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REPRESENTATION OF THE RADIATIVE STRENGTH FUNCTIONS IN THE PRACTICAL MODEL OF CASCADE GAMMA DECAY

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Ву Д. К. и др. Е3-2016-43
Представление радиационных силовых функций в практической модели каскадного гамма-распада

Разработанная в Дубне практическая модель каскадного гамма-распада нейтронного резонанса позволяет из аппроксимации интенсивностей двухквантовых каскадов одновременно определять параметры плотности уровней ядра и парциальных ширин эмиссии продуктов ядерной реакции. В представленном варианте модели минимизирована доля используемых феноменологических представлений. Анализ новых результатов подтвердил полученную ранее зависимость динамики взаимодействия ферми- и бозе-состояний ядра от его формы. Из отношений плотностей уровней вибрационного и квазичастичного типов следует также, что это взаимодействие проявляется в диапазоне энергий связи нейтрона и, вероятно, различается в ядрах с различной четностью нуклонов.

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Representation of the Radiative Strength Functions in the Practical Model of Cascade Gamma Decay

The developed in Dubna practical model of the cascade gamma decay of neutron resonance allows one, from the fitted intensities of the two-step cascades, to obtain parameters both of level density and of partial widths of emission of nuclear reaction products. In the presented variant of the model a part of phenomenological representations is minimized. Analysis of new results confirms the previous finding that dynamics of interaction between Fermi- and Bose-nuclear states depends on the form of the nucleus. It also follows from the ratios of densities of vibrational and quasi-particle levels that this interaction exists at least up to the binding neutron energy and probably differs for nuclei with varied parities of nucleons.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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INTRODUCTION

Parameters of the cascade gamma decay of any high-lying nuclear level (see Figs. 1–3) at any excitation energy are determined only by the level density ρ and by the partial widths Γ of dipole electrical and magnet transitions. Cascade intensity with pure quadrupole transitions is negligible at the nuclear excitation energy of more than a few MeV. For levels excited by primary transitions interval of spins is $\Delta J \leqslant 4$ for any parity.

Investigation of the process of gamma decay is interesting most of all for analysis of interaction dynamics of fermion and boson states of nuclear matter. Valid information is need also for describing the process of fission more correctly. According to [1], energy is divided between excited fission fragments dependent on their level densities. As is seen in Figs. 4–6, level densities, calculated using available models [2], differ greatly from the modern experimental data.

Ordinary gamma spectra and reaction cross sections depend on a $\rho\times\Gamma$ product and this fact completely cuts out the possibility of simultaneous determination of ρ and Γ valid values using such kind of data. This possibility is realized only in experiments on studying the cascade intensities of two sequential gamma transitions. Two-step experiments can decrease the total error of determined ρ and Γ functions up to several dozens of percents as the intensities of two-step cascades include all information about energy of two gamma transitions and any triplets of fixed nuclear levels.

As it is impossible to resolve all individual levels and to determine probabilities of transitions between them by available now spectrometers, information on superfluidity can be obtained from indirect experiments only. At that, both level density ρ and partial widths Γ in any nucleus are fitting functions with a minimal as far as possible number of parameters.

1. POSSIBILITY OF UP-TO-DATE EXPERIMENT AND ITS MODEL REPRESENTATION

The intensities $I_{\gamma\gamma}(E_1)$ of two-step cascades between neutron resonance (or another compound-state) λ and some group of low-lying nuclear levels f through any intermediate levels i for a fixed energy E_1 of primary transition are written

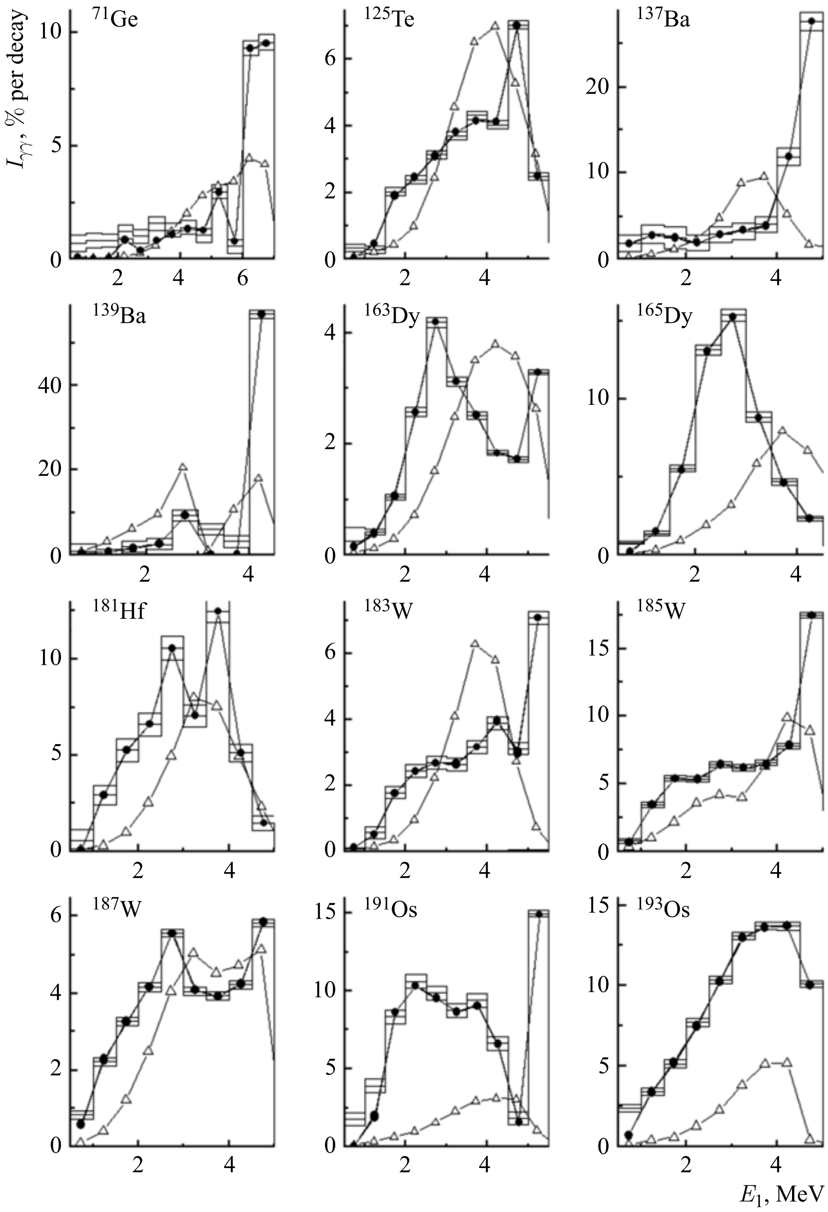

Fig. 1. Dependences of the experimental intensities (histogram with experimental errors) and their best approximations (points) on the energy of primary transition. Triangles are the results of calculations based on statistical model

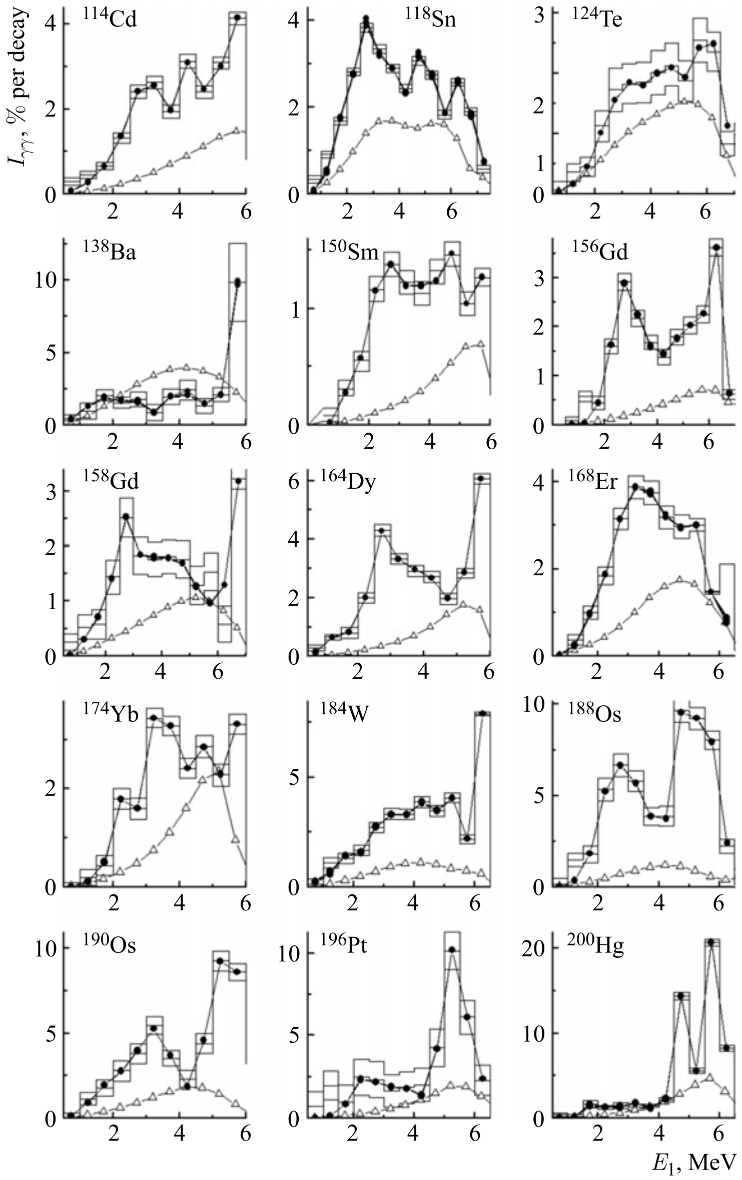

Fig. 2. The same (as in Fig. 1) for even–even nuclei

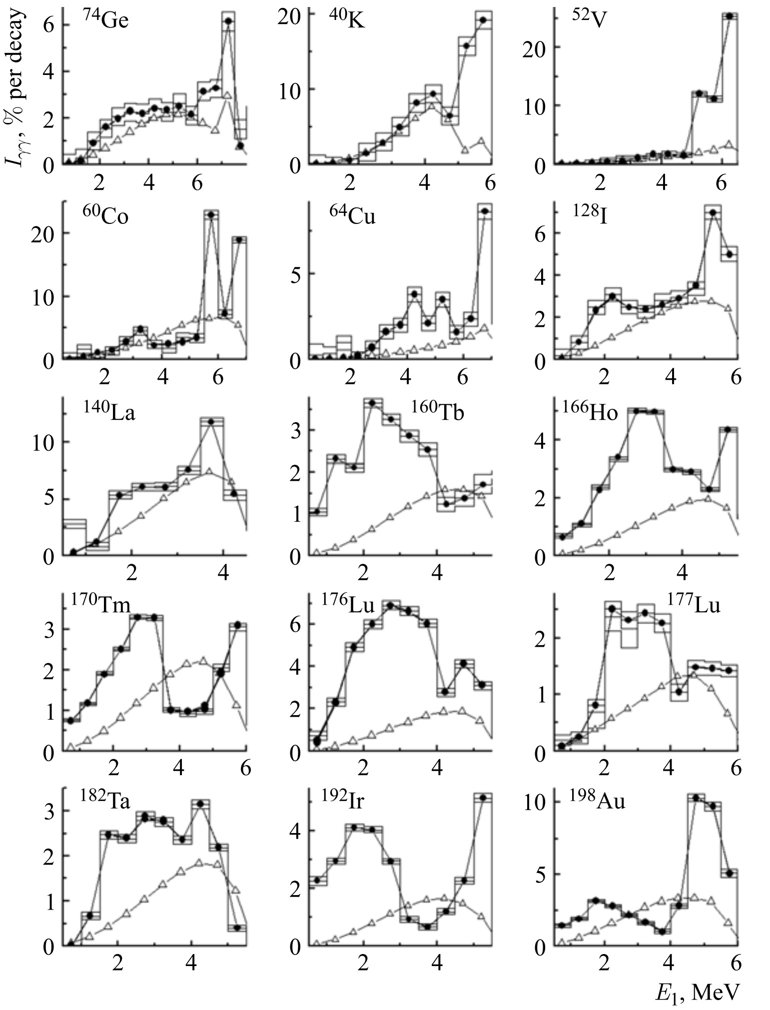

Fig. 3. The same (as in Fig. 1) for ^{74}Ge, ^{177}Lu, and odd–odd nuclei

by a system of equations of type

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f} \sum_{i} \frac{\Gamma_{\lambda i}}{\Gamma_{\lambda}} \frac{\Gamma_{if}}{\Gamma_i} = \sum_{\lambda,f} \frac{\Gamma_{\lambda i}}{\langle\Gamma_{\lambda i}\rangle\, m_{\lambda i}} n_{\lambda i} \frac{\Gamma_{if}}{\langle\Gamma_{if}\rangle m_{if}}, \tag{1}$$

where $m_{\lambda i}$ is the number of levels of excited primary γ transitions in intervals from the energy of initial level λ to the energy of intermediate level i; m_{if} is the number of levels excited by secondary transitions in intervals from the energy

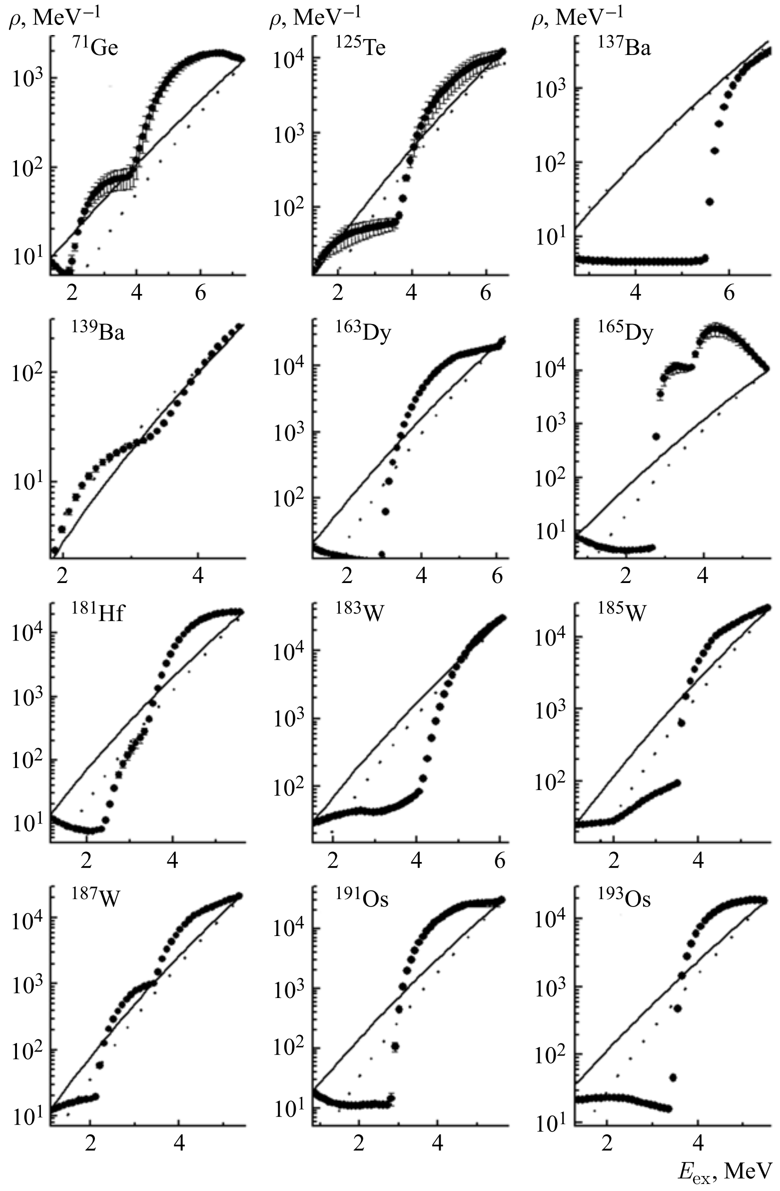

Fig. 4. Average densities of intermediate levels of two-step cascades (points with errors) for even–odd nuclei (fits of the smallest χ^2) depending on the excitation energy. Lines are the data of [19], dotted lines are calculations by model taking into account shell inhomogeneities of single-particle spectrum [11]

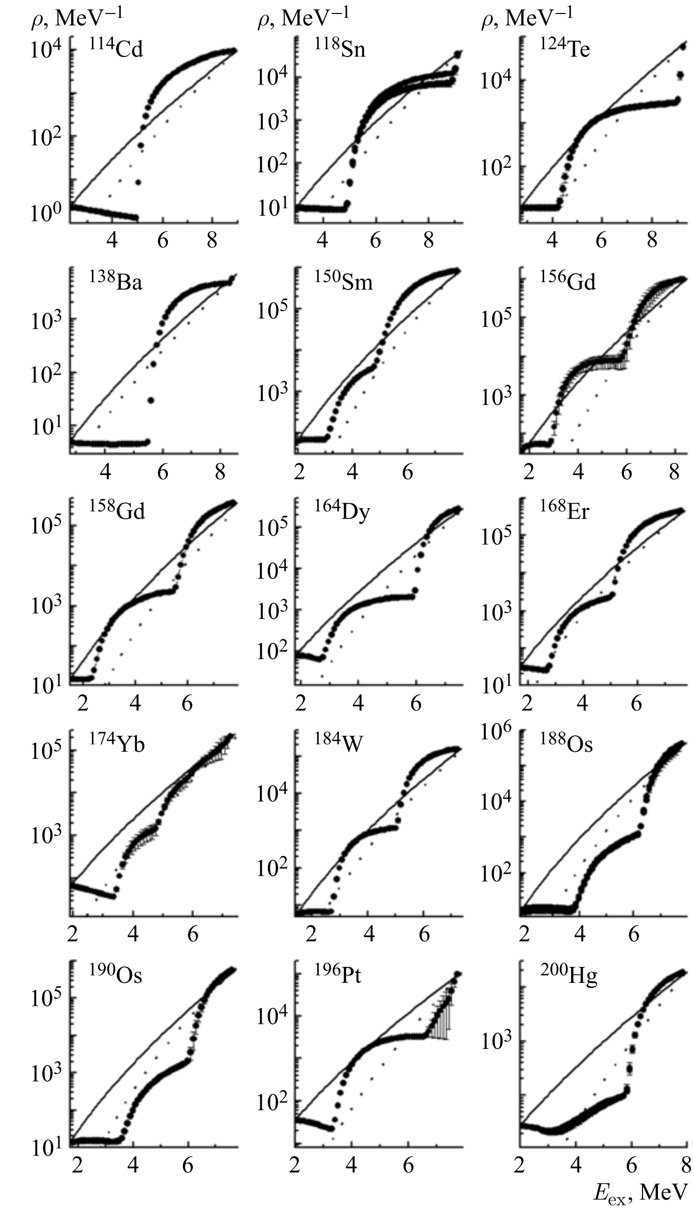

Fig. 5. The same (as in Fig. 4) for even–even nuclei

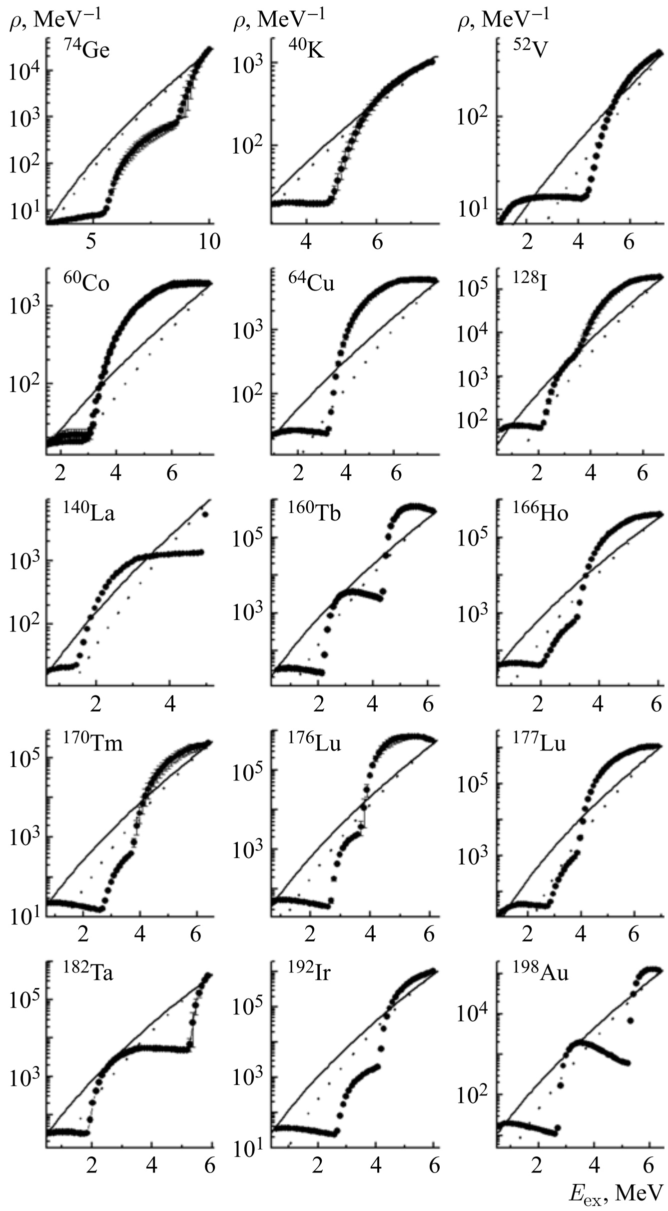

Fig. 6. The same (as in Fig. 4) for ^{74}Ge, ^{177}Lu, and odd–odd nuclei

of intermediate level i to the energy of the final level f; $n_{\lambda i}$ is the number of intermediate cascade levels in small energy intervals. From the system (1), which connects an unknown level number n (or m) and unknown partial widths, a set of p and q parameters of the model functions $\rho = f(p_1, p_2, \ldots)$ and $\Gamma = \varphi(q_1, q_2, \ldots)$ with some uncertainty is determined. The uncertainty is caused by a distortion of available theoretical representations and experimental results. Previous analysis [3] showed that a strong connection between ρ and Γ values in narrow intervals of excitation energies can be included in the model (1). In such a way from two-step cascades it is possible to determine simultaneously parameters of specified ρ and Γ functions at any densities of λ and i levels.

Analysis of the cascade intensities [4, 5] for nuclei of the region $28 \leqslant A \leqslant 200$ showed that obtained level densities cannot be described with the experimental accuracy by models, which ignore influence of boson-states of nuclear matter on ρ function.

The Dubna model is free from using any hypothesis untested by experiment (for example, Porter–Thomas hypothesis [6] about emission widths of nuclear reaction products, hypothesis of Axel–Brink [7, 8] about independency of Γ values from energy of excited level, or Bohr–Mottelson hypothesis [9] about validation of the optical model used for determination of emission probability for nucleon products of reaction). The basis of our model of the cascade gamma decay of nuclear compound-states with excitation energies $E_{\text{ex}} \approx 5{-}10$ MeV is the model of n-quasi-particle levels, balance of entropy and energy of quasi-particle levels [2, 10, 11] and tested model-phenomenological representations about form of energy dependency of radiative strength functions.

A systematic error in any procedure for ρ and Γ determination is always caused by large coefficients of error transfer of measured spectrum δS or cross section $\delta\sigma$ of reaction onto errors of $\delta\rho$ and $\delta\Gamma$ sought parameters. The error value strongly grows at increment of energy of decaying level. It is possible to evaluate this error and to choose a direction for correction of model representation about ρ and Γ only if to compare different model representations of $\rho = f(p_1, p_2, \ldots)$ and $\Gamma = \varphi(q_1, q_2, \ldots)$ functions. For example, comparing a few variants of the practical model [3, 5, 12, 13] we discovered that rate of density change for vibrational levels (given in [12, 13] phenomenologically) is partially or completely determined [4, 5] by pairing energy Δ for the last nucleon in nucleus. Therefore, in proposed variant of our practical model in the coefficient C_{coll} of collective level density increasing [4, 5, 11] E_μ and E_ν parameters (changes of rates of nuclear entropy and of energy of quasi-particle states, respectively) are replaced by united fitting parameter E_u. Thus, C_{coll} coefficient was used in a form

$$C_{\text{coll}} = A_l \exp(\sqrt{(E_{\text{ex}} - U_l)/E_u} - (E_{\text{ex}} - U_l)/E_u) + \beta, \tag{2}$$

where A_l are fitting parameters of vibrational level density above breaking point of each l-th Cooper pair, and U_l are energies of the corresponding breaking thresholds. Parameter $\beta \geqslant 1$ can differ from 1 for deformed nuclei.

The influence of the shell inhomogeneities of a single-particle spectrum [2, 11] on a parameter defining a dependence of level density on excitation energy

$$a(A, E_{\mathrm{ex}}) = \tilde{a}(1 + ((1 - \exp(\gamma E_{\mathrm{ex}}))\delta E/E_{\mathrm{ex}})) \tag{3}$$

(and at the same time on $g = 6a/\pi^2$ parameter of density of n-quasi-particle levels near Fermi-surface [11]) was also taken into account. An asymptotic value $\tilde{a} = 0.114A + 0.162A^{2/3}$ and coefficient $\gamma = 0.054$ were taken from [11]. A shell correction δE calculated from the data of mass defect in a liquid-drop nuclear model [2] was lightly changed to keep an average distance D_λ between resonances of the tested nucleus.

2. ENERGY DEPENDENCE OF THE STRENGTH FUNCTIONS

In the model of the cascade gamma decay for any excited levels and energies of emitted quantum, the form of energy dependence for partial radiative widths must be specified with a good accuracy.

On a base of available models for nucleus of A mass a strength function is determined as $k = \Gamma/(A^{2/3}E_\gamma^3 D_\lambda)$, where E_γ is the energy of the gamma transition. An absolute value of sum of radiative widths for primary $E1$- and $M1$-transitions of cascades (total radiative width) is usually obtained from measured cross sections of the reaction. The expected form of this sum may be found using phenomenological representations or extrapolation of any models to $E_d < E_{\mathrm{ex}} < B_n$ region of excitation (E_d is a point of transition from a set of known levels [14] to a concept of level density function, and B_n is a neutron binding energy in a nucleus).

The main summand of the functions $k(E1, E_\gamma)$ and $k(M1, E_\gamma)$ may be presented as a distribution of strength functions from models type of [15] with additional varied parameters. Variation of these parameters gives a set of functions of $E1$- and $M1$-transitions with a wide area of possible values (as it was done in [12, 13]).

It was experimentally established [16] that an addition to $k(E1, E_\gamma) + k(M1, E_\gamma)$ energy dependence of several peaks ensures a fine description of the cascade intensities. Form of these additional peaks may be found only by empiric way. For example, a description of each of them by two exponents (as in [5, 12]) is convenient to solve a system of nonlinear equations (1), although exponents are not used in theoretical models [2].

Usually for describing a form of peaks of $E1$- and $M1$-strength functions Breit–Wigner or Lorentz distributions are exploited. Asymmetrical Breit–Wigner function is used in theoretical analysis of fragmentation of quasi-particle states at varied locations relative to Fermi-surface [17]. However, variety of results is a trouble for a direct usage of these theoretical representations.

It turned out that application of an asymmetrical Lorentzian curve for description of peaks of the strength functions is simpler. Local peaks of $E1$- and $M1$-strength functions are written by an expression

$$k = W_i \frac{(E_\gamma^2 + (\alpha_i(E_\gamma - E_i)/E_\gamma))\Gamma_i^2}{(E_\gamma^2 - E_i^2)^2 + E_\gamma^2\Gamma_i^2}. \quad (4)$$

Lorentzian curve parameters for each i-th peak are similar to the model [15]: location of the peak center E_i, width Γ_i, amplitude W_i, and asymmetry parameter $\alpha_i \sim T^2$ (T is a nuclear thermodynamic temperature). A value of the expression $\alpha_i(E_\gamma - E_i)/E_\gamma$ grows linearly when an excitation energy increases (from zero in the center of peak to maximum at B_n), and it decreases below if neutron excitation energy reduces.

An essential problem of using Lorentzian curve at fitting is a strong degradation of a convergence of iteration process. As all parameters of (4) are fitted, the possibility of unlimited Γ_i decreasing appears in some fitting paths.

A necessity of phenomenological accounting an influence of sharp local change of level density on the strength functions was discovered already at model-free determination of random functions ρ and Γ [18]. A required correction was done with the help of multiplication of fitted strength functions by ratio

$$M = \rho_{\mathrm{mod}}/\rho_{\mathrm{exp}}, \quad (5)$$

where ρ_{exp} is the best fit for a given iteration, ρ_{mod} is a smooth model functional described both density of neutron resonances and cumulative sum of known levels with E_{ex} below E_d. For ρ_{mod} determination the back shifted Fermi-gas model was chosen. In a given variant of analysis a limitation $1 \leqslant \rho_{\mathrm{mod}}/\rho_{\mathrm{exp}} \leqslant 10$ [12] was used. Sums of dipole strength functions with taking into account such correction and without it are presented in Figs. 10–12.

3. RESULTS

An ambiguity of the system (1) solving appears because of both a strong nonlinearity of the sought functions ρ and Γ and their anticorrelation. There is a noticeable probability of falling into a false minimum of χ^2 which can lead to an essential systematic error of ρ and Γ values. It is possible to evaluate and minimize this uncertainty only if to compare results of different variants of the practical model with various functional ρ and Γ dependences.

The comparison of the results of the given model variant with previous ones showed that a good accuracy is achieved in describing a density of intermediate levels of cascades. The most distortion of density values were found only for ^{137}Ba and ^{182}Ta. At that, for ^{137}Ba the previous variant of fitting [5] most likely gives a large uncertainty. And breaking thresholds of the second and the third pairs for ^{182}Ta in presented variant are 1.6 and 5.8 MeV, but in [5] they are 1.6 and 4.0 MeV, respectively. It means that obtained data on the level density even in the worst case of ^{182}Ta give a picture where principle errors are caused only by ambiguity of the up-to-date representations about gamma-decay process.

A larger accuracy and adequacy of the results would be achieved if not less than $\approx 99\%$ of intensity of primary transitions is separated in experiment from all gamma cascades of compound-state decay. But a comparison of the breaking thresholds for 3–4 Cooper pairs determined from (1) using different functional ρ and Γ dependences showed that a reliable information about the most probable level density and the strength functions of dipole gamma transitions can be extracted even from a convolution of spectrum of primary products of decay of compound-state and dependency of gamma transitions branches coefficients on energy of intermediate level. The obtained results in the last variants of the practical model vary very weakly.

The level densities from back shifted Fermi-gas model [19] and from model with taking into account shell inhomogeneities of single-particle spectrum [11] are presented in Figs. 4–6. It is seen that the second model describes a $d\rho/dE_{\rm ex}$ derivative with a better accuracy than [19] model does. But the level densities calculated using [2] models strongly differ from the ones extracted from (1).

In all realized variants of the practical model [5, 18, 20–22] at step-by-step reduction of a number of fitted parameters a fitting accuracy is kept, and so description of $I_{\gamma\gamma}$ spectra in this paper is practically the same as the ones in [12, 13].

The radiative strength functions of $E1$- and $M1$-transitions and their sums presented in Figs. 7–9 and Figs. 10–12, correspondingly, have no principal distortions with the ones published earlier. But a problem of unambiguous description for observed local peaks of electric and magnetic strength functions remains valid (using exponents [5] or modified Lorentzian curve (4) for this purpose gives closely χ^2).

One needs to append that the data of Figs. 7–12 do not demand to include to strength functions any additional "pygmy-resonances". For a total interpretation of the gamma-decay process theoretical representations (about co-existing quasi-particle levels with vibrational ones and about fragmentation of all nuclear states at $E_{\rm ex}$ growing) are quite enough.

For many nuclei (Figs. 10–12) "plateau" in sum of strength functions of $E1$- and $M1$-transitions coincides with a sum of calculated values from [15] and $k(M1)$ value ($k(M1) = \text{const}$) normalized by $k(M1)/k(E1)$ experimental

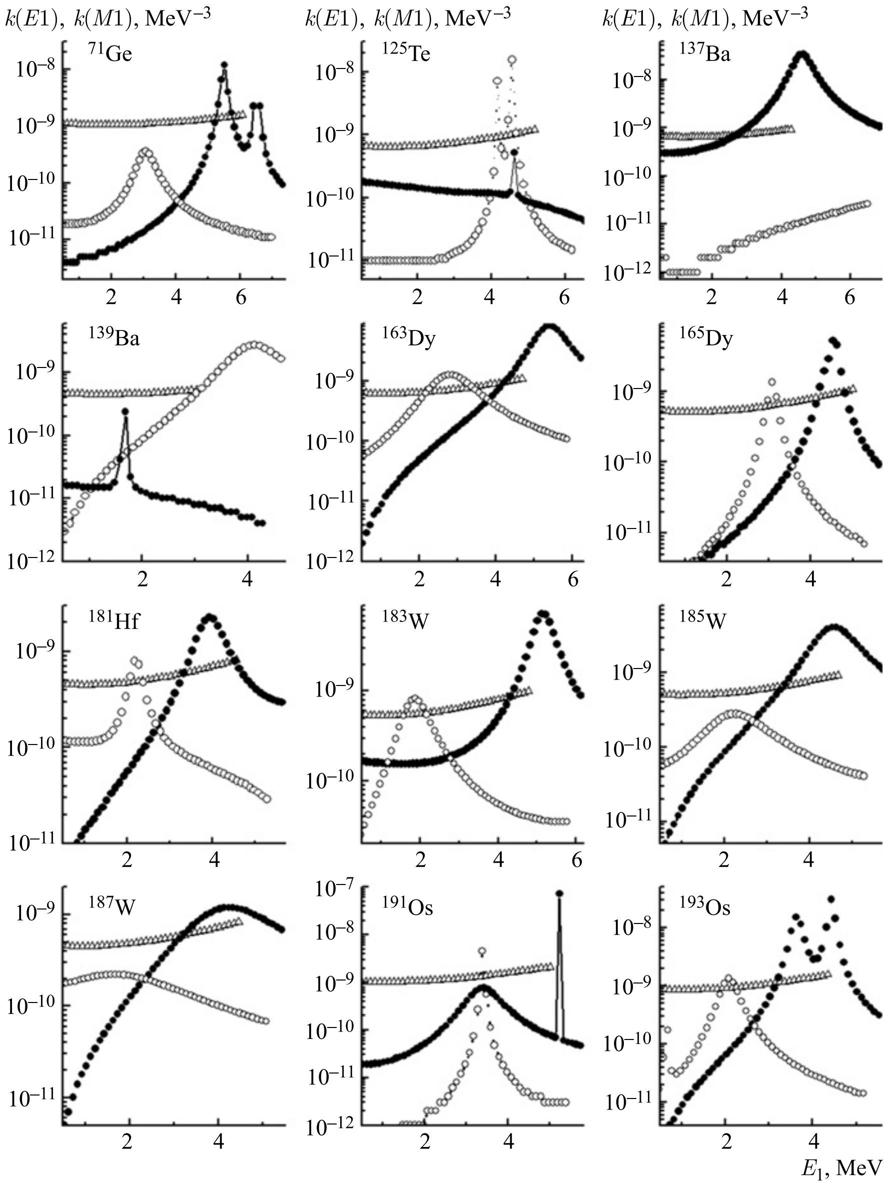

Fig. 7. Strength functions of $E1$-transitions (black points) and of $M1$-transitions (open points) for even–odd nuclei. Triangles are calculations by model [KMF] adding $k(M1) =$ const in $0 < E_1 \leqslant B_n - E_d$ energy range

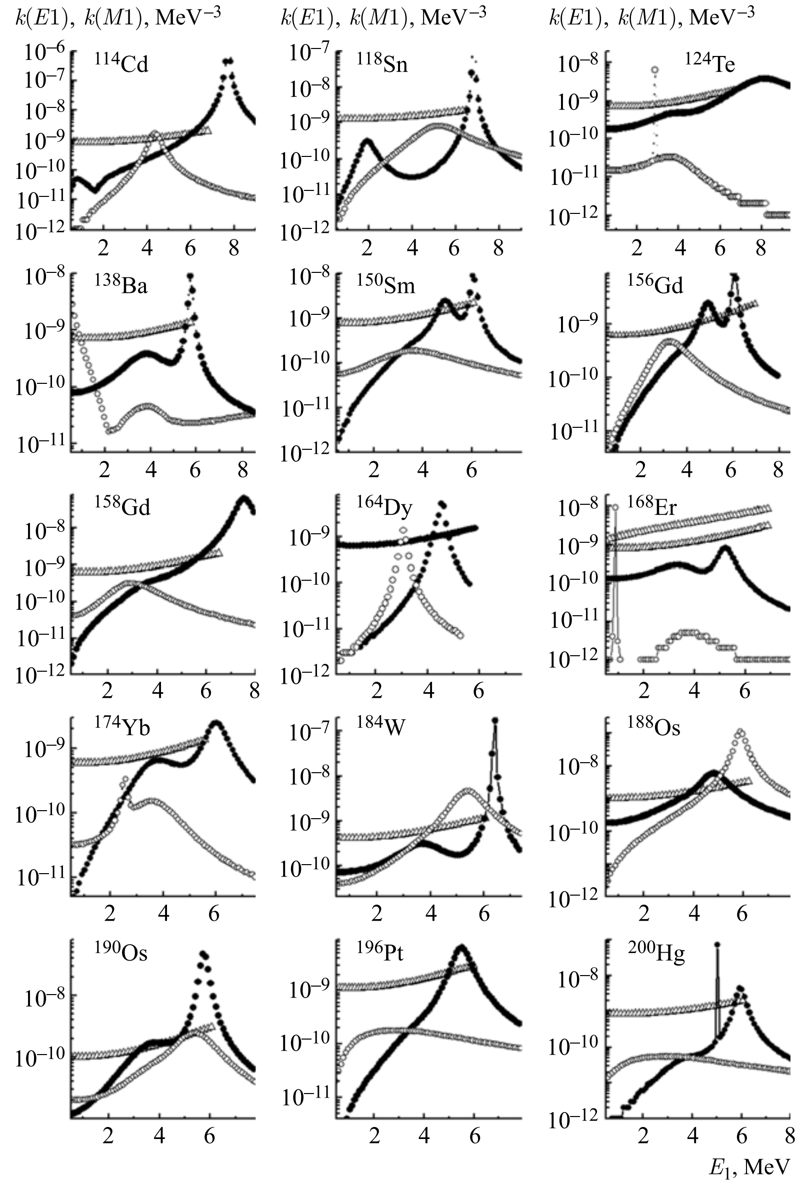

Fig. 8. The same (as in Fig. 7) for even–even nuclei

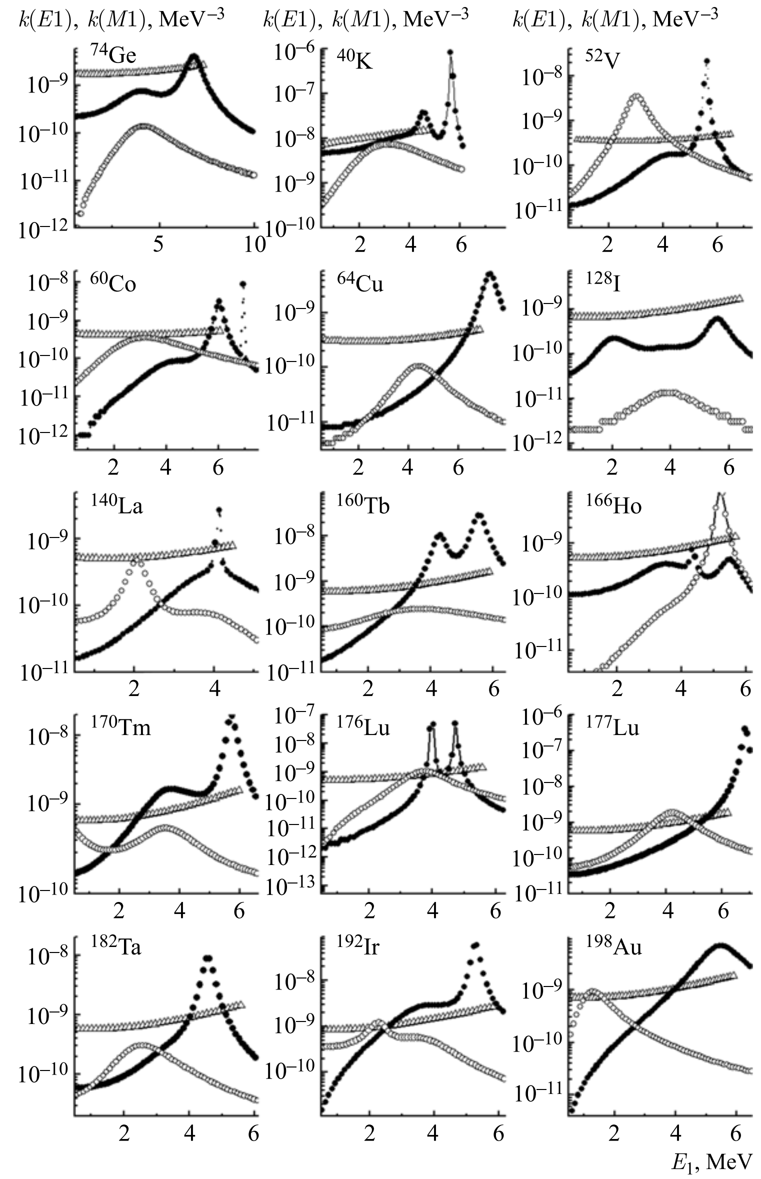

Fig. 9. The same (as in Fig. 7) for ^{74}Ge, ^{177}Lu, and odd–odd nuclei

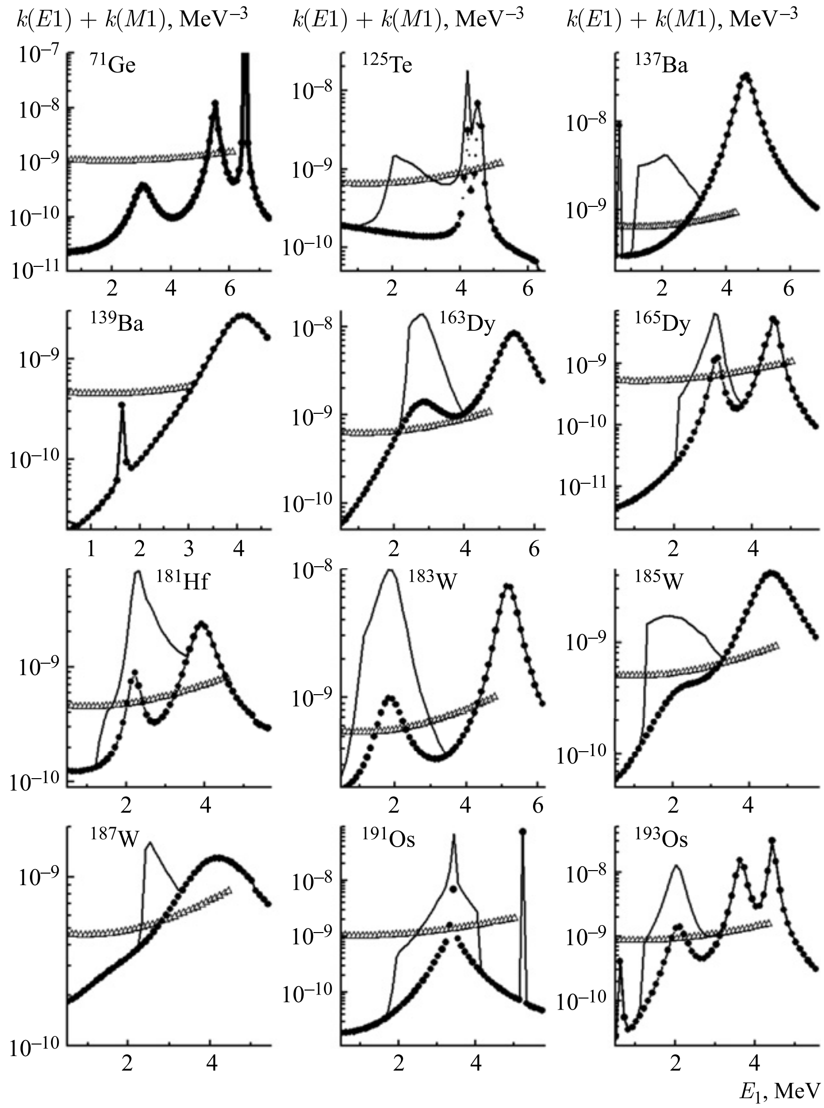

Fig. 10. Sums of strength functions of $E1$- and $M1$-transitions (black points) for even–odd nuclei depending on the energy of primary transition. Lines are fits with taking into account the correction (5). Triangles are calculations by model [15] adding $k(M1) = \mathrm{const}$ for $0 < E_1 \leqslant B_n - E_d$ energy range

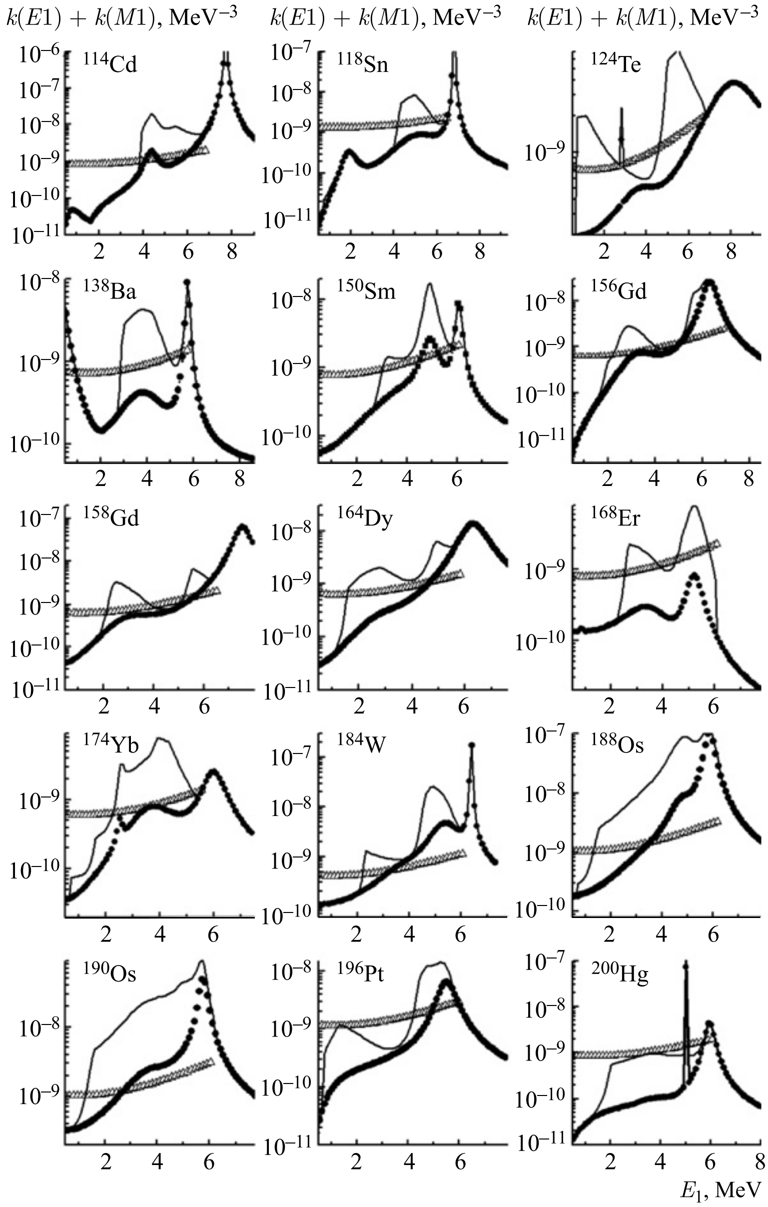

Fig. 11. The same (as in Fig. 10) for ^{74}Ge, ^{177}Lu, and odd–odd nuclei

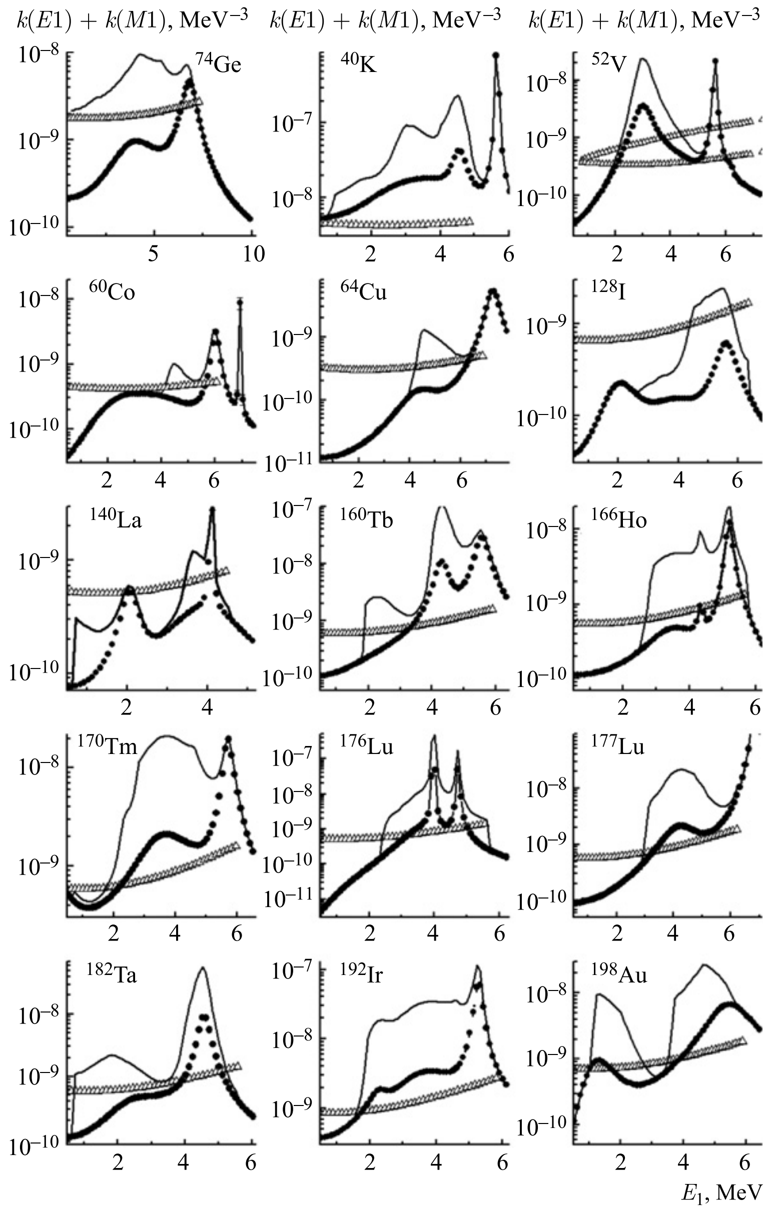

Fig. 12. The same (as in Fig. 7) for ^{74}Ge, ^{177}Lu, and odd–odd nuclei

ratio. An essential decrease of $k(M1) + k(E1)$ sum for small energies of gamma transitions is observed for all tested variants of functional dependences of strength functions. But an existence of asymptotical zero of sums of strength functions does not follow from Dubna model results. At that, a noticeable increase in strength functions of $E1$- or $M1$-transition near B_n and above this energy can be observed at sufficiently high energies of fragmented quasi-particle state. It means that radiative strength functions are not just an extrapolation of giant resonances (it contradicts the Axel–Brink hypothesis [7, 8] used earlier for gamma-spectra calculations).

In Fig. 13, mass dependences of breaking thresholds of the second and the third Cooper pairs are presented. As these values differ for nuclei with various

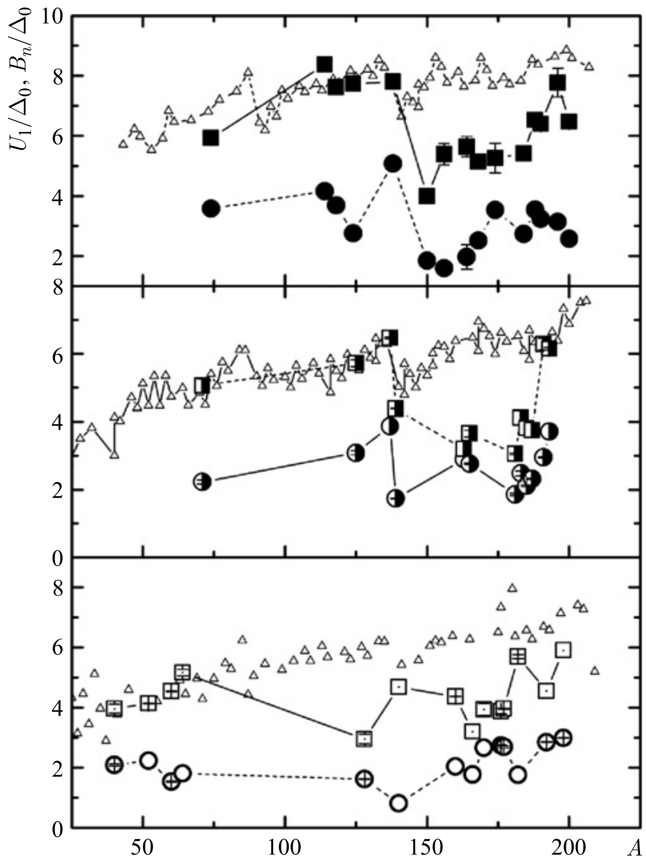

Fig. 13. Mass dependences of breaking thresholds of the second (points) and of the third (squares) Cooper pairs. Black points — even–even nuclei, half-open points — even–odd nuclei, open points – odd–odd compound-nuclei. Triangles are mass dependences of B_n/Δ_0

nucleon parities and depend on an average pairing energy Δ_0, they are shown separately and compared with B_n/Δ_0 (such as in [5]). It follows from this comparison that dependency of breaking thresholds of pairs on form of strength function is weak, and real correlation between ρ and Γ values is insignificant in experiments on the two-step cascade recording.

In Fig. 14, the fits of E_u parameter are shown. One can see practically complete coincidence of E_u fits with Δ_0 value for ≈ 30 nuclei. Causes of E_u scatter for the rest nuclei may be

• errors of normalization of experimental intensities of two-step cascades;

• unaccounted in model [12] possibility of breaking proton pairs together or instead of neutron pairs;

• inaccuracy of phenomenological part of the model;

• variability of Δ_0 experimental values [23].

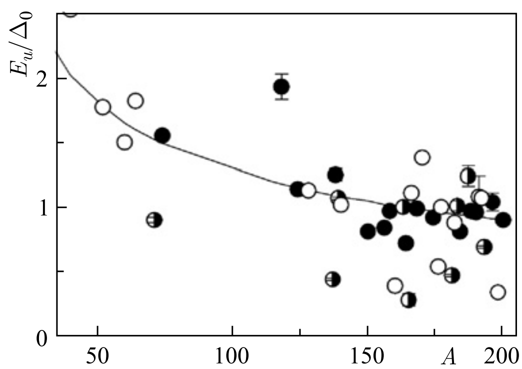

Fig. 14. Dependences of E_u parameter (2) on nuclear mass A. Black points — even–even nuclei, half-open points — even–odd nuclei, open points — odd–odd compound-nuclei

One cannot exclude also a possibility of various ratios of components of quasi-particle and phonon types in wave-function of resonance determined a capture cross section of thermal neutrons by any stable (or long-living) nucleus-target. In the up-to-date models [2], a total level density is equal to sum of densities of quasi-particle levels and collective ones. In Fig. 15, the ratios of collective (practically vibrational only) level density to the total density are presented. Near B_n these ratios are very similar for nuclei with any nucleon parity, but at E_d energy they are noticeable less for even–even nuclei than for even–odd and odd–odd ones.

All tested variants of Dubna model do not give reasons to suppose an existence of drastic changes of nuclear structure in $E_{\rm ex} = B_n$ energy point. Therefore, the data of Fig. 15 allow one to believe that neutron resonances can keep a different type of structure (with a dominance of quasi-particle or phonon components)

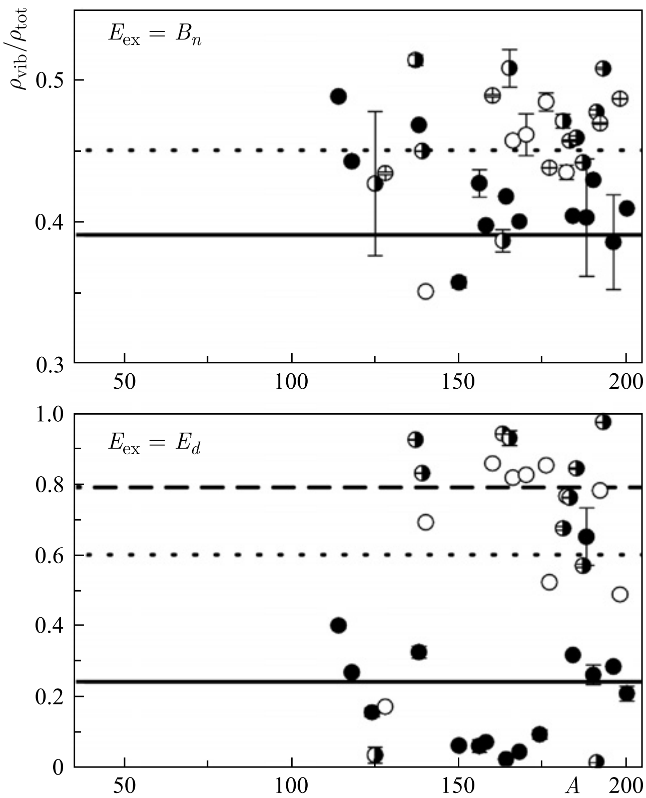

Fig. 15. Mass dependences of the ratio of vibrational level density to the total one near B_n energy (top) and for E_d energy point (bottom). Solid lines — the average of these ratios for even–even nuclei, dashed lines — for even–odd nuclei, and dotted lines — for odd–odd nuclei

of wave-functions and that they belong to some various distributions of reduced neutron resonance widths and total radiative ones.

In [24], an approximation of reduced neutron widths and total radiative widths of neutron resonances was done. At analysis it is supposed that experimental set of these widths is represented by a sum of distributions (from 1 to 4) with varied widths and positions of maximums of neutron amplitudes. For total radiative widths in nuclei with a number of resonances $\geqslant 170$ average parts of two the most intensive distributions are 44 and 34% of overall distribution of total radiative widths (it is close to 40%-part of vibrational levels).

Thus, two completely methodically-independent experiments show that the structure of the wave-functions differs for contiguous levels in a wide range of stable nuclei-targets up to B_n energy (and even at some higher energies).

The existence of nonprincipal distortion between the values of $E1$- and $M1$-strength functions (Fig. 10–12) and results of [5] are caused most likely by different degree of influence on χ^2 of various energy dependences (forms) of partial widths for peaks (4) at energy region of small functional values. At that, form variations for sums of $E1$- and $M1$-strength functions (Figs. 7–9) observed in different nuclei can be interpreted as existence of levels of various structures at excitation energy of 5–10 MeV.

CONCLUSIONS

Direct experimental information on dynamics of breaking 3–4 Cooper pairs of nucleons has been obtained. Systematic uncertainty of determination of breaking thresholds is not more than $\sim$ 1 MeV for the majority of available studied nuclei.

The data extracted with the use of

— model of n-quasi-particle level density [10] for description of sequential 3–4 Cooper pair breaking at energies below 5–10 MeV from Fermi-surface;

— phenomenological representations (2) about energy dependence of density of vibrational levels at the same energy range;

— composition of phenomenological and/or theoretical representations about form of energy dependences of widths of gamma-quanta emission

allow us to suppose that dynamics of interaction between fermion and boson states of nuclear matter depends on the form and parity of nucleon number of studied nucleus.

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