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RADIATIVE CORRECTIONS TO CHIRAL AMPLITUDES IN QUASI-PERIPHERAL KINEMATICS

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Бытьев В. В., Бартош Э., Галынский М. В., Кураев Э. А. E2-2006-42
Радиационные поправки к киральным амплитудам процессов в квазипериферической кинематике

Рассмотрены процессы в квазипериферической кинематике с образованием двух струй. Учтены поправки к подпроцессам образования пары лептонов при взаимодействии реального и виртуального фотонов. Также рассмотрен подпроцесс комптоновского рассеяния с участием виртуального фотона. Учтены вклады от излучения дополнительного мягкого и жесткого фотонов. Все возможные спиральные состояния частиц, участвующих в подпроцессах, приняты во внимание.

Полученные сечения выражены в виде импакт-факторов и записаны через структурные функции, учитывающие лидирующие поправки во всех порядках теории возмущений. Вклад следующего за лидирующим порядка выписан явно в аналитическом виде в форме K-фактора. Обсуждается точность полученных результатов.

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Bytev V. V., Bartoš E., Galynskii M. V., Kuraev E. A. E2-2006-42
Radiative Corrections to Chiral Amplitudes in Quasi-Peripheral Kinematics

Chiral amplitudes for two jets processes in quasi-peripheral kinematics are calculated at the Born and one-loop correction levels. The amplitudes of subprocesses describing interaction of virtual and real photons with creation of a charged fermion pair for various chiral states are considered in detail. Similar results are presented for Compton subprocess with virtual photon.

Contributions of emission of virtual, soft, and hard real additional photons are taken into account explicitly. The relevant cross sections expressed in terms of impact factors are in agreement with structure function approach in the leading logarithmic approximation. Contributions of the next-to-leading terms are presented in an analytical form. Accuracy estimation is discussed.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Much attention was paid during the last decades (see, [1] and references therein) to different processes of kind

$$a_1(p_1,\delta_1)+a_2(p_2,\delta_2)\to \mathrm{jet}_1^{(\lambda_1)}+\mathrm{jet}_2^{(\lambda_2)},\ \ a_{1,2}=e^{\pm},\gamma;\ \ (p_1+p_2)^2=s\gg m_i^2, \tag{1}$$

and $\delta_i,(\lambda_i)$ describe the polarization states of the initial and jet particles. Below we choose $\delta_1=\delta_2=+1$ without loss of generality (see Fig. 1). These processes

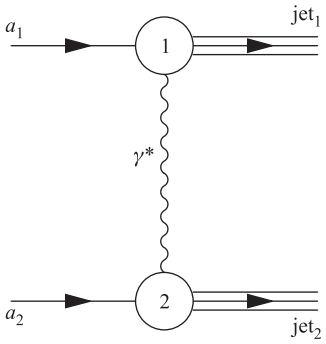

Fig. 1. General diagram for the process

can be studied at high energy collisions of the initial particles in peripheral kinematics, i. e., small angles θ of emission of jet particles to the direction of their parent particle (center-of-mass (c.m.s.) frame of initial particles is implied), see Fig. 2. A remarkable property of nondecreasing of differential and total cross sections on c.m.s. total energy $\sqrt{s}$ in this kinematics is commonly known [2].

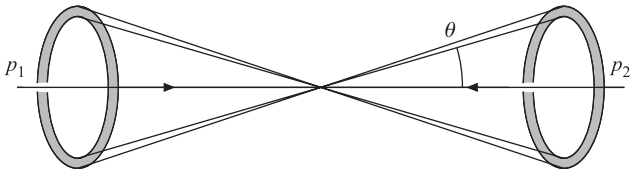

Fig. 2. Kinematics of quasiperipheral process

This property is the consequence of the presence of massless vector particle (photon) in the scattering channel state. The contributions of Feynman diagrams with fermions as well as the interference of amplitudes of these kinds are suppressed compared with photon exchange ones.

As the corresponding cross sections of the relevant QED processes are large numerically, they provide an essential background when studying the effects of weak and strong interactions. Besides, these processes can be used for monitoring and calibration purposes.

Unfortunately, very small emission angles cannot be measured when using in practice. So, we suggest considering the processes (1) in the so-called quasiperipheral (QP) kinematics which implies the values of emission angles to be small compared with unity but much bigger than $m/E = 2m/\sqrt{s}$, where m is characteristic mass of jet's constituents,

$$\frac{2m_i}{\sqrt{s}} \ll \theta_i \ll 1, \qquad m_i^2 \ll -q^2 \ll s, \quad q = -p_1 + \sum p_{i1} = -\sum p_{i2} + p_2, \quad (2)$$

with p_{i1}, p_{i2} — 4-momenta of particles from $\mathrm{jet}_{1,2}$ and q the momentum of the t-channel virtual photon. The QP kinematics provides the independence on energy of differential cross sections but has accuracy of an order of θ^2 — the order of contributions of neglected terms compared to ones considered.

Another important property of QP kinematics is the independence of spin states of $a_1 - \mathrm{jet}_1$ and $a_2 - \mathrm{jet}_2$ blocks of a process. This is a reason why we can put $\delta_{1,2} = +1$. This fact can be seen by using Gribov's form of Green function of exchanged photon with momentum q:

$$\frac{g_{\mu\nu}}{q^2} = \frac{1}{q^2}[g_{\mu\nu\perp} + \frac{2}{s}[p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu}]], \quad (3)$$

which results in form of the amplitude

$$M^{(12)} = -i\frac{4\pi\alpha}{q^2} g_{\mu\nu}(q) J^{1\mu}(q) J^{2\nu}(-q) = \frac{8\pi\alpha i s}{-q^2}\Phi_1\Phi_2, \quad (4)$$

where $J^{1,2}$ — currents associated with blocks 1 and 2 of Feynman diagram (Fig. 1) and their light-cone projections (LP) defined as

$$\Phi_1 = \frac{1}{s} J^{1,\mu} p_{2\mu}, \qquad \Phi_2 = \frac{1}{s} J^{2,\nu} p_{1\nu}. \quad (5)$$

The LP factors Φ^i do not depend on s in the limit $s \to \infty$.

At this stage, we introduce Sudakov's parameterization of 4-momenta

$$p_{i1} = \alpha_i p_2 + x_i p_1 + p_{\perp i1}, \qquad \sum x_i = 1, \qquad \alpha_i = \frac{\vec{p}^{\,2}_{i1}}{s x_i},$$
$$p_{j2} = y_j p_2 + \beta_j p_1 + p_{\perp j2}, \qquad \sum y_j = 1, \qquad \beta_j = \frac{\vec{p}^{\,2}_{j2}}{s y_j},$$
$$q = \alpha p_2 + \beta p_1 + q_\perp, \qquad q^2 \approx -\vec{q}^{\,2}, \qquad p^2_{\perp j} = -\vec{p}^{\,2}_j,$$
$$\sum \vec{p}_{i1} = \vec{q}, \qquad \sum \vec{p}_{i2} = -\vec{q}. \tag{6}$$

Sudakov's longitudinal parameters α, β of the exchanged photon with momentum q are related with jet's invariant mass squares

$$s_1 = (q + p_1)^2 \approx -\vec{q}^{\,2} + s\alpha, \qquad s_2 = (-q + p_2)^2 \approx -\vec{q}^{\,2} - s\beta. \tag{7}$$

Here we use the on-mass shell condition $p^2_{i1} = p^2_{j2} = 0$, conservation law, and introduce the Euclidean two-dimensional vectors $(p_{i\perp} p_{1,2}) = 0$.

We should like to note that the current conservation condition $J^{1\mu} q_\mu = J^{2\nu} q_\nu = 0$ leads to

$$\frac{1}{s} p_2^\mu J^1_\mu = -\frac{1}{s\alpha} q_\perp^\mu J^{1\mu}, \quad \frac{1}{s} p_1^\nu J^2_\nu = -\frac{1}{s\beta} q_\perp^\mu J^{2\mu}. \tag{8}$$

This property of turning to zero of matrix elements at small $\vec{q}$ we use as an important check of calculations (see (4)).

The differential cross section can be written in terms of the Cheng–Wu impact factors [3]:

$$\mathrm{d}\sigma^{(12)} = \frac{\alpha^2}{\pi^2} \frac{\mathrm{d}^2 \vec{q}}{(\vec{q}^{\,2})^2} \int \mathrm{d}\tau_1^{(\lambda_1)} \int \mathrm{d}\tau_2^{(\lambda_2)},$$
$$\int \mathrm{d}\tau_i^{(\lambda_i)} = \int |\Phi_i^{(\lambda_i)}(q)|^2 \mathrm{d}\Gamma_i, \quad i = 1, 2, \tag{9}$$

with

$$\mathrm{d}\Gamma_1 = (2\pi)^4 \int \mathrm{d}s_1 \delta(p_1 + q - \sum p_{i1}) \prod \frac{\mathrm{d}^3 p_{i1}}{2\varepsilon_{i1}(2\pi)^3},$$
$$\mathrm{d}\Gamma_2 = (2\pi)^4 \int \mathrm{d}s_2 \delta(p_2 - q - \sum p_{i2}) \prod \frac{\mathrm{d}^3 p_{i2}}{2\varepsilon_{i2}(2\pi)^3}. \tag{10}$$

Impact factors $\int \mathrm{d}\tau_i$ do not depend on s. For the cases when a jet consists of one, two or three particles we have

$$\int \mathrm{d}\Gamma_1^{(1)} = 2\pi,$$

$$\mathrm{d}\Gamma_1^{(2)} = \frac{\mathrm{d}^2\vec{p}_{11}\mathrm{d}x_1}{2(2\pi)^2 x_1 x_2}, \qquad \vec{p}_{11}+\vec{p}_{21}=\vec{q}, \qquad x_1+x_2=1,$$

$$\mathrm{d}\Gamma_1^{(3)} = \frac{\mathrm{d}^2\vec{p}_{11}\mathrm{d}^2\vec{p}_{21}\mathrm{d}x_1\mathrm{d}x_2}{4(2\pi)^5 x_1 x_2 x_3}, \qquad x_1+x_2+x_3=1, \qquad \vec{p}_{11}+\vec{p}_{21}+\vec{p}_{31}=\vec{q}. \quad (11)$$

For conversion of the initial photon with momentum p_1 and chirality λ to the charged fermion–antifermion pair

$$\gamma^*(q)+\gamma(p_1,\lambda)\to e^+(q_+)+e^-(q_-,\sigma) \quad (12)$$

we accept the description of chiral states of photon and lepton, developed in [4]:

$$\hat{e}^\lambda(p_1) = N_\gamma(\hat{q}_-\hat{q}_+\hat{p}_1\omega_{-\lambda} - \hat{p}_1\hat{q}_-\hat{q}_+\omega_\lambda), \qquad (e^\lambda)^2=0, \qquad (e^\lambda e^{-\lambda})=-1,$$

$$N_\gamma^2 = \frac{2}{s_1\chi_+\chi_-}, \quad s_1=2q_+q_-, \quad \chi_\pm = 2p_1q_\pm, \quad \omega_\sigma = \frac{1+\sigma\gamma_5}{2}, \quad \sigma=\pm 1. \quad (13)$$

Chiral states of fermions are defined as $u^\sigma = \omega_{-\sigma}u, v^\sigma = \omega_\sigma v$. Hereafter, we will imply that chiral states of the subprocess (12) are defined as amplitudes with a definite chiral state $\Phi^{\lambda\sigma}$ of the initial photon (λ) and one electron (σ) from pair. The LP factor of the photon $\Phi^{\lambda\sigma}$ in the Born approximation has the form

$$\Phi_B^{\gamma,++} = N_\gamma f_0 \bar{u}(q_-)\omega_-\hat{q}_+\hat{q}\frac{\hat{p}_2}{s}\omega_+ v(q_+), \quad (14)$$

$$\Phi_B^{\gamma,+-} = N_\gamma f_0 \bar{u}(q_-)\frac{\hat{p}_2}{s}\hat{q}\hat{q}_-\omega_- v(q_+), \qquad f_0 = -i\sqrt{4\pi\alpha}. \quad (15)$$

Note that in the combination $\hat{p}_2\hat{q} = \hat{p}_2\hat{q}_\perp$ we can consider 4-vector q as a two-dimensional one $q_\mu = q_{\perp\mu}$.

The property of the LP factors $\Phi_{1,2}(\vec{q})\to 0$ at $|\vec{q}|\to 0$ is the consequence of gauge invariance, as we have noted above.

The relevant impact factors are

$$\int \mathrm{d}\tau_B^{\gamma,+\pm} = \frac{\alpha}{\pi}\int \frac{d^2\vec{q}_- dx_-}{x_+x_-}\frac{\vec{q}^{\,2}x_\pm^2}{\chi_+\chi_-},$$

$$\chi_\pm = \frac{\vec{q}_\pm^{\,2}}{x_\pm}, \quad x_+ + x_- = 1, \quad \vec{q}_+ + \vec{q}_- = \vec{q}. \quad (16)$$

The LP amplitudes $\Phi_B^{\gamma,-\pm}$ and the corresponding impact factors can be obtained by application of the space reflection operator.

Let us consider the pair production process by photon on electron in the case of definite chiral states of all the particles

$$\gamma(p_1,\lambda=+)+e^-(p_2,\eta)\to e^+(q_+,\mp)+e^-(q_-,\pm)+e^-(p_2',\eta). \quad (17)$$

Using the impact factor of electron-spectator in the lowest order of perturbation theory (PT) with a single-particle jet $e^-(p_2,\eta)+\gamma^*(-q)\to e^-(p_2')$

$$\Phi_2^\eta=\bar u(p_2')\frac{\hat p_1}{s}\omega_\eta u(p_2),\qquad |\Phi_2^\eta|^2=1,\qquad \int d\tau_2=2\pi, \tag{18}$$

we obtain the cross section of pair photoproduction on electron with definite chiral states of the initial photon and positron from the pair (it does not depend on the chiral state of the spectator)

$$\frac{\mathrm{d}\sigma_{B,\eta}^{\gamma,++}}{\mathrm{d}\Gamma}=\frac{\mathrm{d}\sigma_{B,\eta}^{\gamma,--}}{\mathrm{d}\Gamma}=\frac{2x_+^2\alpha^3}{\pi^2\vec q^{\,2}\chi_+\chi_-},\qquad \frac{\mathrm{d}\sigma_{B,\eta}^{\gamma,+-}}{\mathrm{d}\Gamma}=\frac{\mathrm{d}\sigma_{B,\eta}^{\gamma,-+}}{\mathrm{d}\Gamma}=\frac{2x_-^2\alpha^3}{\pi^2\vec q^2\chi_+\chi_-},$$

$$\mathrm{d}\Gamma=\frac{\mathrm{d}^2\vec q\,\mathrm{d}^2\vec q_-\,\mathrm{d}x_-}{x_+x_-}. \tag{19}$$

Also, we put here for consideration the process of single-photon emission at high-energy electrons peripherical scattering

$$e^-(p_2,\eta)+e^-(p_1,\sigma=+)\to e^-(p_1',+)+\gamma(k_1,\lambda=\pm)+e^-(p_2',\eta) \tag{20}$$

with definite chiral states

$$\hat e^\lambda(k_1)=N_1(\hat p_1'\hat p_1\hat k_1\omega_{-\lambda}-\hat k_1\hat p_1'\hat p_1\omega_\lambda),$$

$$N_1^2=\frac{2}{u\chi\chi'},\qquad u=2p_1p_1',\qquad \chi=2p_1k_1,\qquad \chi'=2p_1'k_1. \tag{21}$$

The LP factor of the electron $\Phi^{e,\sigma\lambda}$ is

$$\Phi_B^{e,++}=N_1f_0\bar u(p_1')\omega_-\hat p_1\hat q\frac{\hat p_2}{s}\omega_+u(p_1),\quad \Phi_B^{e,+-}=-N_1f_0\bar u(p_1')\frac{\hat p_2}{s}\hat q\hat p_1'\omega_+u(p_1). \tag{22}$$

The corresponding impact factors are

$$\int\mathrm{d}\tau_B^{e,++}=\frac{\alpha}{\pi}\int\frac{\mathrm{d}^2\vec k_1\,\mathrm{d}x_1}{x_1x'}\frac{\vec q^{\,2}}{\chi\chi'},\qquad \int\mathrm{d}\tau_B^{e,+-}=\frac{\alpha}{\pi}\int\frac{\mathrm{d}^2\vec k_1\,\mathrm{d}x_1}{x_1x'}\frac{\vec q^{\,2}(x')^2}{\chi\chi'}, \tag{23}$$

where x_1,x' are the energy fractions of photon and the scattered electron from the jet. From the conservation law and on-mass shell conditions we have

$$x_1+x'=1,\quad \vec p_1{}'+\vec k_1=\vec q,\quad \chi=\frac{\vec k_1^2}{x_1},\quad \chi'=\frac{1}{x_1x'}(x_1\vec p-x'\vec k_1)^2. \tag{24}$$

The differential cross sections have the form

$$\frac{\mathrm{d}\sigma_{B,\eta}^{e,++}}{\mathrm{d}\Gamma}=\frac{\mathrm{d}\sigma_{B,\eta}^{e,--}}{\mathrm{d}\Gamma}=\frac{2\alpha^3}{\vec q^{\,2}\pi^2\chi\chi'},\qquad \frac{\mathrm{d}\sigma_{B,\eta}^{e,+-}}{\mathrm{d}\Gamma}=\frac{\mathrm{d}\sigma_{B,\eta}^{e,-+}}{\mathrm{d}\Gamma}=\frac{2\alpha^3(x')^2}{\vec q^2\pi^2\chi\chi'},$$

$$\mathrm{d}\Gamma = \frac{\mathrm{d}^2\vec{k}_1\,\mathrm{d}x_1\mathrm{d}^2\vec{q}}{x_1 x'}. \tag{25}$$

Our paper is organized in the following way. In Sec. 2 we consider the virtual (in the one-loop approximation) and soft real photon emission contribution to the photon impact factor. In Sec. 3 the similar calculations are presented for the electron impact factor. In Sects. 4 and 5 we consider the emission of an additional hard photon in collinear and noncollinear kinematics. Some general remarks are given in Conclusion. We discuss in particular the validity of the structure function approach in the leading and next-to-leading approximations. The relevant one-loop integrals are listed in Appendix A. Appendix B contains the explicit expressions for nonleading contributions arizing from virtual and soft real photon emission. These nonleading contributions expressed in terms of K-factor turn out to be the quantities of an order of unity for typical experimental conditions.

2. PHOTON IMPACT FACTOR: VIRTUAL AND SOFT PHOTON CONTRIBUTION

All diagrams (see Figs. 3 and 4) we can divide into several types, some of which (Figs. 3, a, d, and 4, e, h) can be obtained by simple exchanges of chirality and 4-momenta of particles:

$$Re[\Phi^{\gamma,++}_{+,i}(\Phi^{\gamma,++}_{\mathrm{Born}})^*] = Re[\Phi^{\gamma,+-}_{-,i}(\Phi^{\gamma,+-}_{\mathrm{Born}})^*(q_- \to q_+, q_+ \to q_-)], \tag{26}$$

where $i = \Sigma V, V, B$ — the self-energy, vertex, and box-type Feynman diagram contribution.

Here, the low subscript describes the absorbtion of virtual photon by electron (–) (Fig. 3, a–d) or positron (+) line (Fig. 4, e–h).

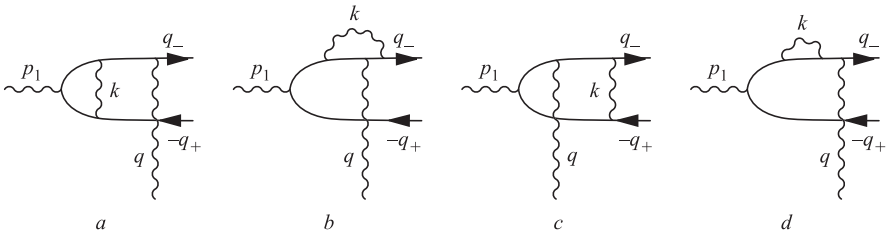

Fig. 3. Photon impact factor diagram (I)

One class of RC to electron impact factor consists of the renormalized electron mass operator and the vertex function with only one off-mass shell electron or positron (see Figs. 3, a, d and 4, f, g). Its contribution can be written in the form [5]

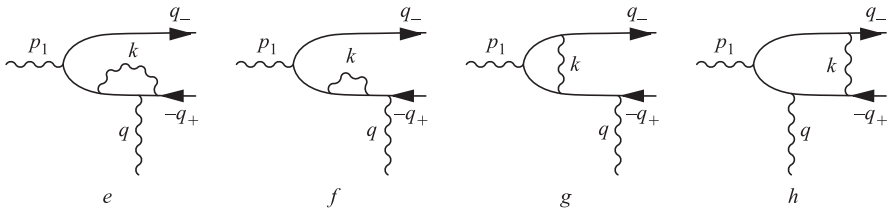

Fig. 4. Photon impact factor diagram (II)

$$\Phi^{\gamma,\lambda\sigma}_{-,\Sigma V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\bar{u}(q_-)\frac{\hat{p}_2}{s}\frac{\hat{p}_1-\hat{q}_+}{-\chi_+}\times$$
$$\times\left[2(\frac{3}{2}-l_l+\frac{1}{2}l_+)\hat{e}^\lambda+\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{\gamma^\mu(-\hat{q}_++\hat{p}_1-\hat{k})\hat{e}^\lambda(-\hat{q}_+-\hat{k})\gamma^\mu}{(0)(\bar{2})(q)}\right]\omega^\sigma v(q_+),$$
$$\Phi^{\gamma,\lambda\sigma}_{+,\Sigma V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\bar{u}(q_-)\left[2(\frac{3}{2}-l_l+\frac{1}{2}l_-)\hat{e}^\lambda+\right.$$
$$\left.+\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{\gamma^\mu(\hat{q}_--\hat{k})\hat{e}^\lambda(\hat{q}_--\hat{p}_1-\hat{k})\gamma^\mu}{(0)(2)(\bar{q})}\right]\times$$
$$\times\frac{\hat{q}_--\hat{p}_1}{-\chi_-}\frac{\hat{p}_2}{s}\omega^\sigma v(q_+),\qquad l_l=\ln\frac{m^2}{\lambda^2},\quad l_\pm=\ln\frac{\chi_\pm}{m^2},\quad(27)$$

with the denominators $(0),(2),(\bar{2}),(q),(\bar{q})$ are defined below (see Appendix A). After integration we obtain

$$\Phi^{\gamma,+-}_{-,\Sigma V} = -f_0N_\gamma\frac{\alpha}{2\pi}(l_+-\frac{1}{2})\bar{u}(q_-)\frac{\hat{p}_2}{s}\hat{p}_1\hat{q}_-\omega_+v(q_+),$$
$$\Phi^{\gamma,++}_{+,\Sigma V} = f_0N_\gamma\frac{\alpha}{2\pi}(l_--\frac{1}{2})\bar{u}(q_-)\hat{q}_+\hat{p}_1\frac{\hat{p}_2}{s}\omega_-v(q_+).\qquad(28)$$

After multiplying the relevant Born amplitude we obtain [6]

$$2\Phi^{\gamma,+\pm}_{\mp,\Sigma V}(\Phi^{\gamma,+\pm}_{\mathrm{Born}})^* = \frac{8\alpha^2}{\chi_+\chi_-}x_\pm(\vec{q}\vec{q}_\pm)(l_\mp-\frac{1}{2}).\qquad(29)$$

We should like to note that the ΣV contribution does not satisfy the gauge condition (turning to zero as $\vec{q}^2$ at small $\vec{q}$). We will see later that the total sum satisfies the gauge condition.

The contribution of the vertex functions with a virtual photon can be written in the form:

$$\Phi^{\gamma,\lambda\sigma}_{-,V} =$$
$$= \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{\bar{u}(q_-)\gamma_\mu(\hat{q}_- - \hat{k})\frac{\hat{p}_2}{s}(\hat{q}_- - \hat{q} - \hat{k})\gamma_\mu(\hat{p}_1 - \hat{q}_+)\hat{e}^\lambda\omega^\sigma v(q_+)}{(0)(2)(q)(-\chi_+)},$$

$$\Phi^{\gamma,\lambda\sigma}_{+,V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}$$
$$\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{\bar{u}(q_-)\hat{e}^\lambda(\hat{q}_- - \hat{p}_1)\gamma_\mu(-\hat{q}_+ + \hat{q} - \hat{k})\frac{\hat{p}_2}{s}(-\hat{q}_+ - \hat{k})\gamma_\mu\omega^\sigma v(q_+)}{(0)(\bar{2})(\bar{q})(-\chi_-)}. \quad (30)$$

Using the list of integrals (see Appendix A for designations) we obtain

$$2\Phi^{+-}_{-,V}(\Phi^{+-}_B)^* = -2|\Phi^{+-}_B|^2\frac{\alpha}{2\pi}\left[-\frac{1}{2}L - \frac{1}{4} - \frac{\chi_+ + 2\vec{q}^{\,2}}{2a}l_+ + \frac{3\vec{q}^{\,2}}{2a}l_q - \vec{q}^{\,2}J_{02q}\right],$$
$$2\Phi^{++}_{+,V}(\Phi^{++}_B)^* = -2|\Phi^{++}_B|^2\frac{\alpha}{2\pi}\left[-\frac{1}{2}L - \frac{1}{4} - \frac{\chi_- + 2\vec{q}^{\,2}}{2\tilde{a}}l_- + \frac{3\vec{q}^{\,2}}{2\tilde{a}}l_q - \vec{q}^{\,2}J_{0\bar{2}\bar{q}}\right],$$
$$a = \chi_+ - \vec{q}^{\,2}, \quad \tilde{a} = \chi_- - \vec{q}^{\,2}. \quad (31)$$

The other contributions $\Phi^{++}_{-,V}(\Phi^{++}_B)^*$, $\Phi^{+-}_{+,V}(\Phi^{+-}_B)^*$, $\Phi^{\gamma,+-}_{+,\Sigma V}(\Phi^{+-}_B)^*$, $\Phi^{\gamma,++}_{-,\Sigma V}(\Phi^{++}_B)^*$ are equal to zero. We remind that we work in the framework of the unrenormalized field theory. The regularization procedure consists in the replacement of ultraviolet cut-off logarithm $L = \ln\dfrac{\Lambda^2}{m^2}$ as $L \to 2l_l - \dfrac{9}{2}$ (see [5]).

The most complicated case is the calculation of the box-type contribution. It can be written down in the form (see Fig. 3, c and Fig. 4, h)

$$\Phi^{\gamma,\lambda\sigma}_{-,box} =$$
$$= \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{\bar{u}(q_-)\gamma^\mu(\hat{q}_- - \hat{k})\frac{\hat{p}_2}{s}(\hat{q}_- - \hat{q} - \hat{k})\hat{e}^\lambda(-\hat{q}_+ - \hat{k})\gamma^\mu\omega^\sigma v(q_+)}{(0)(\bar{2})(2)(q)},$$

$$\Phi^{\gamma,\lambda\sigma}_{+,box} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}$$
$$\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{\bar{u}(q_-)\gamma^\mu(\hat{q}_- - \hat{k})\hat{e}^\lambda(\hat{q}_- - \hat{p}_1 - \hat{k})\frac{\hat{p}_2}{s}(-\hat{q}_+ - \hat{k})\gamma^\mu\omega^\sigma v(q_+)}{(0)(\bar{2})(2)(\bar{q})}. \quad (32)$$

All the details about loop calculations and relevant integrals can be found in Appendix A. It is worth mentioning that in the case of the box-type contribution both chiral amplitudes $\lambda = +1$, $\sigma = \pm 1$ are nonzero.

An additional real soft photon emission contribution to the LP factor has the standard form

$$\Phi_{soft}^{\gamma,\lambda\sigma\eta} = \Phi_B^{\gamma,\lambda\sigma}\sqrt{4\pi\alpha}(\frac{q_-}{q_-k} - \frac{q_+}{q_+k})e(k)^\eta. \tag{33}$$

The corresponding contribution to the impact factor is

$$\int \frac{\mathrm{d}^3k}{16\omega\pi^3}\sum_\eta |\Phi_{\mathrm{soft}}^{\gamma,\lambda\sigma\eta}|^2|_{\omega<\Delta\varepsilon\ll\varepsilon_\gamma}. \tag{34}$$

Here ε_γ is the energy of the initial electron in c.m.s. The result is

$$\mathrm{d}\tau_{soft}^{\gamma,\lambda\sigma} = \frac{\alpha}{\pi}\mathrm{d}\tau_B^{\gamma,\lambda\sigma}\left[(l_s-1)(l_l+\ln\frac{(\Delta)^2}{x_+x_-}) + \frac{1}{2}l_s^2 - \frac{1}{2}\ln^2\frac{x_+}{x_-} - \frac{\pi^2}{6}\right],$$
$$\Delta = \frac{\Delta\varepsilon}{\varepsilon_\gamma}, \quad l_s = \ln\frac{s_1}{m^2}. \tag{35}$$

We use here the smallness of the angle between 3-momenta of pair components in the center-of-mass frame. Smallness of emission angles provides the possibility to perform angular integration in (34) in frame S_0 which coinside with c.m.s. frame [7].

After summing all contributions (27), (30), (32) and adding the soft photon contribution we explicitly see the cancellation of an auxiliary parameter λ and squared large logarithm

$$2[\mathrm{d}\tau_{\pm,\mathrm{box}}^{\gamma,+\pm} + \mathrm{d}\tau_{\mp,\Sigma V}^{\gamma,+\pm} + \mathrm{d}\tau_{\mp,V}^{\gamma,+\pm}] + \mathrm{d}\tau_{\mathrm{soft}}^{\gamma,+\pm} =$$
$$= \frac{\alpha}{2\pi}\mathrm{d}\tau_B^{\gamma\pm}[(l_s-1)(4\ln\Delta + 3 - 2\ln(x_+x_-)) + K_{SV}^{\gamma,\pm}]. \tag{36}$$

Here, we can see that leading logarithm contribution (containing the factor (l_s-1)) is proportional to the Born cross section, so our calculation is in agreement with the structure function approach predictions, namely the leading logarithm contribution is exactly the Δ part of the evolution equation kernel (see (VI)). All nonleading terms are gathered in the so-called K-factor.

Due ro gauge-invariance the right part of (36) including K-factor turns to zero as $\vec{q}^{\,2} \to 0$. This fact provides an important check of our calculation.

The $K_{SV}^{\gamma,\pm}$ is presented in the analytical form in Appendix B.

The contribution from emission of hard photon which eliminate the Δ-dependence can be written as a sum of two parts, one from collinear and the other from noncollinear kinematics. It will be considered below.

3. ELECTRON IMPACT FACTOR: VIRTUAL AND SOFT PHOTON CONTRIBUTION

In the same way we calculate the electron impact factor. All diagrams (see Figs. 5 and 6) are divided into six types, the contribution of three of them to LP (Fig. 5, c, d and Fig. 6, f, h) can be obtained by a simple exchange

$$Re[\Phi^{e,+\mp}_{i,\mathrm{contr}}(\Phi^{e,+\mp}_{\mathrm{Born}})^*] = Re[\Phi^{e,+\pm}_{f,\mathrm{contr}}(\Phi^{e,+\pm}_{\mathrm{Born}})^*(p_1 \to -p_1', p_1' \to -p_1)], \quad (37)$$

where the low subscripts correspond to interaction of virtual photon with the initial (i) or scattered (f) electron, and $\mathrm{contr} = \Sigma V, V, \mathrm{box}$ — the self energy, vertex, and box-type Feynman diagram contributions.

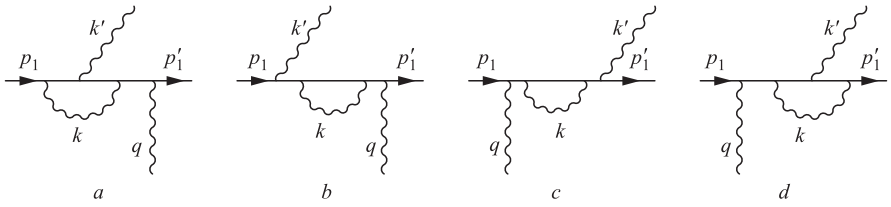

Fig. 5. Electron impact factor diagrams (I)

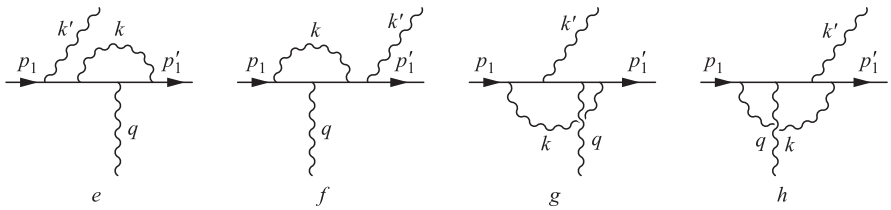

Fig. 6. Electron impact factor diagrams (II)

We take into account the contributions from self-energy (Fig. 5, a, b), vertex (Fig. 6, e) and box-type (Fig. 6, g) FD amplitudes

$$\Phi^{e,+\lambda}_{i,\Sigma V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2(-\chi)}\bar{u}(p_1')\frac{\hat{p}_2}{s}(\hat{p}_1 - \hat{k}_1)\times$$
$$\times\left[\int \frac{\mathrm{d}^4k}{i\pi^2}\frac{\gamma_\mu(\hat{p}_1 - \hat{k}_1 - \hat{k})\hat{e}^\lambda(\hat{p}_1 - \hat{k})\gamma_\mu}{(0)_e(1)_e(q)_e} + 2(\frac{1}{2}\ln\frac{\chi}{m^2} - 2\ln\frac{m}{\lambda} + \frac{3}{2})\hat{e}^\lambda\right]\omega_+ u(p_1), \quad (38)$$

$$\Phi^{e,+\lambda}_{i,V} =$$
$$\frac{(4\pi\alpha)^{3/2}}{16\pi^2(-\chi)}\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{\bar u(p_1')\gamma_\mu(\hat p_1'-\hat k)\frac{\hat p_2}{s}(\hat p_1'-\hat k-\hat q)\gamma_\mu(\hat p_1-\hat k_1)e^\lambda\omega_+u(p_1)}{(0)_e(2)_e(q)_e},$$
$$\Phi^{e,+\lambda}_{i,\mathrm{box}} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{\bar u(p_1')\gamma_\mu(\hat p_1'-\hat k)\frac{\hat p_2}{s}(\hat p_1'-\hat k-\hat q)\hat e^\lambda\gamma_\mu\omega_+u(p_1)}{(0)_e(1)_e(2)_e(q)_e}.$$

The first two contribute (see details in Appendix A)

$$2\Phi^{e,+-}_{i,V}(\Phi^{e,+-}_B)^* =$$
$$= -2|\Phi^{e,+-}_B|^2\frac{\alpha}{2\pi}\left[-\frac{1}{2}L-\frac{1}{4}-\vec q^{\,2}I_{(0)_e(2)_e(q)_e}+\frac{3\vec q^{\,2}}{2d}\ln\frac{\vec q^{\,2}}{m^2}-\frac{\chi+2\vec q^{\,2}}{2d}\ln\frac{\chi}{m^2}\right],$$
$$2\Phi^{e,+-}_{i,\Sigma V}(\Phi^{e,+-}_B)^* =$$
$$= \frac{8\alpha^2}{\chi\chi'}(\ln\frac{\chi}{m^2}-\frac{1}{2})x'[x'k_1-x\vec p_1\,']\vec q,\quad d=\chi-q^2,\quad l_{\chi'}=\ln\frac{\chi'}{m^2}-i\pi. \quad (39)$$

The contribution for other polarization can be obtained by substitution (37).

The soft photon contribution has the standard form (the energy of soft photon does not exceed $\Delta\varepsilon$)

$$\mathrm{d}\tau^{e,+\pm}_{soft} = \mathrm{d}\tau^{e,+\pm}_B\frac{\alpha}{\pi}[(\ln\frac{u}{m^2}-1)(2\ln\frac{m}{\lambda}+2\ln\Delta-\ln x')+\frac{1}{2}\ln^2\frac{u}{m^2}-\frac{1}{2}\ln^2x'-\frac{\pi^2}{6}], \quad (40)$$

where $\Delta=\dfrac{\Delta\varepsilon}{\varepsilon}$, ε is the energy of the initial electrons in c.m.s. frame. We can express the contribution to electron impact factors with a definite chiral state:

$$2(\mathrm{d}\tau^{e,+\pm}_{i,\Sigma V}+\mathrm{d}\tau^{e,+\pm}_{i,V}+\mathrm{d}\tau^{e,+\pm}_{i,\mathrm{box}}+\mathrm{d}\tau^{e,+\pm}_{f,\mathrm{box}})+\mathrm{d}\tau^{+\pm}_{e,\mathrm{soft}} =$$
$$= \mathrm{d}\tau^{e,+\pm}_B\frac{\alpha}{2\pi}[(\ln\frac{u}{m^2}-1)(4\ln\Delta+3-2\ln x')+K^{e,+\pm}_{SV}]. \quad (41)$$

Again here we can see the cancellation of the auxiliary «photon mass» parameter λ and agreement with the prediction of the structure function approach. The $K^{e,+\pm}_{SV}$ is presented in the analytical form in Appendix B.

4. COLLINEAR KINEMATICS OF ADDITIONAL HARD PHOTON EMISSION CONTRIBUTION

For appropriate consideration of RC to impact factors, we have to consider additional hard collinear photon emission. It is convenient to distinguish the collinear and noncollinear kinematics of emission of hard photon. For this aim

we introduce an auxiliary small parameter $\theta_0 \ll 1$. Collinear kinematics corresponds to the the case when the photon emission angle θ to the direction of motion of some charged particle (initial or final) do not exceed θ_0. Noncollinear kinematics corresponds to large emission angles $\theta > \theta_0$. Chiral amplitudes in noncollinear kinematics can be calculated using the methods developed by the CALCUL collaboration [4]. The contribution from collinear kinematics can be obtained using the quasi-real electron method developed in [8]. The total sum does not depend on the parameter θ_0. Cancellation of the θ_0 dependence is some check of our calculations. The nonleading contributions from additional hard photon emission essentially depend on the experimental setup. We do include it in the K-factors in the structure function picture of impact factors.

Using the quasi-real electron method [8] for contribution to photon impact factor in collinear kinematics we obtain

$$d\tau_{\rm coll}^{\gamma,+\lambda} = \frac{\alpha}{2\pi} \int\limits_{x_-(1+\Delta)}^{1} \frac{dz_-}{z_-} \left[\frac{1+\tilde{x}_-^2}{1-\tilde{x}_-} (l_s + r_- + \ln\theta_0^2 - 1) + 1 - \tilde{x}_- \right] d\tau_B^{\gamma,+\lambda}(\frac{q_-}{z_-}, q_+)$$

$$+ \frac{\alpha}{2\pi} \int\limits_{x_+(1+\Delta)}^{1} \frac{dz_+}{z_+} \left[\frac{1+\tilde{x}_+^2}{1-\tilde{x}_+} (l_s + r_+ + \ln\theta_0^2 - 1) + 1 - \tilde{x}_+ \right] d\tau_B^{\gamma,+\lambda}(q_-, \frac{q_+}{z_+}), \tag{42}$$

where the first term in square brackets corresponds to the emission of hard photon along electron and the second one — along positron from the pair created. Besides, we use the notation $\tilde{x}_\pm = \dfrac{x_\pm}{z_\pm}$ and

$$l_s = \ln\frac{2q_+q_-}{m^2} = \ln\frac{2E^2 x_+ x_- (1-c_\gamma)}{m^2}, \quad r_\pm = \ln\frac{x_\pm}{2x_\mp(1-c_\gamma)}, \tag{43}$$

the quantity c_γ is the cosine of the angle between pair momenta (the center-of-mass frame of colliding beams is implied). The «shifted» photon impact factor (conservation law reads as $p_1 + q = \dfrac{1}{z_-}q_- + \dfrac{1}{z_+}q_+$) is

$$\mathrm{d}\tau_B^{\gamma,+\pm}(\frac{q_+}{z_+}, \frac{q_-}{z_-}, x_\pm) = \frac{\alpha}{\pi}\frac{\tilde{x}_\pm^2 \vec{q}^{\,2}}{\vec{q}_+^{\,2}\vec{q}_-^{\,2}} \mathrm{d}^2 q_- \mathrm{d}\tilde{x}_-, \quad \tilde{x}_+ + \tilde{x}_- = 1, \quad \vec{q} = \frac{1}{z_-}\vec{q}_- + \frac{1}{z_+}\vec{q}_+. \tag{44}$$

A similar method can be applied to the problem of calculating a contribution from the collinear kinematics of the photon emission for the impact factor of

electron. The result is

$$d\tau_{\rm coll}^{e,+\lambda} = \frac{\alpha}{2\pi}\int\limits_0^{1-\Delta} dz_1\left[\frac{1+z_1^2}{1-z_1}(l_u+l_1+\ln\theta_0^2-1)+1-z_1\right]d\tau_B^{e,+\lambda}(p_1z_1,p_1')+$$

$$+\frac{\alpha}{2\pi}\int\limits_{x'(1+\Delta)}^{1}\frac{dz_2}{z_2}\left[\frac{1+(\frac{x'}{z_2})^2}{1-\frac{x'}{z_2}}(l_u+l_2+\ln\theta_0^2-1)+1-\frac{x'}{z_2}\right]{\rm d}\tau_B^{e,+\lambda}(p_1,\frac{1}{z_2}p_1'), \tag{45}$$

where the first term in the square brackets describes the emission from the initial and the emission from the scattered electron. Here we use the following designations:

$$l_u=\ln\frac{2p_1p_1'}{m^2}=\ln\frac{2E^2x'(1-c_e)}{m^2},\quad l_1=\ln\frac{z_1^2}{2x'(1-c_e)},\quad l_2=\ln\frac{x'}{2z_2^2(1-c_e)}, \tag{46}$$

c_e is the cosine of the angle between the initial and the scattered electrons momenta.

The «shifted» electron impact factor in the Born approximation (the conservation law reads as $z_1p_1+q=\dfrac{1}{z_2}p_1'+k_1$) is

$$d\tau_B^{e,+\pm}(p_1z_1,\frac{1}{z_2}p_1')=\frac{\alpha}{\pi}\frac{\vec{q}^{\,2}}{\chi\chi'}\eta^{\pm}\frac{z_2d^2kdx_1}{x_1x'},\quad \eta^+=z_1^2,\quad \eta^-=(\frac{x'}{z_2})^2,$$

$$\chi=\frac{z_1}{x}\vec{k}_1^2,\quad \chi'=\frac{z_2(\vec{p}_1'x_1-\vec{k}_1\frac{x'}{z_2})^2}{x_1x'},\quad x_1+\frac{x'}{z_2}=1,\quad \vec{q}=\vec{k}_1+\frac{1}{z_2}\vec{p}_1'. \tag{47}$$

The terms containing «large» logarithms l_s-1, l_u-1 will be included in lepton nonsinglet structure functions in the Drell–Yan form of impact factors, whereas the rest terms contribute to the relevant K-factors. So we can rewrite these formulae in the term of structure function approach (chiral indices are suppressed) for the electron impact factor

$${\rm d}\tau^{e,\rm coll}=\int\limits_0^1{\rm d}z_1\left[P_\theta(z_1)\frac{\alpha}{2\pi}(l_u-1)+\frac{\alpha}{\pi}K_{\rm coll}^{i,e}\right]{\rm d}\tau^{i,e}(z_1p_1,p_1')+{\rm d}\tau_{\rm comp}^{i,e}+$$

$$+\int\limits_0^1\frac{{\rm d}z_2}{z_2}\left[P_\theta(\frac{x'}{z_2})\frac{\alpha}{2\pi}(l_u-1)+\frac{\alpha}{\pi}K_{\rm coll}^{f,e}\right]{\rm d}\tau^{f,e}(p_1,\frac{p_1'}{z_2})+{\rm d}\tau_{\rm comp}^{f,e}, \tag{48}$$

and for the photon impact factor we have

$$\mathrm{d}\tau^{\gamma,\mathrm{coll}} = \int\limits_0^1 \frac{\mathrm{d}z_-}{z_-}\left[P_\theta(\frac{x_-}{z_-})\frac{\alpha}{2\pi}(l_s-1)+\frac{\alpha}{\pi}K_{\mathrm{coll}}^{-,\gamma}\right]\mathrm{d}\tau(\frac{q_-}{z_-},q_+)+\mathrm{d}\tau_{\mathrm{comp}}^{-,\gamma}+$$

$$+\int\limits_0^1 \frac{\mathrm{d}z_+}{z_+}\left[P_\theta(\frac{x_+}{z_+})\frac{\alpha}{2\pi}(l_s-1)+\frac{\alpha}{\pi}K_{\mathrm{coll}}^{+,\gamma}\right]\mathrm{d}\tau(q_-,\frac{q_+}{z_+})+\mathrm{d}\tau_{\mathrm{comp}}^{+,\gamma}, \quad (49)$$

where

$$P_\theta(z)=\frac{1+z^2}{1-z}\theta(1-z-\Delta), \quad (50)$$

and nonleading contributions equal (photon impact factor)

$$\mathrm{d}\tau_{\mathrm{comp}}^{-,\gamma} = \frac{\alpha}{2\pi}\int\limits_0^1 \frac{\mathrm{d}z_-}{z_-}P_\theta(\frac{x_-}{z_-})\mathrm{d}\tau_B^\gamma(\frac{q_-}{z_-},q_+)\ln\theta_0^2,$$

$$K_{\mathrm{coll}}^{-,\gamma} = \frac{1}{2}\int\limits_{x_-}^1 \frac{\mathrm{d}z_-}{z_-}P_\theta(\frac{x_-}{z_-})(r_-+1-\tilde{x}_-)\mathrm{d}\tau_B^\gamma(\frac{q_-}{z_-},q_+),$$

$$\mathrm{d}\tau_{\mathrm{comp}}^{+,\gamma} = \frac{\alpha}{2\pi}\int\limits_0^1 \frac{\mathrm{d}z_+}{z_+}P_\theta(\frac{x_+}{z_+})\mathrm{d}\tau_B^\gamma(q_-,\frac{q_+}{z_+})\ln\theta_0^2,$$

$$K_{\mathrm{coll}}^{+,\gamma} = \frac{1}{2}\int\limits_{x_+}^1 \frac{\mathrm{d}z_+}{z_+}P_\theta(\frac{x_+}{z_+})(r_+-1-\tilde{x}_+)\mathrm{d}\tau_B^\gamma(q_-,\frac{q_+}{z_+}), \quad (51)$$

and (electron impact factor)

$$\mathrm{d}\tau_{\mathrm{comp}}^{i,e} = \frac{\alpha}{2\pi}\int\limits_0^1 \mathrm{d}z_1 P_\theta(z_1)\mathrm{d}\tau_B^e(z_1p_1,p_1')\ln\theta_0^2,$$

$$K_{\mathrm{coll}}^{i,e} = \frac{1}{2}\int\limits_0^1 \mathrm{d}z_1 P_\theta(z_1)(l_1+1-z_1)\mathrm{d}\tau_B^e(z_1p_1,p_1'),$$

$$\mathrm{d}\tau_{\mathrm{comp}}^{f,e} = \frac{\alpha}{2\pi}\int\limits_{x'}^1 \frac{\mathrm{d}z_2}{z_2}P_\theta(\frac{x'}{z_2})\mathrm{d}\tau_B^e(p_1,\frac{p_1'}{z_2})\ln\theta_0^2,$$

$$K_{\mathrm{coll}}^{f,e} = \frac{1}{2}\int\limits_{x'}^1 \frac{\mathrm{d}z_2}{z_2}P_\theta(\frac{x'}{z_2})(l_2+1-\frac{x'}{z_2})\mathrm{d}\tau_B^e(p_1,\frac{p_1'}{z_2}). \quad (52)$$

Terms with $\ln\theta_0^2$ will be compensated with additional noncollinear photon emission.

5. NONCOLLINEAR HARD PHOTON EMISSION CONTRIBUTION

Contribution to the electron impact factor from the channel of the double Compton scattering process

$$e(p_1,\lambda_1)+\gamma^*(q)\to\gamma(k_1,\lambda_1)+\gamma(k_2,\lambda_2)+e(p_1',\lambda_e),$$
$$u=2p_1p_1',\quad \chi_i=2k_ip_1,\quad \chi_i'=2k_ip_1', \tag{53}$$

with the emission of both the final electrons outside the narrow cone $\theta>\theta_0$ can be calculated using the chiral amplitude technique [4]. The result is

$$\mathrm{d}\tau^{e\gamma\gamma}_{\lambda_e\lambda_1\lambda_2}=\frac{\alpha^3}{2\pi^2}|m^{\lambda_e}_{\lambda_1\lambda_2}|^2\frac{\mathrm{d}^2k_1\mathrm{d}^2k_2\mathrm{d}x_1\mathrm{d}x_2}{x_1x_2x'}|_{\theta_{1,2}>\theta_0},$$
$$x'=1-x_1-x_2,\quad \vec{q}=\vec{k}_1+\vec{k}_2+\vec{p}_1', \tag{54}$$

with

$$|m^+_{++}|^2=\frac{4\vec{q}^{\,2}u}{\chi_1\chi_2\chi_1'\chi_2'}\quad |m^+_{--}|^2=\frac{4(x')^2\vec{q}^{\,2}u}{\chi_1\chi_2\chi_1'\chi_2'},\quad |m^+_{-+}|^2=|m^+_{+-}|^2,$$
$$|m^+_{+-}|^2=\frac{4}{u^2\chi_1\chi_2\chi_1'\chi_2'}Tr\hat{p}_1'B^+_{+-}\omega_+\hat{p}_1\tilde{B}^+_{+-}, \tag{55}$$

with

$$B^+_{+-}=-\frac{1}{(p_1+q)^2}\hat{p}_1\hat{k}_1\hat{p}_1'\hat{p}_1\hat{k}_2(\hat{p}_1+\hat{q})\frac{\hat{p}_2}{s}-\frac{1}{(p_1'-q)^2}\frac{\hat{p}_2}{s}(\hat{p}_1'-\hat{q})\hat{k}_1\hat{p}_1'\hat{p}_1\hat{k}_2\hat{p}_1'+$$
$$+\hat{p}_1(\hat{p}_1'+\hat{k}_1)\frac{\hat{p}_2}{s}(\hat{p}_1-\hat{k}_2)\hat{p}_1'. \tag{56}$$

It was explicitly shown [9] that the quantity B^+_{+-} turns to zero as $|\vec{q}|\to 0$, whose property is the consequence of gauge invariance implement for the virtual photon with momentum q.

For the aim of checking the θ_0 dependence cancellation for the sum of collinear and noncollinear kinematic contributions we put below the limiting expressions for $|m^+_{ij}|^2$ for the emission of real photon kinematics

$$\theta_1>\theta_0,\quad \theta_1\to\theta_0,\quad \theta_2\gg\theta_0, \tag{57}$$

with θ_1 being the angle of emission of photon with momentum k_1 to the initial or final electron momenta. These limiting values are

$$
\begin{aligned}
(|m^+_{+-}|^2+|m^+_{++}|^2)_{\chi_1\to 0} &= \frac{4\vec{q}^2}{\chi_1}\frac{((x')^2+(1-x_1)^2)}{x_1(1-x_1)^2\chi_2\chi_2'},\\
(|m^+_{+-}|^2+|m^+_{++}|^2)_{\chi_1'\to 0} &= \frac{4\vec{q}^2}{\chi_1'}\frac{x'}{x_1\chi_2\chi_2'}[1+(1-x_2)^2],\\
(|m^+_{-+}|^2+|m^+_{--}|^2)_{\chi_1\to 0} &= \frac{4\vec{q}^2}{\chi_1}\frac{1}{x_1\chi_2\chi_2'}[(x')^2+(1-x_1)^2],\\
(|m^+_{-+}|^2+|m^+_{--}|^2)_{\chi_1'\to 0} &= \frac{4\vec{q}^2}{\chi_1'}\frac{(x')^3}{x_1(1-x_2)^2\chi_2\chi_2'}[1+(1-x_2)^2].
\end{aligned}
\tag{58}
$$

At small emission angles we can express all the invariants in terms of angular two-dimensional vectors in the plane transversal to the beams axis

$$
\vec{k}_1 = Ex_1\vec{\theta}_1, \quad \vec{p}_1' = Ex'\vec{\theta}', \quad \int\limits_{\theta_1>\theta_0}\frac{d^2k_1}{\chi_1} = \pi x_1\ln\frac{1}{\theta_0^2}+\ldots;
$$

$$
\int\frac{d^2k_1}{\chi_1'} = \frac{x_1}{x'}\int\limits_{|\vec{\theta}_1-\vec{\theta}'|>\theta_0}\frac{d^2\theta_1}{(\vec{\theta}_1-\vec{\theta}')^2} = \frac{\pi x_1}{x'}\ln\frac{1}{\theta_0^2}+\ldots. \tag{59}
$$

One can be convinced explicitly in the absence of θ_0 dependence in the sum of collinear kinematics and the summed on final-state hard photon chiral states of noncollinear contributions to electron impact factor

$$
\mathrm{d}\tau^e_{\mathrm{hard},nc} = \sum_{\lambda_1\lambda_2}(\mathrm{d}\tau^{e\gamma\gamma}_{+\lambda_1\lambda_2}+\mathrm{d}\tau^{i,e}_{\mathrm{comp}}+\mathrm{d}\tau^{f,e}_{\mathrm{comp}}). \tag{60}
$$

Its value however depends essentially on the experimental photon detection setup. Similar calculations of the photon impact factor in the noncollinear kinematics of the emission of hard photon

$$
\begin{gathered}
\gamma(k,\lambda_\gamma)+\gamma^*(q)\to e^-(q_-,\lambda_-)+e^+(q_+,-\lambda_+)+\gamma(k_1,\lambda_1),\\
s_1 = 2q_-q_+, \quad \chi_\pm = 2kq_\pm, \quad \chi_{1\pm} = 2k_1q_\pm
\end{gathered}
\tag{61}
$$

with chiral amplitudes defined as $m^{\lambda_\gamma}_{\lambda_1\lambda_-}$ gives

$$
\begin{gathered}
\mathrm{d}\tau^{e^+e^-\gamma}_{\lambda_\gamma\lambda_1\lambda_-} = \frac{\alpha^3}{2\pi^2}|m^{\lambda_\gamma}_{\lambda_1\lambda_-}|^2\frac{\mathrm{d}^2q_-\mathrm{d}^2q_+\mathrm{d}x_+\mathrm{d}x_-}{x_1x_+x_-},\\
x_1 = 1-x_+-x_-, \quad \vec{q} = \vec{q}_-+\vec{q}_++\vec{k}_1,
\end{gathered}
\tag{62}
$$

with

$$|m^+_{++}|^2 = \frac{4\vec{q}^{\,2}s_1x_+^2}{\chi_-\chi_{1-}\chi_+\chi_{1+}}, \quad |m^+_{+-}|^2 = \frac{4\vec{q}^{\,2}s_1x_-^2}{\chi_-\chi_{1-}\chi_+\chi_{1+}},$$
$$|m^+_{--}(k,k_1)|^2 = |m^+_{-+}(-k_1,-k)|^2,$$
$$|m^+_{-+}|^2 = \frac{4}{s_1^2\chi_-\chi_{1-}\chi_+\chi_{1+}} Tr\hat{q}_-A^+_{-+}\omega_+\hat{q}_+\tilde{A}^+_{-+} \tag{63}$$

and

$$A^+_{-+} = \frac{s_1}{(q_+-q)^2}\hat{k}\hat{q}_+\hat{k}_1(-\hat{q}_+ + \hat{q})\frac{\hat{p}_2}{s} - \frac{s_1}{(q_- - q)^2}\frac{\hat{p}_2}{s}(\hat{q}_- - \hat{q})\hat{k}\hat{q}_-\hat{k}_1$$
$$- \hat{q}_+(\hat{q}_- - \hat{k})\frac{\hat{p}_2}{s}(\hat{q}_+ + \hat{k}_1)\hat{q}_-. \tag{64}$$

Again, the proportionality $A^+_{-+} \sim |\vec{q}|$ at small $|\vec{q}|$ was demonstrated in [9].

To check the cancellation of the θ_0 dependence, we put the limiting values of $|m^+_{\lambda_-\lambda_1}|^2$ in the limit of emission angles close to the momentum directions of one of the charged particles

$$(|m^+_{++}|^2 + |m^+_{+-}|^2)_{\chi_{1-}\to 0} = \frac{4\vec{q}^{\,2}}{\chi_{1-}}\frac{(x_+)^2x_-}{x_1(1-x_+)^2\chi_+\chi_-}[x_-^2 + (1-x_+)^2];$$
$$(|m^+_{-+}|^2 + |m^+_{--}|^2)_{\chi_{1-}\to 0} = \frac{4\vec{q}^{\,2}}{\chi_{1-}}\frac{x_-}{x_1\chi_+\chi_-}[x_-^2 + (1-x_+)^2];$$
$$(|m^+_{-+}|^2 + |m^+_{--}|^2)_{\chi_{1+}\to 0} = \frac{4\vec{q}^{\,2}}{\chi_{1-}}\frac{x_+x_-}{x_1(1-x_-)^2\chi_+\chi_-}[x_+^2 + (1-x_-)^2];$$
$$(|m^+_{++}|^2 + |m^+_{+-}|^2)_{\chi_{1+}\to 0} = \frac{4\vec{q}^{\,2}}{\chi_{1-}}\frac{x_-}{x_1\chi_+\chi_-}[x_+^2 + (1-x_-)^2]. \tag{65}$$

One can be convinced in cancellation of θ_0 dependence in the sum of collinear kinematics and the summed on hard photon chiral states of noncollinear contributions to photon impact factor:

$$\mathrm{d}\tau^\gamma_{\mathrm{hard,nc}} = \mathrm{d}\tau^{e^+e^-\gamma}_{+\lambda,\lambda} + \mathrm{d}\tau^{-,\gamma}_{\mathrm{comp}} + \mathrm{d}\tau^{+,\gamma}_{\mathrm{comp}}. \tag{66}$$

The numerical value of $\mathrm{d}\tau^\gamma_{\mathrm{hard,nc}}$ as well depends on experimental setup and will not be considered here.

6. DISCUSSION AND CONCLUSIONS

We have obtained that the impact factors of both electron and photon in LLA can be written in the partonic form of the Drell–Yan process in terms of structure functions for any chiral states of the initial and final particles

$$
\begin{aligned}
&(\mathrm{d}\tau_B + \mathrm{d}\tau_{SV} + \sum \mathrm{d}\tau_{\mathrm{hard}})^{\gamma,\,\lambda\sigma}(q_-, q_+) = \\
&\quad = \int\limits_{x_-}^{1} \frac{\mathrm{d}z_-}{z_-} \int\limits_{x_+}^{1} \frac{\mathrm{d}z_+}{z_+} D(\frac{x_-}{z_-}, l_s) D(\frac{x_+}{z_+}, l_s) \mathrm{d}\tau_B^{\gamma,\,\lambda\sigma}(\frac{q_-}{z_-}, \frac{q_+}{z_+}) \times \\
&\qquad \times (1 + \frac{\alpha}{\pi}[K_{SV}^{\gamma,\,\lambda\sigma} + K_{\mathrm{coll}}^{-,\gamma} + K_{\mathrm{coll}}^{+,\gamma} + K_{\mathrm{ncol}}^{\gamma}]), \\
&\qquad (\mathrm{d}\tau_B + \mathrm{d}\tau_{SV} + \sum \mathrm{d}\tau_{\mathrm{hard}})^{e,\,\sigma\lambda}(p_1, p_1') = \\
&\quad = \int\limits_{0}^{1} \mathrm{d}z_1 \int\limits_{x'}^{1} \frac{\mathrm{d}z_2}{z_2} D(z_1, l_u) D(\frac{x'}{z_2}, l_u) \mathrm{d}\tau_B^{e,\,\sigma\lambda}(z_1 p_1, \frac{p_1'}{z_2}) \\
&\qquad\qquad \times (1 + \frac{\alpha}{\pi}[K_{SV}^{e,\,\sigma\lambda} + K_{\mathrm{coll}}^{i,e} + K_{\mathrm{coll}}^{f,e} + K_{\mathrm{ncoll}}^{e}]).
\end{aligned}
$$

Here we suppress the chirality indices, and D is nonsinglet structure function of fermion [6]

$$
\begin{aligned}
D(z, l) &= \delta(z-1) + \frac{\alpha}{2\pi}(l-1)P^{(1)}(z) + \ldots \qquad (67) \\
P^{(1)}(z) &= \left(\delta(1-z)(2\ln\Delta + \frac{3}{2}) + \theta(1-z-\Delta)\frac{1+z^2}{1-z} \right)_{\Delta\to 0}.
\end{aligned}
$$

The explicit form of K_{SV} is presented in Appendix B for the definite chiral states. The explicit form of K_{coll} is given above (see (51), (52)). The form of K_{ncol} (after proper regularization compensating the divergent terms in limit $\theta_0 \to 0$) strongly depends on the details of experiment-tagging the additional hard photon. The nonleading terms are free from infrared and collinear divergences (do not depend on λ, Δ and θ_0).

The terms containing the vector product arise due to the nonzero imaginary part of the LP amplitudes.

We can be convinced in validity of gauge-invariance check: the squares of chiral amplitudes in the Born approximation as well as one-loop corrected ones tend to zero as $|\vec{q}|^2$ at small $|\vec{q}|$.

The electron impact factor has also contributions from pair production channels [9] which are not considered here.

The accuracy of the formulae given above are determined by the omitted terms (2)

$$1 + \mathcal{O}\left(\frac{m^2}{s_1^2}, \frac{m^2}{u^2}, \left(\frac{\alpha}{\pi}\right)^2\right). \tag{68}$$

APPENDIX A

Here we put the asymptotic expressions for part of scalar, vector and tensor integrals corresponding to the absorption of virtual photon by electron from the pair created in $\gamma(p_1)\gamma^*(q)$ collisions.

We give first the scalar integrals with two, three, and four (different) denominators

$$\begin{aligned}
&(0) = k^2 - \lambda^2, \\
&(2) = (q_- - k)^2 - m^2 + i0 = k^2 - 2q_-k + i0, \\
&(\bar{2}) = (-q_+ - k)^2 - m^2 + i0, \\
&(q) = (p_1 - q_+ - k)^2 - m^2 + i0.
\end{aligned} \tag{69}$$

The loop momentum integrals with the denominator $(\bar{q}) = (q_- - p_1 - k)^2 - m^2$ instead of $(q) = (p_1 - q_+ - k)^2 - m^2$, including scalar, vector and tensor ones can be obtained from the ones listed below by means of the replacement (26)

$$q_- \to -q_+, \quad q_+ \to -q_-, \quad p_1 \to -p_1, \quad \chi_\pm \to \chi_\mp, \quad (2) \to (\bar{2}), \quad (q) \to (\bar{q}). \tag{70}$$

So we can restrict ourselves by consideration only the integrals with denominators (0), (2), $(\bar{2})$, (q).

We use the same conservation law, on-mass shell conditions and the kinematic invariants as for the photon impact factor (12, 13)

$$s_1 + \vec{q}^2 = \chi_+ + \chi_-. \tag{71}$$

Two denominator scalar integrals are defined as

$$I_{ij} = \int \frac{d^4k}{i\pi^2} \frac{1}{(i)(j)}.$$

The explicit expressions for them are

$$\begin{aligned}
&I_{02} = L + 1, \qquad &&I_{2q} = L - l_q + 1, \qquad &&I_{0q} = L - l_+ + 1, \\
&I_{0\bar{2}} = L + 1, \qquad &&I_{2\bar{2}} = L - L_s + 1, \qquad &&I_{\bar{2}q} = L - 1.
\end{aligned} \tag{72}$$

Here and below we use the notation

$$L=\ln\frac{\Lambda^2}{m^2},\quad l_\pm=\ln\frac{\chi_\pm}{m^2},\quad l_q=\ln\frac{\vec q^{\,2}}{m^2}, L_s=\ln\frac{s_1}{m^2}-i\pi=l_s-i\pi,$$
$$l_l=\ln\frac{m^2}{\lambda^2},\quad \mathrm{Li}2z=-\int\limits_0^z\frac{dx}{x}\ln(1-x). \tag{73}$$

Remind once more that we imply all the kinematic invariants to be greater than electron mass squared $s_1\sim\vec q^{\,2}\sim\chi_\pm\gg m^2$ and present below the asymptotic expressions systematically omitting the terms of order m^2/s_1 and similar ones.

The tree denominator scalar integrals $I_{ijk}=\int\frac{\mathrm{d}^4k}{i\pi^2(i)(j)(k)}$ are

$$I_{0\bar 2q}=-\frac{1}{2\chi_+}\Big[l_+^2+\frac{2\pi^2}{3}\Big],\qquad I_{02\bar 2}=\frac{1}{2s_1}\Big[l_s^2+2l_sl_l-\frac{4\pi^2}{3}-\mathrm{i}\pi(2l_s+2l_l)\Big],$$
$$I_{2\bar 2q}=-\frac{1}{2(s_1+\vec q^{\,2})}\Big[l_q^2-l_s^2+\pi^2+2\mathrm{i}\pi l_s\Big],$$
$$I_{02q}=\frac{1}{\chi_+-\vec q^{\,2}}\Big[l_q(l_q-l_+)+\frac{1}{2}(l_q-l_+)^2+2Li21-\frac{\chi_+}{\vec q^{\,2}}\Big]. \tag{74}$$

The four denominator integral $I_{02\bar 2q}=\int\frac{\mathrm{d}^4k}{\mathrm{i}\pi^2(0)(2)(\bar 2)(q)}$ has the form

$$I_{02\bar 2q}=\frac{1}{s_1\chi_+}\Big[l_q^2-2l_+l_s-l_sl_l+2\mathrm{Li}21+\frac{\vec q^{\,2}}{s_1}+$$
$$+\frac{\pi^2}{6}+\mathrm{i}\pi\big(2l_++l_l-2\ln(1+\frac{\vec q^{\,2}}{s_1})\big)\Big]. \tag{75}$$

Now we describe the vector integrals

$$I_r^\mu=\int\frac{\mathrm{d}^4kk^\mu}{r}=a_r^+q_+^\mu+a_r^-q_-^\mu+a_r^1p_1^\mu \tag{76}$$

with $r=(ij),(ijk),(ijkl)$, where $i,j,k,l=(0),(2),(\bar 2),(q)$.

For the vector integrals, with two denominators we have (we put only nonzero coefficients)

$$a_{2q}^-=a_{2q}^1=-a_{2q}^+=\frac{1}{2}\Big(L-l_q+\frac{1}{2}\Big),\quad a_{0q}^1=-a_{0q}^+=\frac{1}{2}\Big(L-l_++\frac{1}{2}\Big),$$
$$a_{2\bar 2}^-=-a_{2\bar 2}^+=\frac{1}{2}\Big(L-L_s+\frac{1}{2}\Big),\qquad a_{\bar 2q}^1=-\frac{1}{2}a_{2q}^+=\frac{1}{2}\Big(L-\frac{3}{2}\Big),$$
$$a_{0\bar 2}^-=\frac{1}{2}L-\frac{1}{4},\qquad a_{0\bar 2}^+=-\frac{1}{2}L+\frac{1}{4}, \tag{77}$$

and the coefficients for the vector integrals with three denominators are

$$
\begin{aligned}
&a^{-}_{02q} = \frac{1}{a}\Big(\chi_+ I_{02q} + \frac{2\chi_+}{a} l_+ - \frac{\vec{q}^{\,2} + \chi_+}{a} l_q\Big), && a^{+}_{02q} = -a^{1}_{02q} = \frac{1}{a}\Big(l_+ - l_q\Big),\\
&a^{1}_{0\bar{2}q} = \frac{1}{\chi_+}\Big(-l_+ + 2\Big), && a = \chi_+ - \vec{q}^{\,2},\\
&a^{+}_{0\bar{2}q} = -I_{0\bar{2}q} - \frac{1}{\chi_+} l_+, && a^{-}_{02\bar{2}} = -a^{+}_{02\bar{2}} = \frac{1}{s_1} L_s,\\
&a^{-}_{2\bar{2}q} = \frac{1}{c}\Big(L_s - l_q\Big), && a^{+}_{2\bar{2}q} = -I_{2\bar{2}q} + \frac{1}{c}\Big(L_s - l_q\Big),\\
&a^{1}_{2\bar{2}q} = \frac{s_1}{c} I_{2\bar{2}q} + \frac{1}{c}\Big(-l_q + 2\Big) - \frac{2s_1}{c^2}\Big(L_s - l_q\Big), && c = s_1 + \vec{q}^{\,2} = \chi_+ + \chi_-.
\end{aligned}
\tag{78}
$$

Finally, the coefficient of the vector integral with 4 denominators has the form

$$
\begin{aligned}
&a^{1} = \frac{s_1}{d}\Big(\chi_+ A + \chi_- B - s_1 C\Big), && a^{+} = \frac{\chi_-}{d}\Big(\chi_+ A - \chi_- B + s_1 C\Big),\\
&a^{-} = \frac{\chi_+}{d}\Big(-\chi_+ A + \chi_- B + s_1 C\Big), && d = 2 s_1 \chi_+ \chi_-,\\
&A = I_{2\bar{2}q} - I_{0\bar{2}q}, && B = I_{02q} - I_{2\bar{2}q},\\
&C = I_{02q} - I_{02\bar{2}} - \chi_+ I_{02\bar{2}q}.
\end{aligned}
\tag{79}
$$

We parameterized the second rank tensor integrals in the form

$$
\begin{aligned}
I^{\mu\nu}_r = \int \frac{\mathrm{d}^4 k}{\mathrm{i}\pi^2} \frac{k_\mu k_\nu}{r} = \Big[& a^g_r g + a^{11}_r p_1 p_1 + a^{++}_r q_+ q_+ + a^{--}_r q_- q_- + \\
& + a^{1+}_r (p_1 q_+ + q_+ p_1) + a^{1-}_r (p_1 q_- + q_- p_1) + a^{+-}_r (q_+ q_- + q_- q_+)\Big]_{\mu\nu}.
\end{aligned}
\tag{80}
$$

The coefficients for tensor integral with four denominators are (we suppressed the index $02\bar{2}q$)

$$
\begin{aligned}
&a^{1+} = \frac{1}{\chi_+}\Big(A_6 + A_7 - A_{10}\Big), && a^{+-} = \frac{1}{s_1}\Big(A_2 + A_6 - A_{10}\Big),\\
&a^{1-} = \frac{1}{\chi_-}\Big(A_2 + A_7 - A_{10}\Big), && a^{11} = \frac{1}{\chi_-}\Big(A_1 - s_1 a^{1+}\Big),\\
&a^{--} = \frac{1}{s_1}\Big(A_5 - \chi_+ a^{1-}\Big), && a^{++} = \frac{1}{s_1}\Big(A_3 - \chi_- a^{1+}\Big),\\
&a^{g} = \frac{1}{2}\Big(A_{10} - A_2 - \chi_+ a^{1+}\Big),
\end{aligned}
\tag{81}
$$

with

$$
\begin{aligned}
&A_1 = a^1_{2\bar{2}q} - a^1_{0\bar{2}q}, && A_6 = a^+_{02q} - a^+_{2\bar{2}q},\\
&A_2 = a^-_{2\bar{2}q}, && A_7 = a^1_{02q} - \chi_+ a^1,\\
&A_3 = a^+_{2\bar{2}q} - a^+_{0\bar{2}q}, && A_8 = a^-_{02q} - a^-_{02\bar{2}} - \chi_+ a^-,\\
&A_4 = a^1_{02q} - a^1_{2\bar{2}q}, && A_9 = a^+_{02q} - a^+_{02\bar{2}} - \chi_+ a^+,\\
&A_5 = a^-_{02q} - a^-_{2\bar{2}q}, && A_{10} = I_{2\bar{2}q}.
\end{aligned}
\tag{82}
$$

One can verify that the checking relations

$$
A_4 = \chi_+ a^{11} + s_1 a^{1-}, \quad A_8 = \chi_- a^{--} + \chi_+ a^{+-}, \quad A_9 = \chi_+ a^{++} + \chi_- a^{+-}
\tag{83}
$$

for the above coefficients (82) are fulfilled.

The coefficients entering into the tensor integral $I^{\mu\nu}_{02q}$ are

$$
\begin{aligned}
&a^g_{02q} = \frac{1}{4}L + \frac{3}{8} + \frac{\vec{q}^{\,2}}{4a} l_q - \frac{\chi_+}{a} l_+,\\
&a^{+-}_{02q} = -a^{1-}_{02q} = \frac{1}{2a}\Big[\frac{\chi_+}{a}(l_+ - l_q) - 1\Big],\\
&a^{++}_{02q} = a^{11}_{02q} = -a^{1+}_{02q} = \frac{1}{2a}(l_q - l_+),\\
&a^{--}_{02q} = \frac{1}{a^2}\Big[\chi_+^2 I_{02q} + \frac{3\chi_+^2}{a} l_+ + \frac{(\vec{q}^{\,2})^2 - 4\vec{q}^{\,2}\chi_+ - 3\chi_+^2}{2a} l_q + \frac{\vec{q}^{\,2} - 3\chi_+}{2}\Big].
\end{aligned}
\tag{84}
$$

The coefficients entering into the tensor integral $I^{\mu\nu}_{02\bar{2}}$ are

$$
a^g_{02\bar{2}} = \frac{1}{4}(L - L_s) + \frac{3}{8}, \quad a^{++}_{02\bar{2}} = a^{--}_{02\bar{2}} = \frac{1}{2s_1}(L_s - 1), \quad a^{+-}_{02\bar{2}} = -\frac{1}{2s_1},
\tag{85}
$$

and the coefficients for the tensor integral $I^{\mu\nu}_{0\bar{2}q}$ are

$$
\begin{aligned}
&a^g_{0\bar{2}q} = \frac{1}{4}(L - l_+) + \frac{3}{8}, && a^{1+}_{0\bar{2}q} = \frac{1}{\chi_+}\Big(l_+ - \frac{5}{2}\Big),\\
&a^{11}_{0\bar{2}q} = \frac{1}{2\chi_+}(-l_+ + 2), && a^{++}_{0\bar{2}q} = I_{0\bar{2}q} + \frac{1}{2\chi_+}(3l_+ - 1).
\end{aligned}
\tag{86}
$$

In the case of the tensor integral $I^{\mu\nu}_{2\bar{2}q}$ they have the form

$$
\begin{aligned}
&a^{g}_{2\bar{2}q} = \frac{1}{2}\Big[\frac{1}{2}L + \frac{3}{4} - \frac{s_1}{2c}L_s - \frac{\vec{q}^{\,2}}{2c}l_q\Big], \qquad a^{--}_{2\bar{2}q} = -\frac{1}{2c}(l_q - L_s),\\
&a^{++}_{2\bar{2}q} = I_{2\bar{2}q} + \frac{3}{2c}(l_q - L_s), \qquad\qquad a^{+-}_{2\bar{2}q} = \frac{1}{2c}(l_q - L_s),\\
&a^{1-}_{2\bar{2}q} = \frac{1}{c}\Big[-\frac{1}{2} + \frac{s_1}{2c}L_s - \frac{s_1}{2c}l_q\Big],\\
&a^{1+}_{2\bar{2}q} = \frac{1}{c}\Big[-\frac{5}{2} - s_1 I_{2\bar{2}q} + \frac{5s_1}{2c}L_s + \frac{2\vec{q}^{\,2} - 3s_1}{2c}l_q\Big],\\
&a^{11}_{2\bar{2}q} = \frac{1}{c^2}\Big[4s_1 + \vec{q}^{\,2} + s_1^2 I_{2\bar{2}q} - \frac{3s_1^2}{c}L_s + \frac{3s_1^2 - (\vec{q}^{\,2})^2 - 4s_1\vec{q}^{\,2}}{2c}l_q\Big].
\end{aligned}
\tag{87}
$$

The checking equations for the coefficients (87) could be obtained after multiplying $I^{\mu\nu}_{2\bar{2}q}$ by $2(q_+ + q_-)^\nu$ or $2p_1^\nu$, using the relations $2k\cdot(q_+ + q_-) = (\bar{2}) - (2)$, $2p_1\cdot k = (\bar{2}) - (q) - \chi_+$ and using the vector integrals (76). They have the form

$$
\begin{aligned}
&2a^{g}_{2\bar{2}q} + s_1 a^{--}_{2\bar{2}q} + c a^{1-}_{2\bar{2}q} + s_1 a^{+-}_{2\bar{2}q} = a^{-}_{2q} - a^{-}_{\bar{2}q},\\
&2a^{g}_{2\bar{2}q} + s_1 a^{++}_{2\bar{2}q} + c a^{1+}_{2\bar{2}q} + s_1 a^{+-} = a^{+}_{2q} - a^{+}_{\bar{2}q},\\
&c a^{11}_{2\bar{2}q} + s_1 a^{1+}_{2\bar{2}q} + s_1 a^{1-}_{2\bar{2}q} = a^{1}_{2q} - a^{1}_{\bar{2}q}.
\end{aligned}
\tag{88}
$$

Integrals for calculation of the electron impact factor with the denominators

$$
\begin{aligned}
&(0)_e = k^2 - \lambda^2, \qquad\qquad (1)_e = (p_1 - k)^2 - m^2 + \mathrm{i}0,\\
&(2)_e = (p_1' - k)^2 - m^2 + \mathrm{i}0,\\
&(q)_e = (p_1 - k_1 - k)^2 - m^2 + \mathrm{i}0
\end{aligned}
\tag{89}
$$

can be obtained from the cited above by the substitution

$$
\int \frac{d^4k}{i\pi^2}\,\frac{1,\,k,\,kk}{(0\,1\,2\,q)_e} = \mathcal{P}(q_- \to p_1', q_+ \to -p_1, p_1 \to -k_1, q \to q)\int \frac{d^4k}{i\pi^2}\,\frac{1,\,k,\,kk}{(0\,2\,\bar{2}\,q)}.
\tag{90}
$$

An additional set of relevant integrals for the electron impact factor can be obtained by the substitution (37).

APPENDIX B

The explicit expression for the photon K-factor are (the case of two different polarization):

$$
\frac{1}{2}K^{\gamma+-}_{SV} = -\frac{1}{2}\ln^2\frac{x_+}{x_-} - \frac{3}{4}l_{qs}^2 + l_{qs}l_{ps} + \frac{1}{2}l_{qs}l_{ms} + \frac{1}{4}l_{ms} + \frac{3}{4}l_{qs} + \frac{1}{2}l_{qs} - \frac{3}{4}
$$

$$+\frac{3}{2}\mathrm{Li21}+\frac{\vec{q}^{\,2}}{s_1}-\mathrm{Li21}-\frac{\chi_+}{\vec{q}^{\,2}}-\frac{1}{2}\mathrm{Li21}-\frac{\chi_-}{\vec{q}^{\,2}}+x_+\frac{\chi_-x_+-2\chi_+-2s_1}{4x_-^2s_1}+$$
$$\Big(\frac{\chi_-x_+}{\chi_+x_-}-\frac{(x_+\chi_--s_1)^2}{2x_-^2\chi_+^2}\Big)\Big(\frac{1}{2}l_{qs}^2-l_{qs}l_{ms}+\frac{\pi^2}{3}-\mathrm{Li21}+\frac{\vec{q}^{\,2}}{s_1}+\mathrm{Li21}-\frac{\chi_-}{\vec{q}^{\,2}}\Big)$$
$$-x_+\frac{(l_{ms}-l_{qs})}{x_-^2}\Big(\frac{a^2x_+-2s_1x_-\chi_-}{4\tilde{a}^2}+\frac{4\chi_--s_1-2x_+\chi_-}{2\tilde{a}}-\frac{x_+\chi_-^2}{2\tilde{a}\chi_+}+$$
$$\frac{2x_+\chi_--s_1}{2\chi_+x_+}\Big)+\frac{\pi}{2x_-^2\chi_+}[\vec{q}_+\vec{q}_-]_z\left[\frac{l_{ms}-\ln\left(1+\frac{\vec{q}^{\,2}}{s_1}\right)}{\chi_+}(s_1-x_+\chi_-+x_-\chi_+)\right.$$
$$\left.+\frac{s_1+3x_-\chi_+-x_+\chi_-}{c}\right]+\frac{l_{qs}}{2x_-^2}\Big(\frac{\chi_+^2+s_1^2+2x_+s_1\chi_+}{c^2}-\frac{2x_-s_1+\chi_+}{c}+$$
$$\frac{s_1^2}{\chi_+c}+x_+\frac{-2s_1+x_+\chi_-}{\chi_+}\Big)+\frac{\chi_+^2+s_1^2+2x_+s_1\chi_+}{2x_-^2s_1c}+\frac{\chi_+^2}{2s_1\vec{q}^{\,2}x_-^2}\Big(\frac{1}{2}x_+^2-1\Big)$$
$$\frac{\chi_+x_+}{4\tilde{a}s_1\vec{q}^{\,2}x_-^2}\Big(x_+\chi_+^2+\vec{q}^{\,2}(x_+\chi_--2\chi_+)\Big)+\frac{1}{\vec{q}^{\,2}x_-}\Big(\frac{1}{4}\chi_--\frac{\vec{q}^{\,2}}{2}+$$
$$\chi_+x_--\frac{1}{2}s_1x_--\frac{1}{4}x_+^2s_1+\frac{1}{4}x_+^2\chi_+\Big),\quad(91)$$

$$\frac{1}{2}K_{SV}^{\gamma++}=-\frac{1}{2}\ln^2\frac{x_+}{x_-}-\frac{3}{4}l_{qs}^2+l_{qs}l_{ms}+\frac{1}{2}l_{qs}l_{ps}+\frac{1}{4}(l_{ps}+l_{qs})-\mathrm{Li21}-\frac{\chi_-}{\vec{q}^{\,2}}$$
$$+\frac{3}{2}\mathrm{Li21}+\frac{\vec{q}^{\,2}}{s_1}-\frac{1}{2}\mathrm{Li21}-\frac{\chi_+}{\vec{q}^{\,2}}-1+\frac{2x_-x_+\chi_-\chi_+-(x_-\chi_+-s_1)^2}{2\chi_-^2x_+^2}$$
$$\Big(\frac{1}{2}l_{qs}^2-l_{qs}l_{ps}-\mathrm{Li21}+\frac{\vec{q}^{\,2}}{s_1}+\mathrm{Li21}-\frac{\chi_+}{\vec{q}^{\,2}}+\frac{\pi^2}{3}\Big)$$
$$-\frac{l_{ps}-l_{qs}}{x_+^2}\left[\frac{x_-(-2s_1\chi_+x_++x_-\tilde{a}^2)}{4a^2}+\frac{x_-(4x_+s_1\chi_+-x_+s_1^2-x_-\tilde{a}^2)}{2as_1}-\right.$$
$$\left.\frac{(s_1-x_-\chi_+)^2}{2\chi_-s_1}\right]+\frac{l_{qs}}{x_+^2}\left[\frac{2x_+s_1\chi_++\tilde{a}^2}{2c^2}+\frac{s_1^2}{2c\chi_-}-\frac{x_+\vec{q}^{\,2}+s_1x_-+\frac{1}{2}\chi_-}{c}-\right.$$
$$\left.\frac{x_-}{\chi_-}(s_1-\frac{1}{2}x_-\chi_+)-\frac{1}{2}-x_+x_-\right]+\pi\frac{[\vec{q}_+\vec{q}_-]_z}{2x_+^2}\left[\frac{x_+\chi_-+s_1-x_-\chi_+}{\chi_-^2}\right.$$

$$\left(l_{ps}-\ln\left(1+\frac{\vec{q}^{\,2}}{s_1}\right)\right)+\frac{2x_++1}{c}-\frac{x_-}{\chi_-}+\frac{s_1}{\chi_- c}\Bigg]-\frac{1}{x_+^2}\Bigg(x_-\frac{2s_1-x_-\vec{q}^{\,2}}{\tilde{a}}+$$
$$\frac{x_+x_-}{2}-\frac{2x_+s_1\chi_++(\chi_+-s_1)^2}{2cs_1}\Bigg)+\frac{1}{\vec{q}^{\,2}x_+^2}\Bigg(\frac{x_-^2s_1^2}{4a}+\frac{\vec{q}^{\,2}}{4}+\frac{5}{4}\chi_+$$
$$+\frac{3}{2}s_1x_+-s_1-x_+\chi_+-\frac{1}{2}x_+^2s_1\Bigg)-\frac{\chi_+^2}{2s_1x_+^2\vec{q}^{\,2}}, \quad (92)$$

$$a=\chi_+-\vec{q}^{\,2}, \quad \tilde{a}=\chi_--\vec{q}^{\,2}, \quad c=\chi_++\chi_-,$$
$$l_{qs}=\ln\frac{\vec{q}^{\,2}}{s_1}, \quad l_{ps}=\ln\frac{\chi_+}{s_1}, \quad l_{ms}=\ln\frac{\chi_-}{s_1}. \quad (93)$$

The explicit expression for electron K-factor is

$$K_{SV}^{e,+-}=-\ln^2x'-\frac{3}{2}\ln^2\frac{\vec{q}^{\,2}}{u}+2\ln\frac{\chi}{u}\ln\frac{\vec{q}^{\,2}}{u}+\ln\frac{\chi'}{u}\ln\frac{\vec{q}^{\,2}}{u}+\frac{1}{2}\ln\frac{\chi'}{\vec{q}^{\,2}}+3\ln\frac{\vec{q}^{\,2}}{u}$$
$$-2\mathrm{Li}21-\frac{\chi}{\vec{q}^{\,2}}+3\mathrm{Li}21-\frac{\vec{q}^{\,2}}{u}-\mathrm{Li}21+\frac{\chi'}{\vec{q}^{\,2}}-\frac{\pi^2}{4}-2$$
$$-\frac{1}{(x')^2(\chi')^2}\left(\frac{1}{2}\ln^2\frac{\vec{q}^{\,2}}{u}-\ln\frac{\vec{q}^{\,2}}{u}\ln\frac{\chi'}{u}-\mathrm{Li}21-\frac{\vec{q}^{\,2}}{u}+\mathrm{Li}21+\frac{\chi'}{\vec{q}^{\,2}}+\frac{\pi^2}{12}\right)$$
$$\times\left(d^2-2x'(-u\chi'+u^2+\chi\chi')+(x')^2u^2\right)$$
$$-\frac{1}{2(x')^2\vec{q}^{\,2}}\left(-u+\chi+2x'\chi-(x')^2u-3(x')^2\chi+\frac{u^2}{\tilde{d}}\right)+\frac{(\chi')^2x^2}{(x')^2\vec{q}^{\,2}u}\left(1+\frac{\vec{q}^{\,2}}{c}\right)$$
$$-\frac{1}{(x')^2}\left(\frac{x(-u+2\chi+ux')}{c}+\frac{1}{2}-x'-\frac{\chi'-2u+2x'u}{2\tilde{d}}\right)$$
$$-\frac{1}{(x')^2}\ln\frac{\chi'}{\vec{q}^{\,2}}\left(\frac{x'(2d+x'u)}{\chi'}-\frac{x'(4\chi'-u)}{\tilde{d}}-\frac{d^2(-2u-3\chi)}{2\tilde{d}^2\chi}+\frac{x'u\chi'}{\tilde{d}^2}\right)$$
$$-\frac{1}{(x')^2}\ln\frac{\vec{q}^{\,2}}{u}\left(\frac{u\,x}{\chi(\chi')^2}-\frac{x((\chi-u)^2x-2\chi u\,x')}{c^2}-\frac{2u\,x\,x'}{c}+\frac{(u^2-\chi^2)x^2}{c\chi}\right)$$
$$+4\pi\frac{[\vec{p}_1',\vec{k}_1]_z}{(x')^2}\left(\ln\frac{\vec{q}^{\,2}}{u}-\ln\frac{\chi'}{\vec{q}^{\,2}}\right)\frac{\chi'-u-x'(\chi-u)}{\chi^2}+2\pi\frac{[\vec{p}_1',\vec{k}_1]_z}{(x')^2}$$
$$\left(\frac{d(-2u-3\chi)}{\tilde{d}^2\chi}+x'\frac{-2u+\chi}{\chi\tilde{d}}\right), \quad (94)$$

where

$$d=\chi-\vec{q}^{\,2}, \quad \tilde{d}=u+\chi, \quad c=u-\vec{q}^{\,2}. \quad (95)$$

For $K_{SV}^{e,++}$ we have

$$
\begin{aligned}
K_{SV}^{e,++} = &-\ln^2 x' - \frac{3}{2}\ln^2\frac{\vec{q}^{\,2}}{u} + 2\ln\frac{\vec{q}^{\,2}}{u}\ln\frac{\chi'}{u}\ln\frac{\vec{q}^{\,2}}{u} + \ln\frac{\chi}{u} + \frac{1}{2}\ln\frac{\chi}{u} + \frac{5}{2}\ln\frac{\vec{q}^{\,2}}{u} \\
&- 2\mathrm{Li}21 + \frac{\chi'}{\vec{q}^{\,2}} + 3\mathrm{Li}21 - \frac{\vec{q}^{\,2}}{u} - \mathrm{Li}21 - \frac{\chi}{\vec{q}^{\,2}} - \frac{\pi^2}{4} - \frac{3}{2} \\
&- \frac{1}{(\chi')^2}\left(-\ln^2\frac{\vec{q}^{\,2}}{u} + \ln\frac{\chi}{u}\ln\frac{\vec{q}^{\,2}}{u} - \frac{\pi^2}{12} + \frac{(\chi')^2}{c^2}\ln\frac{\vec{q}^{\,2}}{u} + \left(\ln\frac{\chi}{u} - \ln\frac{\vec{q}^{\,2}}{u} + \frac{\chi'}{c}\right)\frac{\chi'}{u}\right. \\
&\left. + \mathrm{Li}21 - \frac{\vec{q}^{\,2}}{u} - \mathrm{Li}21 - \frac{\chi}{\vec{q}^{\,2}}\right)\left(-u^2x^2 + 2x'\chi\tilde{d} + (x')^2\chi(-2u-\chi)\right) \\
&+ \frac{\chi^2(1-2x')}{u\,c}\left(1 + \frac{u}{c}\ln\frac{\vec{q}^{\,2}}{u}\right) - \frac{2x'\chi}{u}\left(-\ln\frac{\chi}{u} - \frac{\chi'}{c} + \ln\frac{\vec{q}^{\,2}}{u}\left(1 - \frac{u\chi'}{c^2}\right)\right) \\
&- \frac{x'}{d}\left(u\,x - \frac{1}{2}x'\chi\right) + \frac{x'}{u\,d}\ln\frac{\chi}{\vec{q}^{\,2}}\left(u^2 + 4\chi u - x'\tilde{d}^2 - \frac{u(-2\chi u + x'\tilde{d}^2)}{2d}\right) \\
&- \ln\frac{\vec{q}^{\,2}}{u}\left(x'\frac{-2ux + x'\chi}{\chi'} + \frac{(2u(1-(x')^2) - \chi x^2}{c} - \frac{u^2x^2}{c\chi'}\right) \\
&- \frac{1}{\vec{q}^{\,2}}\left(-\frac{1}{2}\chi + x'u + 3x'\chi - 2(x')^2u - \frac{5}{2}(x')^2\chi - \frac{d\,x^2u^2}{2}\right) + \frac{x^2\chi^2}{u\vec{q}^{\,2}}. \quad (96)
\end{aligned}
$$

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