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ABOUT DIRECT $CP$ VIOLATION
IN THE SYSTEM OF $K^0$ MESONS
About Direct CP Violation in the System of $K^0$ mesons

This work is devoted to computation of the parameter of direct $CP$ violation by the weak interactions in the system of $K^0$ mesons at $K^0_{1-}$, $K^0_{2-}$-meson mixings and oscillations via $K_S$-, $K_L$-meson states.

The investigation has been performed at the Veksler and Baldin Laboratory of High Energy Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 2012
1. INTRODUCTION

A phenomenological analysis of $K^0$-meson processes was done in work [1] (see also [2]). There nonunitary transformation and nonorthogonal states were used in obtaining $K_S$, $K_L$ states. It was supposed that these states arise at $CP$ violation. The expressions for these states have the following form:

\[ K_S = (K^0_1 + \varepsilon_0 K^0_2) / \sqrt{1 + |\varepsilon_0|^2}, \]

\[ K_L = (K^0_2 + \varepsilon_0 K^0_1) / \sqrt{1 + |\varepsilon_0|^2}, \]  

(1)

and, on the contrary,

\[ K^0_1 = (K_S - \varepsilon_0 K_L) \sqrt{1 + |\varepsilon_0|^2} / (1 - |\varepsilon_0|^2), \]

\[ K^0_2 = (K_L - \varepsilon_0 K_S) \sqrt{1 + |\varepsilon_0|^2} / (1 - |\varepsilon_0|^2). \]

(2)

Writing the wave function of $K_L$, $K_S$ mesons in the form

\[ K_S = \frac{1 - \varepsilon_0}{\sqrt{2(1 + |\varepsilon_0|^2)}} e^{-i m_S t - \frac{\Gamma_S t}{2}}, \]

\[ K_L = \frac{1 - \varepsilon_0}{\sqrt{2(1 + |\varepsilon_0|^2)}} e^{-i m_L t - \frac{\Gamma_L t}{2}}, \]

(3)

and putting expression (3) into expression (2) and then taking the first term of (2) in the quadratic form on the absolute value, we obtain ($\hbar = 1$)

\[ |K^0_1|^2 = \frac{|1 - \varepsilon_0|^2}{2(1 - |\varepsilon_0|^2)} \times \left( e^{-\Gamma_S t} + |\varepsilon_0|^2 e^{-\Gamma_L t} - 2|\varepsilon_0| e^{-i(m_S + m_L) t} \cos((m_L - m_S) t) \right). \]  

(4)

In expression (4) a cross term appears which is responsible for oscillations. This term can be interpreted as oscillations between $K_S$, $K_L$ states; i.e., these states are
nonorthogonal ones. It is necessary to stress that in this approach it is supposed that at long distances from the source of $K^0$ mesons there mainly present $K_L$ mesons and with probability $|\epsilon_0|$ also appear $K_S$ mesons.

In the framework of quantum mechanics, if the states are wave vectors, expression (3) has to be written in the following form:

$$K_S(t) = \frac{1 - \epsilon_0}{\sqrt{2(1 + |\epsilon_0|^2)}} e^{-im_S t - \frac{i\epsilon_S}{2} - \Gamma_S t} K_S(0),$$

$$K_L(t) = \frac{1 - \epsilon_0}{\sqrt{2(1 + |\epsilon_0|^2)}} e^{-im_L t - \frac{i\epsilon_L}{2} - \Gamma_L t} K_L(0),$$

then after taking it in the quadratic form on the absolute value we get

$$|K^0|^2 = \frac{|1 - \epsilon_0|^2}{2(1 - |\epsilon_0|^2)} \left(e^{-\Gamma_S t} + |\epsilon_0|^2 e^{-\Gamma_L t}\right).$$

For description of processes in the system of $K^0$ mesons in our previous work [3] the standard theory of oscillations was used.

In the system of $K^0$ mesons a sufficiently complex process takes place. At first strangeness is violated in weak interactions and, as a consequence of it, $K^0$, $\bar{K}^0$ mesons are transformed into superpositions of $K_1^0$, $K_2^0$ mesons ($K_1^0$, $K_2^0$ mesons are eigenstates of the weak interactions violating the strangeness and they have definite $CP$ parities). Then follow oscillations of $K^0 \leftrightarrow \bar{K}^0$ mesons. Probability for $K^0$, $\bar{K}^0$-meson oscillations is given by the following expression:

$$P(K^0 \to \bar{K}^0) = \frac{1}{4} \left[ e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2 e^{-\frac{(\Gamma_S + \Gamma_L) t}{2}} \cos ((E_L - E_S) t) \right].$$

In the weak interactions there also exists violation of $CP$ parity. Then $K_1^0$, $K_2^0$ mesons become superposition states of $K_S$, $K_L$ mesons ($K_S$, $K_L$-meson states are eigenstates of weak interactions violating $CP$ parity). As a result, there appear oscillations between $K_1^0 \leftrightarrow K_2^0$ mesons. Just as a result of such transitions, there arise two pion decays at big distances from $K^0$ sources. The expression for probability of $K_2^0$-meson transition into $K_1^0$ has the following form:

$$P(K_2^0 \to K_1^0) = \frac{1}{4} \sin^2 2\beta \left[ e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2 e^{-\frac{(\Gamma_S + \Gamma_L) t}{2}} \cos ((E_L - E_S) t) \right].$$

If we take into account that $\cos \beta \simeq 1$, $\sin \beta \simeq \epsilon$, we get ($\epsilon = \epsilon^2$, $\epsilon_0 \sim \epsilon$)

$$P(K_2^0 \to K_1^0) = \epsilon^2 \left[ e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2 e^{-\frac{(\Gamma_S + \Gamma_L) t}{2}} \cos ((E_L - E_S) t) \right].$$
and probability for $P(K_1^0 \rightarrow K_1^0)$ transitions is
\[
P(K_1^0 \rightarrow K_1^0) = \left[ e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2\epsilon e^{-\frac{(\Gamma_S + \Gamma_L) t}{2}} \cos ((E_L - E_S)t) \right],
\]
and $P(K_2^0 \rightarrow K_2^0)$ is
\[
P(K_2^0 \rightarrow K_2^0) = \left[ e^{2}e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2\epsilon e^{-\frac{(\Gamma_S + \Gamma_L) t}{2}} \cos ((E_L - E_S)t) \right],
\]
These oscillations arise against the background of $K_0 \rightarrow l^- \pi^+ \nu_l$, $\bar{K}_0 \rightarrow l^+ \pi^- \bar{\nu}_l$, $K_1^0 \rightarrow 2\pi$, $K_2^0 \rightarrow 3\pi$ and others decays.

Now we write out some expressions which we will use later. The connection between $K_1^0$, $K_2^0$ and $K_0$, $\bar{K}_0$ states and also $K_S^0$, $K_L^0$ states are given by the expressions [3]
\[
K_1^0(t) = \cos \beta e^{-iE_S t}K_S^0(0) + \sin \beta e^{-iE_L t}K_L^0(0),
\]
\[
K_2^0(t) = -\sin \beta e^{-iE_S t}K_S^0(0) + \cos \beta e^{-iE_L t}K_L^0(0).
\]
We can also connect $K_S^0$, $K_L^0$-meson states with the $K^0$, $\bar{K}^0$-meson states. Then
\[
K_1^0 = \frac{K_0 - \bar{K}_0}{\sqrt{2}}, \quad K_2^0 = \frac{K_0 + \bar{K}_0}{\sqrt{2}},
\]
\[
K_1^0 = \frac{1}{\sqrt{2}}[(\cos \beta - \sin \beta)K_S + (\sin \beta + \cos \beta)K_L],
\]
\[
\bar{K}_1^0 = \frac{1}{\sqrt{2}}[-(\sin \beta + \cos \beta)K_S + (\cos \beta - \sin \beta)K_L],
\]
at the inverse transformation we get
\[
K_S = \frac{1}{\sqrt{2}}[(\cos \beta - \sin \beta)K_1^0 - (\cos \beta + \sin \beta)\bar{K}_1^0],
\]
\[
K_L = \frac{1}{\sqrt{2}}[(\cos \beta + \sin \beta)K_1^0 + (\cos \beta - \sin \beta)\bar{K}_1^0].
\]
We can simplify the above expressions by taking into account that $\sin \beta \ll 1$ and then $\cos \beta \simeq 1$. Then expression (13) get the following form:
\[
K_S = \frac{1}{\sqrt{2}}[(1 - \epsilon)K_1^0 - (1 + \epsilon)\bar{K}_1^0],
\]
\[
K_L = \frac{1}{\sqrt{2}}[(1 + \epsilon)K_1^0 + (1 - \epsilon)\bar{K}_1^0].
\]
where we replaced $\sin \beta$ by $\epsilon$. 3
2. DIRECT CP VIOLATION IN THE SYSTEM OF $K^0$ MESONS

2.1. Old Result Obtained in [1]. In work [1], where it is supposed that at big distances there are only $K_S, K_L$ mesons, an expression was obtained for direct CP violation in the system of $K^0$ mesons. For this aim they compute the decay probability of $K_S, K_L$ mesons into two pions assuming that CPT invariance takes place. Since pions are bosons, full wave functions have to be invariant at their transposition. Isospin of pion is equal to one $I = 1$ and final state at $K_S-, K_L-$meson decay must have isospin $I = I_3 = 0$ or $I = 2, I_3 = 0$ (at transition of $K_{SL} \to 2\pi$ the rule $\Delta S = 1/2$ is realized). Then there appear the following 4 states:

\[ \langle \pi \pi, I = 0 | H_W | K_S \rangle, \quad \langle \pi \pi, I = 2 | H_W | K_S \rangle, \]
\[ \langle \pi \pi, I = 0 | H_W | K_L \rangle, \quad \langle \pi \pi, I = 2 | H_W | K_L \rangle. \]  \hfill (15)

Using of Klebsh–Gordon coefficients, we can write two pion states in the form

\[ \langle \pi^+ \pi^- | = \frac{1}{\sqrt{3}} \langle \pi \pi, I = 2 | + \frac{2}{\sqrt{3}} \langle \pi \pi, I = 0 |, \]
\[ \langle \pi^0 \pi^0 | = \frac{2}{\sqrt{3}} \langle \pi \pi, I = 2 | - \frac{1}{\sqrt{3}} \langle \pi \pi, I = 0 |, \]  \hfill (16)

where $\pi^+ \pi^- = (\pi^+ \pi^0 + \pi^0 \pi^-)/\sqrt{2}$. We will take into account that there arise the following phase shifts due to pion interactions in final state $- e^{i\delta_0}, e^{i\delta_2}$ for $I = 0, I = 2$. We can then rewrite expression (16) in the following form:

\[ \langle \pi^+ \pi^- | = \sqrt{\frac{1}{3}} e^{i\delta_2} \langle \pi \pi, I = 2 | + \sqrt{\frac{2}{3}} e^{i\delta_0} \langle \pi \pi, I = 0 |, \]
\[ \langle \pi^0 \pi^0 | = \frac{2}{\sqrt{3}} e^{i\delta_2} \langle \pi \pi, I = 2 | - \frac{1}{\sqrt{3}} e^{i\delta_0} \langle \pi \pi, I = 0 |. \]  \hfill (17)

Decay amplitudes are determined by the following expressions:

\[ A_0 = \langle \pi \pi, I = 0 | H_W | K^0 \rangle, \]
\[ A_2 = \langle \pi \pi, I = 2 | H_W | K^0 \rangle. \]  \hfill (18)

Analogous amplitudes for $\bar{K}^0$ can be obtained by using CPT transformation, then $|K^0 \rangle \to - |\bar{K}^0 \rangle$ and

\[ \langle \pi \pi, I = 0 | \to |\pi \pi, I = 0 \rangle, \]
\[ \langle \pi \pi, I = 2 | \to |\pi \pi, I = 2 \rangle. \]  \hfill (19)
Then on supposition of CPT invariance we get
\[
\langle \pi\pi, I = 0 | H_W | K^0 \rangle = -A_0^*,
\]
\[
\langle \pi\pi, I = 2 | H_W | K^0 \rangle = -A_2^*.
\] (20)

The primary state of kaon beam is some superposition of \( K^0, \bar{K}^0 \) mesons which have isospin \( I = 1/2 \). Therefore, transitions described by the amplitude \( A_2 \) have \( \Delta I = 3/2 \) and this violates the rule \( \Delta I = 1/2 \). It is known that the transitions with \( \Delta I = 3/2 \) are suppressed by the factor 1/20.

Using expressions (14)–(20), we can write the amplitudes of observable values via \( A_0, A_2 \) and \( CP \)-violating parameter \( \varepsilon_0 \) in the following form:
\[
\langle \pi^+\pi^- | H_W | K_S \rangle = \frac{1}{\sqrt{6}} \left\{ (A_2 + A_2^*) e^{i\delta_2} + \frac{4}{\sqrt{2}} A_0 e^{i\delta_0} \right\} + \varepsilon_0 (A_2 - A_2^*) e^{i\delta_2},
\]
\[
\langle \pi^0\pi^0 | H_W | K_S \rangle = \frac{1}{\sqrt{3}} \left\{ (A_2 + A_2^*) e^{i\delta_2} - \sqrt{2} A_0 e^{i\delta_0} \right\} + \varepsilon_0 [(A_2 - A_2^*) e^{i\delta_2}],
\] (21)
\[
\langle \pi^+\pi^- | H_W | K_L \rangle = \frac{1}{\sqrt{6}} \left\{ (A_2 - A_2^*) e^{i\delta_2} + \varepsilon_0 \left[ (A_2 + A_2^*) e^{i\delta_2} + \frac{4}{\sqrt{2}} A_0 e^{i\delta_0} \right] \right\},
\]
\[
\langle \pi^0\pi^0 | H_W | K_L \rangle = \frac{1}{\sqrt{3}} \left\{ [(A_2 - A_2^*) e^{i\delta_2}] + \varepsilon_0 [(A_2 + A_2^*) e^{i\delta_2} - \sqrt{2} A_0 e^{i\delta_0}] \right\}.
\]

If we neglect the second-order terms of the small values \( \varepsilon_0 \) and \( |A_2| \), then from (21) we get
\[
\eta^{+-} \approx \varepsilon_0 + \varepsilon'_0,
\]
\[
\eta^{00} \approx \varepsilon_0 - 2\varepsilon'_0,
\] (23)

where
\[
\varepsilon'_0 = \frac{1}{\sqrt{2}} \text{Im} \left( \frac{A_2}{A_0} \right) e^{i(\pi/2 + \delta_2 - \delta_0)}.
\] (24)

The value \( \varepsilon'_0 \) is a direct \( CP \)-violating term which does not appear at indirect \( CP \) violation in the system of \( K^0, \bar{K}^0 \) mesons [1].

At big distance ($t \geq 6\tau_S$) all primary $K_S$ mesons have time to decay and then there will be present only $K_S$ mesons which are created at $K_S^0$ oscillations. From expressions for $K_S^0$, $K_L^0$-meson oscillations we see that there cannot appear direct $CP$ violation. Direct $CP$ violation can appear only at direct decays of $K_2^0$ mesons. Now we consider the case of direct $CP$ violation when the standard theory of oscillations is used.

For this aim we will use expressions (14)–(20). The expression for amplitudes of $K_S^0$, $K_L^0$-meson decays into two pions can be written (by using $A_0$, $A_2$ and $CP$-violating parameter $\beta$) in the following form [3] (here we suppose that at transition $K_2^0 \rightarrow K_1^0$ the $K_L$ state is generated):

\[
\langle \pi^+ \pi^- | H_W | K_1^0 \rangle = \frac{1}{\sqrt{6}} \left( (A_2 + A_2^*) e^{i\delta_2} + \frac{4}{\sqrt{2}} A_0 e^{i\delta_0} \right),
\]

\[
\langle \pi^0 \pi^0 | H_W | K_1^0 \rangle = \frac{1}{\sqrt{3}} \left[ (A_2 + A_2^*) e^{i\delta_2} - \sqrt{2} A_0 e^{i\delta_0} \right],
\]

\[
\langle \pi^+ \pi^- | H_W | K_L \rangle = \frac{1}{\sqrt{6}} \left\{ \cos \beta (A_2 - A_2^*) e^{i\delta_2} + \right.
\]

\[+ \sin \beta \left[ (A_2 + A_2^*) e^{i\delta_2} + \frac{4}{\sqrt{2}} A_0 e^{i\delta_0} \right] \left\}, \right.

\[
\langle \pi^0 \pi^0 | H_W | K_L \rangle = \frac{1}{\sqrt{3}} \left\{ \cos \beta [(A_2 - A_2^*) e^{i\delta_2}] + \right.
\]

\[+ \sin \beta [(A_2 + A_2^*) e^{i\delta_2} - \sqrt{2} A_0 e^{i\delta_0}] \right\}. \tag{25}
\]

Taking into account that $\cos \beta \approx 1$ and introducing the notation $\sin \beta = \epsilon$ (then the parameter of $CP$ violation is $\epsilon = \sin^2 \beta$), we can rewrite expression (25) in the following form:

\[
\langle \pi^+ \pi^- | H_W | K_1^0 \rangle = \frac{1}{\sqrt{6}} \left( (A_2 + A_2^*) e^{i\delta_2} + \frac{4}{\sqrt{2}} A_0 e^{i\delta_0} \right),
\]

\[
\langle \pi^0 \pi^0 | H_W | K_1^0 \rangle = \frac{1}{\sqrt{3}} \left[ (A_2 + A_2^*) e^{i\delta_2} - \sqrt{2} A_0 e^{i\delta_0} \right], \tag{26}
\]

\[
\langle \pi^+ \pi^- | H_W | K_L \rangle = \frac{1}{\sqrt{6}} \left\{ (A_2 - A_2^*) e^{i\delta_2} + \epsilon \left[ (A_2 + A_2^*) e^{i\delta_2} + \frac{4}{\sqrt{2}} A_0 e^{i\delta_0} \right] \right\},
\]

\[
\langle \pi^0 \pi^0 | H_W | K_L \rangle = \frac{1}{\sqrt{3}} \left\{ [(A_2 - A_2^*) e^{i\delta_2}] + \epsilon [(A_2 + A_2^*) e^{i\delta_2} - \sqrt{2} A_0 e^{i\delta_0}] \right\}. \tag{26}
\]
The expression for relation between amplitudes (for experimentally observable values) for $K_L \to \pi^+\pi^-$ and $K^0_1 \to \pi^+\pi^-$ decays and the expression for relation between amplitudes of their decay into two neutral pions look as follows:

\[
\eta^+_- = \frac{\langle \pi^+\pi^- | H_W | K^0_L \rangle}{\langle \pi^+\pi^- | H_W | K^0_1 \rangle},
\]
\[
\eta^{00} = \frac{\langle \pi^0\pi^0 | H_W | K^0_L \rangle}{\langle \pi^0\pi^0 | H_W | K^0_1 \rangle}.
\]  

(27)

If we neglect the second-order terms of $\varepsilon$ and $|A_2|$ (since they are small values), then from (25) or (26) we get

\[
\eta^+_- \approx \varepsilon + \varepsilon',
\]
\[
\eta^{00} \approx \varepsilon - 2\varepsilon',
\]

(28)

where

\[
\varepsilon' = \frac{1}{\sqrt{2}} \text{Im} \left( \frac{A_2}{A_0} \right) e^{i(\pi/2 + \delta_2 - \delta_0)}.
\]

It is necessary to keep in mind that in the approach where the standard theory of oscillations is used [3] $\varepsilon = \sin \beta$ and then the parameter of CP violation $\epsilon$ is $\epsilon = \sin^2 \beta$ in contrast to the old result where these parameters are the same. The value $\varepsilon'$ is a new direct CP-violating parameter which does not coincide with the old direct CP-violating parameter $\varepsilon'_0$ [1].

Now ratios between experimentally observable values, in contrast to the old case, are given by the following relations:

\[
|\eta^+_-|^2 = \frac{\langle \pi^+\pi^- | H_W | K^0_L \rangle^2}{\langle \pi^+\pi^- | H_W | K^0_1 \rangle^2} \approx (\varepsilon + \varepsilon')^2 = \varepsilon^2 + 2\varepsilon\varepsilon',
\]
\[
|\eta^{00}|^2 = \frac{\langle \pi^0\pi^0 | H_W | K^0_L \rangle^2}{\langle \pi^0\pi^0 | H_W | K^0_1 \rangle^2} \approx (\varepsilon - 2\varepsilon')^2 = \varepsilon^2 - 4\varepsilon\varepsilon'.
\]  

(29)

In the above expressions we neglected the term $\varepsilon^2$, supposing that $\varepsilon^2 \ll \varepsilon'^2$. We remind that $\varepsilon^2$ is the parameter of CP violation and $\epsilon = \varepsilon^2$. Then

\[
\frac{|\eta^{00}|^2}{|\eta^+_-|^2} \approx \frac{\varepsilon^2 - 4\varepsilon\varepsilon'}{\varepsilon^2 + 2\varepsilon\varepsilon'} = 1 - \frac{6\varepsilon'}{\varepsilon},
\]

or

\[
R = \frac{|\eta^{00}|^2}{|\eta^+_-|^2} \approx 1 - 6\frac{\varepsilon'}{\sqrt{\varepsilon}},
\]  

(30)

where $\epsilon$ is the parameter of CP violation.
In work [4] a value for \(\text{Re}\left(\frac{\epsilon'}{\epsilon}\right)\) was obtained and it is equal to

\[
\text{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{1 - R}{6} = (14.7 \pm 2.2) \cdot 10^{-4}.
\]

Taking into account that [5] \(\epsilon = 2.23 \cdot 10^{-3}\) (\(\sqrt{\epsilon} = 4.72 \cdot 10^{-2}\)), for the old case we have

\[
\epsilon' = 32.78 \cdot 10^{-7}. \tag{31}
\]

For our case for \(\epsilon'\) we obtain

\[
\epsilon' = 69.38 \cdot 10^{-6}. \tag{32}
\]

3. CONCLUSION

In work [3], in the framework of the standard theory of oscillations, we considered \(K^0\)-, \(K^0\)-meson mixings and oscillations via \(K^0_L\)-, \(K^0_S\)-meson states at strangeness violation by the weak interactions and \(K^0\)-, \(K^0\)-meson mixings and oscillations via \(K_S\)-, \(K_L\)-meson states at \(CP\) violation by the weak interactions without and with taking into account decay widths. It was realized in the framework of the masses mixing scheme.

In this work we computed the parameter of direct \(CP\) violation by the weak interactions at \(K^0\)-, \(K^0\)-meson mixings and oscillations via \(K_S\)-, \(K_L\)-meson states in the framework of the above-mentioned approach. This direct \(CP\) violation appears owing to the presence of \(CP\)-violation term with isospin \(I = 2\).

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Received on January 30, 2012.
Редактор Е. И. Кравченко

Подписано в печать 23.04.2012.
Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 0,69. Уч.-изд. л. 0,91. Тираж 375 экз. Заказ № 57628.

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