum output of single protons in the forward direction at $z \simeq 1.4\lambda_T$. The dependences of ε and ε_1 on z for different values of k_p are illustrated in Fig.1.

The description of angular distributions by Eq.(9) is correct only for elastic scattering in some region of t. At $-t > 0.05$ (GeV/c)^2, elastic pp and quasielastic pN scatterings predominate over pC elastic scattering. So, the slope parameter of $b = 7$ was chosen for estimations. Taking into account that the contribution of elastic and quasi-elastic processes to (1) does not exceed 50% in the few GeV region, these calculations can be used only for some qualitative estimations. In Fig.2 $d\varepsilon/dt$ and $d\varepsilon_1/dt$ are given for $z = \lambda_T$ and $z = 1.5\lambda_T$. One can see that the output of single-scattered events goes down slightly with increasing target thickness above λ_T while the output of rescattered events grows significantly. At $-t > 0.5$ (GeV/c)^2, rescattered events become predominant.

The approximation of the analyzing power, measured in [11], shows that the analyzing power has its maximum at $t \simeq -0.1$ (GeV/c)^2 and practically does not depend on beam momentum. The maximum values, A_y^{max}, are inversely proportional to the laboratory momenta. Due to energy losses in material (which do not depend on the beam momentum in the few GeV region), one can expect a small increasing of A_y with target thickness. With values of $dE/dz \simeq 2$ MeV cm^2/g for any material, and defining the effective beam momentum as the momentum at half target thickness, we have for single-scattered events

$$\langle A_y\rangle(z) = \langle A_y\rangle(0)\left(1 + 0.001 z p^{-1}\right), \tag{12}$$

where z is the target thickness in cm and p is the beam momentum in GeV/c. One can see that this effect diminishes with increasing beam momentum. The rescattering effects lead to decreasing A_y with target thickness. The summary effect was estimated with the help of a Monte-Carlo simulation. The depolarization factors κ (polarization transfer coefficients) during each scattering were taken to be 0.5 and 0.7 in all the region of t (very rough model). New parameters of A_y for each target thickness were searched for by fitting of (10), where initial values of $A_y(t)$ are multiplied by the free coefficient, to the accumulated (t, φ) plots. The initial values of $A_y(t)$ were taken from approximation of data [11] in the form

$$A_y(t,p) = \frac{\sum_{j=1}^{5} a_j t^{j/2}}{p}.$$

Keeping in mind that Eq.(10) is correct only for single-scattered events and having an estimation of ratio between single-scattered and other events, presented in Fig.1, only events with $-t < 0.5$ $(\mathrm{GeV/c})^2$ were included in the fit. The ratios $\langle A_y^2\rangle(z)/\langle A_y^2\rangle(0)$ are plotted in Fig.3.

The dependence of the figure of merit on z can be written as

$$\frac{d\mathcal{F}^2}{dz} = \varepsilon\frac{d\langle A_y^2\rangle}{dz} + \langle A_y^2\rangle\frac{d\varepsilon}{dz}. \tag{13}$$

From Fig.3 it is clear that $d\langle A_y^2\rangle/dz < 0$ at the beam momenta higher than 3.5 GeV/c. At $z > z_m$, both terms in (13) are negative, and this condition sets the upper limit of experimental search for the optimum target thickness. The calculations of the figure of merit by the Monte-Carlo simulation are illustrated in Fig.4. They show that increasing $\mathcal{F}^2(z)$ at $z > \lambda_T$ is either negligible or absent.

4 Optimum thickness for two targets

Proceeding from the assumption that the maximum of the figure of merit is reached at a target thickness close to λ_T, let us calculate the maximum output of single-scattered events when two analyzing targets (with track detection between them) are used [7].

In this case, the output of single-scattered events can be written in the form

$$\varepsilon_{1,2} = \frac{z_1}{\lambda_p} e^{-z_1/\lambda_T} + \frac{z_2}{\lambda_p} e^{-z_1/\lambda_T} e^{-z_2/\lambda_T}. \tag{14}$$

When the length of the second target is fixed at an arbitrary value, the local maximum of $\varepsilon_{1,2}$ is reached at

$$z_1 = \lambda_T (1 - e^{-z_2/\lambda_T}). \tag{15}$$

The global maximum takes place when $z_2 = \lambda_T$. The corresponding thickness for the first target is $z_1 = 0.63\lambda_T$. The efficiency of two targets is illustrated in Fig.5, where the output for two targets is normalized to the maximum output of single-scattered events for one target (taking place at $z = \lambda_T$). For each z_2, the best values of z_1 were taken according to (15). These values are shown in Fig.5 by a dashed line.

The efficiency of two targets reaches 1.44 in comparison with one target. A real improvement can be larger if in a certain region of t the depolarization is either negligible or known. Then the part of events scattered in the first target can be included in the analysis of scatterings in the second target.

5 Conclusions

An experimental search for an optimum target thickness for the reaction (1) makes sense in the region $\lambda_T < z < 1.5\lambda_T$, but one can hardly expect sizable effects in comparison with the target thickness of λ_T.

Assuming that the best thickness for one target is λ_T, the optimum thicknesses for two targets are $z_1 = 0.63\lambda_T$ and $z_2 = \lambda_T$.

At $-t > 0.5$ $(\mathrm{GeV/c})^2$, the output of rescattered events exceeds the output of single-scattered events, and the analysis of data in this region with the help of (10) is hardly correct. Besides, systematic errors in this region can be very sensitive to the target thickness.

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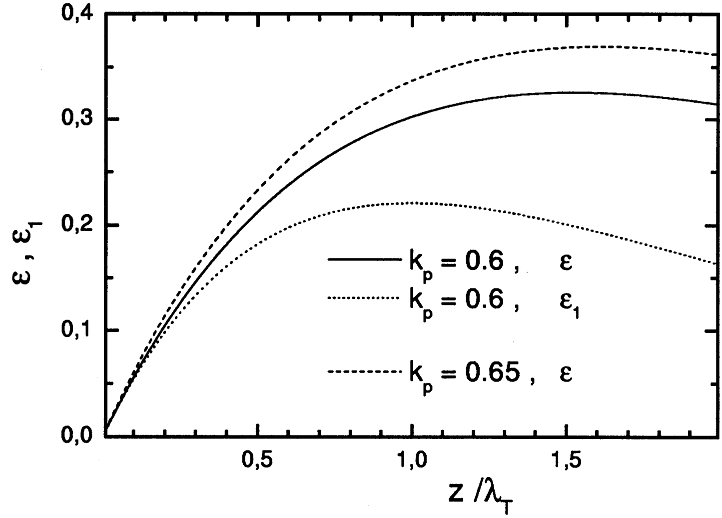

Figure 1: Total output (ε) and output of single-scattered events (ε_1) for different intensities of the process

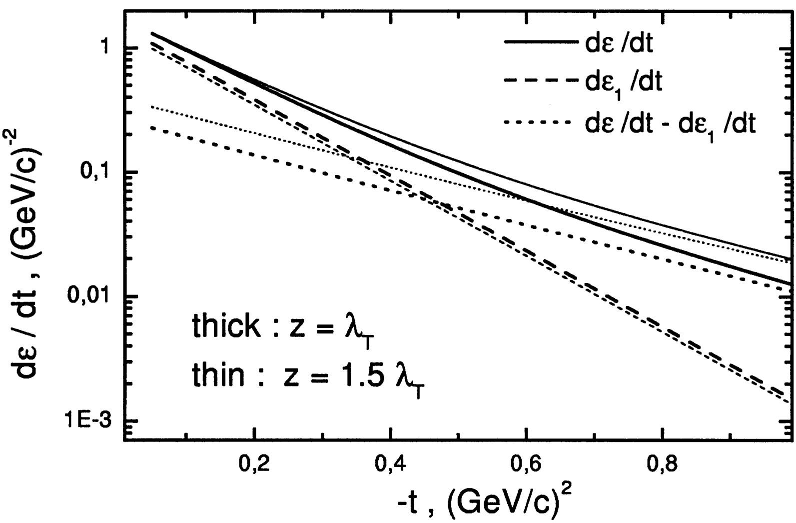

Figure 2: t-distribution of total output ($\frac{d\varepsilon}{dt}$), of single-scattered ($\frac{d\varepsilon_1}{dt}$) and of all other events ($\frac{d\varepsilon}{dt} - \frac{d\varepsilon_1}{dt}$) for two target thicknesses

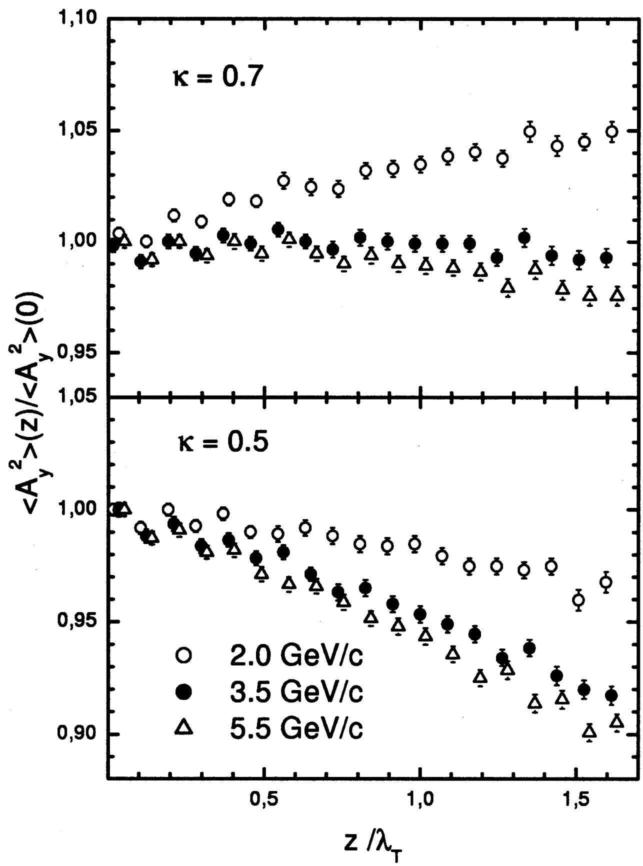

Figure 3: Dependence of analyzing power on target thickness for different beam momenta and depolarization factors

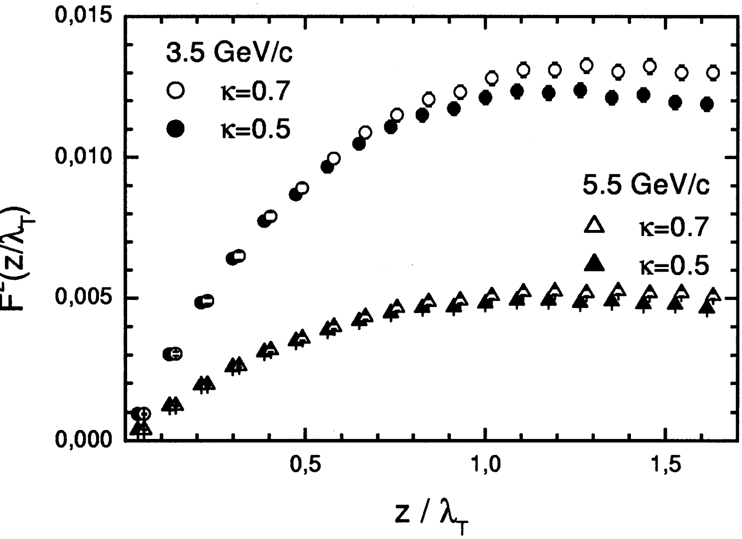

Figure 4: Dependence of figure of merit on target thickness for different beam momenta and depolarization factors at $k_p = 0.6$

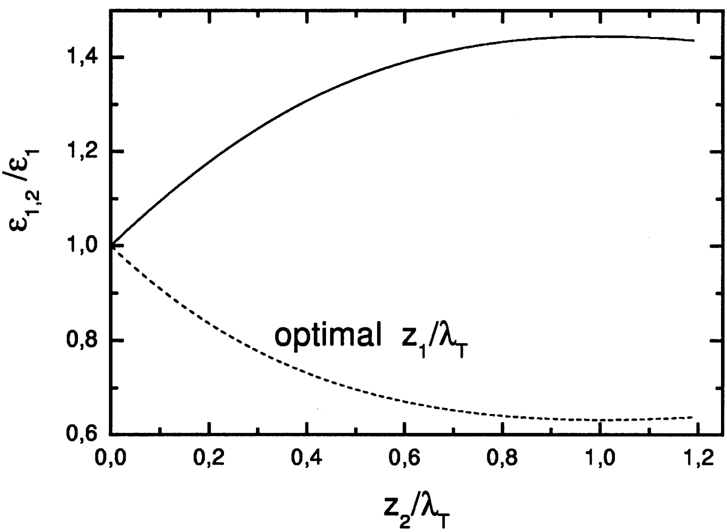

Figure 5: Output of single-scattered events for two targets and optimal thickness of the first target for each thickness of the second target

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Ситник И. М. E13-2003-61
Об оптимальной толщине мишеней для поляриметров

Поляриметры с довольно толстыми мишенями — широко распространенное средство для измерения поляризации протонов. Однако вопрос об оптимальной толщине мишени все еще является предметом дискуссии. В данной статье сделана попытка рассчитать наиболее общие параметры, связанные с этой проблемой, в области нескольких ГэВ.

Работа выполнена в Лаборатории высоких энергий им. В. И. Векслера и А. М. Балдина ОИЯИ.

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Optimum Target Thickness for Polarimeters

Polarimeters with thick targets are a tool to measure the proton polarization. But the question about the optimum target thickness is still the subject of discussion. An attempt to calculate the most common parameters concerning this problem, in a few GeV region, is made in this paper.

The investigation has been performed at the Veksler and Baldin Laboratory of High Energies, JINR.

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