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TWO DIFFERENT DISTANCE SCALES OF q^2 AND V_{QCD} POTENTIAL OF PROTONS FROM π^-C INTERACTIONS AT 40 GeV/c

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Два разных масштаба в q^2 и потенциале V_{QCD} протонов
в π^-C-взаимодействиях при 40 ГэВ/c

В этой статье мы представили формулу для квадрата четырехимпульсов q^2 как функцию двух разных масштабов расстояний r_1 и r_2. Эта процедура требует учета дополнительной производной от формулы для q^2 (или от $\alpha_s(q^2)$). Мы рассчитали силу взаимодействия, используя потенциал $V_{\mathrm{QCD}}(r_2)$, и влияние на нее параметра r_2.

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Two Different Distance Scales of q^2 and V_{QCD} Potential
of Protons from π^-C Interactions at 40 GeV/c

In this paper we have expressed the formula for the four-momentum squared q^2 as a function of two different distance scales r_1 and r_2. This procedure necessitates taking into account an additional derivative from the formula for q^2 (or the running coupling constant of $\alpha_s(q^2)$). We have calculated the force using the potential $V_{\mathrm{QCD}}(r_2)$. The impact of r_2 parameter on the force has also been studied.

The investigation has been performed at the Veksler and Baldin Laboratory of High Energy Physics, JINR.

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INTRODUCTION

Investigations of the multiparticle production processes in hadron–nucleon (hN), hadron–nucleus (hA) and nucleus–nucleus (AA) interactions at high energies and large momentum transfers play a very important role in understanding the strong interaction mechanism and the inner quark–gluon structure of nuclear matter.

In the multiparticle production processes at high energies, the secondary particles are produced at different scattering angles with different values of momenta. Values of the scattering angle and momentum are mainly described by the square of the 4-dimensional transferred momentum q^2.

It is well known that in hA and AA collisions at high energies and large momentum transfers in comparison with hN interactions, the secondary particles are produced in the region kinematically forbidden for hN interactions. These particles are called cumulative because the production of these particles requires more than one nucleon mass from the target.

The hA and AA interactions at high energies are regarded as a unique tool to study the nuclear matter behavior under extreme conditions.

This paper is dedicated to the analysis of the formula for q^2, and we have shown that this formula can be written in the equivalent form depending on two different distance scales. So, on the basis of the experimental distributions of these two distances we assume that these distances presumably correspond to the extension and to the particle formation length of the multiparticle production process at high energies.

The description of the quark inside hadrons and nucleus remains an important problem of elementary particle physics. The asymptotic quantum chromodynamics (QCD) gives an opportunity to investigate quark interactions at small distances by using the standard perturbation theory. The quark dynamics at large distances (at small momentum transfers or in the confinement region) is not described by the perturbation theory. So, for this purpose other approaches are used: the potential model, string models, bag models, lattice calculations and so on.

The potential $V_{\mathrm{QCD}}(r_2)$ consists of terms corresponding to short- and long-range parts. It means that this potential gives the behavior of multiparticle production process at high energies in both short- and long-range physical parts together.

In this paper we have carried out the calculation of the potential $V_{\rm QCD}(r_2)$ with two differences in comparison with other models. The first one is applied in the calculation of the parameter r_2 of the formula obtained in this paper. As a result of this procedure, the calculation has been simplified.

The second difference is related with the calculation of the force which is determined as the first-order derivative of the potential $V_{\rm QCD}(r_2)$ by the parameter r_2. We have written and used the formula for q^2 depending on the parameter r_2.

This necessitates an additional derivative from the coupling constant of the strong interaction $\alpha_s(r_2)$ which is included in the formula for the potential $V_{\rm QCD}(r_2)$. As a result of this procedure, a new term appears in the formula for the force $F(r_2)$. This effect has been taken into account in this paper.

1. EXPERIMENTAL METHOD

The experimental material was obtained using the Dubna 2-meter propane (C_3H_8) bubble chamber exposed to π^- mesons with a momentum of 40 GeV/c from the Serpukhov accelerator. The advantage of the bubble chamber experiment in this paper is that the distributions are obtained under the condition of 4π geometry of secondary π^- mesons.

The average error of the momentum measurements is $\sim 12\%$, and the average error of the angular measurements is $\sim 0.6\%$.

In connection with the identification problem between energetic protons and π^+ mesons, the protons with a momentum more than ~ 1 GeV/c are included in π^+ mesons. The average boundary momentum from which protons are detected in this experiment is ~ 150 MeV/c. So, the secondary protons with a momentum from ~ 150 MeV/c to ~ 1 GeV/c are used for proton distributions. In this paper, we have taken the following reaction:

$$\pi^- + {\rm C} \longrightarrow p + X. \qquad (1)$$

8791 π^-C interactions have been used in this analysis; 12441 protons have been registered in these interactions.

2. ABOUT THE CUMULATIVE NUMBER n_c

The variable n_c called the cumulative number in the fixed target experiment is determined by the following formula [1, 2]:

$$n_c = \frac{P_a P_c}{P_a P_b} = \frac{E_c - \beta_a P_c^{||}}{m_p}, \qquad (2)$$

where P_a, P_b and P_c are the four-dimensional momenta of incident, target and secondary particles under consideration; E_c and $P_c^{||}$ are the energy and longitudinal momentum of the secondary particle; $\beta_a = P_a/E_a$ is the velocity of the incident particle. At high energy experiment $\beta_a \cong 1$ and m_p is the proton mass.

From formula (2) we see that the variable n_c is a relative invariant and dimensionless. Furthermore, this variable gives us an opportunity to know which particles in one event under consideration are produced in the cumulative region ($n_c > 1$) and vice versa.

3. THE SQUARE OF 4-MOMENTUM TRANSFER q^2 AND THE COUPLING CONSTANT OF STRONG INTERACTION $\alpha_s(r)$

As already mentioned in the introduction, the square of the transferred momentum t plays a very important role in the multiparticle production process at high energies.

The square of the transferred momentum t (or q^2) is determined by the following formula:

$$t = q^2 = -(P_a - P_c)^2 = 2E_a(E_c - \beta_a P_c^{||}) - (m_a^2 + m_c^2), \tag{3}$$

where E_a and m_a are the energy and mass of the incident particle, m_c is the rest mass of the secondary particles under consideration, and the other notations are the same as in subsection 2.

Formula (3) may be written in the following form using formula (2):

$$q^2 = 2E_a m_p n_c - (m_a^2 + m_c^2). \tag{3$'$}$$

We would like to note that formula (3′) gives the explicit dependence on the target mass, $m_t = m_p n_c$, which is required from the target to produce the secondary particle under consideration; i.e., this formula gives the explicit form of the mass dependence.

From the other hand, it is well known that the inverse mass gives a distance. So, now we have an opportunity to write formula (3′) on the distance in the form

$$q^2 = \frac{2E_a}{r_1} - (m_a^2 + m_c^2), \tag{3$''$}$$

where

$$r_1 = \frac{1}{m_p n_c} = \frac{\lambda_c^p}{n_c} = \frac{0.21\,\text{fm}}{n_c}. \tag{4}$$

Formulae (3′) and (3″) show that the square of the transferred momentum q^2 can be expressed by the target mass $m_p n_c$ or by the distance r_1 which is determined as the inverse target mass $(m_p n_c)^{-1}$.

From the other hand, the running coupling constant $\alpha_s(q^2)$ of the strong interaction is determined as follows:

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln \dfrac{q^2}{\Lambda^2}}, \tag{5}$$

where $\beta_0 = 11 - (2/3)N_f$ is the parameter described by a number of quark flavors N_f, Λ is the cut parameter of QCD. From formula (5) we see that the coupling constant $\alpha_s(q^2)$ has the quadratic dependence on the cut parameter Λ. In connection with this, the formula for q^2 is required to be the same quadratic dependence on the distance scale. This can be done easily. Multiplying and dividing the first term of equation (3′) by proton mass m_p, we get the following, equivalent to formula (3″):

$$q^2 = \frac{2E_a}{r_2^2 m_p} - (m_a^2 + m_c^2), \tag{3‴}$$

where

$$r_2^2 = \frac{1}{m_p^2 n_c} \quad \left(\text{or} \;\; r_2 = \frac{1}{m_p \sqrt{n_c}}\right). \tag{6}$$

So, we have obtained the result that the formula for q^2 (see formulae (3″), (3‴)) may be determined by two different distance scales r_1 and r_2.

Furthermore, from formulae (4) and (6) we see that these two different distances are simply connected with each other:

$$r_1 = \frac{r_2}{\sqrt{n_c}} \;\; \text{or} \;\; r_2 = r_1 \sqrt{n_c}\,. \tag{7}$$

This connection means that if we know one of these two parameters at the given value of the variable n_c, then the other one can be calculated using one of the formulae (7).

So, we have shown that the formula for q^2 can be expressed by two different distance scales r_1 and r_2. This means that formula (5) for the running coupling constant $\alpha_s(q^2)$ of the strong interaction can be expressed by these two distance scales r_1 and r_2 in the form

$$\alpha_s(r_1) = \frac{4\pi}{\beta_0 \ln \dfrac{q^2(r_1)}{\Lambda^2}} = \frac{4\pi}{\beta_0 \ln \left(\dfrac{2E_a}{r_1 \Lambda^2} - \dfrac{m_a^2 + m_c^2}{\Lambda^2}\right)}, \tag{5′}$$

or

$$\alpha_s(r_2^2) = \frac{4\pi}{\beta_0 \ln \dfrac{q^2(r_2^2)}{\Lambda^2}} = \frac{4\pi}{\beta_0 \ln \left(\dfrac{2E_a}{m_p r_2^2 \Lambda^2} - \dfrac{m_a^2 + m_c^2}{\Lambda^2}\right)}. \tag{5″}$$

So, we have shown that formula (5) for the running coupling constant can be expressed by the parameter r_1 or r_2 independently.

Now we will try to analyze and understand the physical meaning of these two parameters using the experimental data from π^-C interactions at 40 GeV/c.

Figure 1 shows the cumulative number n_c distribution of the secondary protons produced in π^-C interactions at 40 GeV/c.

From this distribution we see that many protons ($\sim 40\%$ of all protons) are produced in the cumulative particle production region $n_c > 1$ and this distribution continues until $n_c \approx 2.3$. The average value of the cumulative number is $n_c = 0.9612 \pm 0.0017$ [2].

Figure 2 shows the distance r_1 of the protons produced in π^-C interactions at 40 GeV/c. The average value of the distance is $\langle r_1 \rangle = 0.2278 \pm 0.0004$.

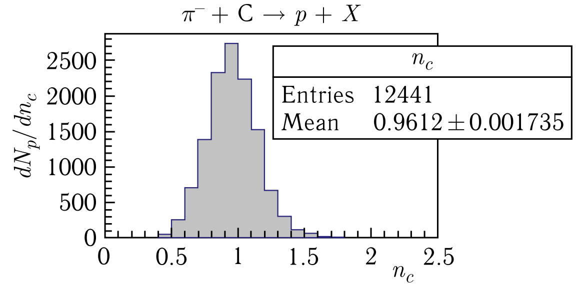

Fig. 1. Cumulative number n_c distribution of the secondary protons

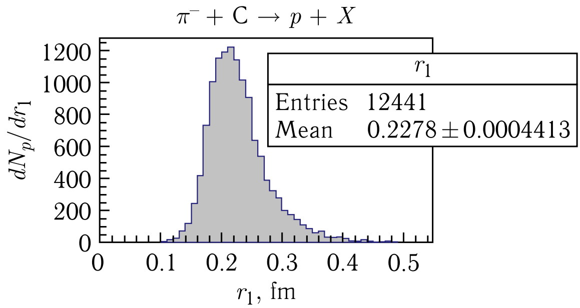

Fig. 2. Distance r_1 distribution of the secondary protons

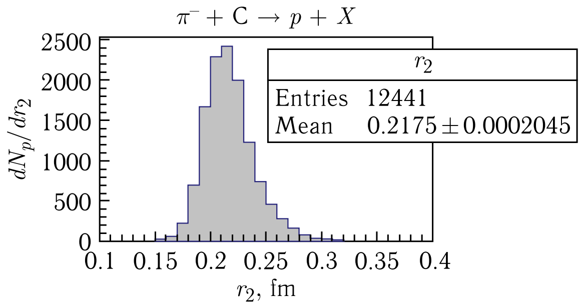

Fig. 3. Distance r_2 distribution of the secondary protons

Figure 3 shows the distribution of the distance r_2 of the protons produced in $\pi^-\mathrm{C}$ interactions at 40 GeV/c. The average value of the distance is $\langle r_2 \rangle = 0.2175 \pm 0.0002$.

Figure 4 shows dependences of the parameters r_1 and r_2 on the cumulative number n_c. As shown in subsection 3, these two distances r_1 and r_2 are connected with each other (see formula (7) and Fig. 4). From formulae (4) and (6) for the parameters r_1 and r_2 we see that $r_1 = r_2$ at $n_c = 1$; $r_1 > r_2$ at $n_c < 1$, and $r_1 < r_2$ at $n_c > 1$.

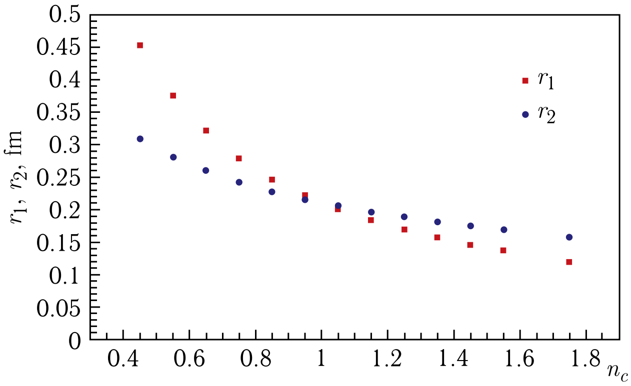

Fig. 4. Distances r_1 and r_2 as a function of the variable n_c

4. THE QCD POTENTIAL $V_{\mathrm{QCD}}(r_2)$

The QCD potential $V_{\mathrm{QCD}}(r_2)$ is often taken to be of the following form:

$$V_{\mathrm{QCD}}(r_2) = -\frac{4}{3}\frac{\alpha_s(q^2)}{r_2} + kr_2,$$

where the minus sign $(-)$ means the attractive interaction between quarks, 4/3 is the color factor, $k = 1$ GeV/fm is string tension, r_2 is the distance between quarks constituting hadrons, and $\alpha_s(q^2)$ is the running coupling constant of the strong interaction. In this paper we have carried out the calculations using only the distance r_2.

Figure 5 shows the dependence of the QCD potential V_{QCD} on the distance r_2. The parameter r_2 has been calculated by formula (6).

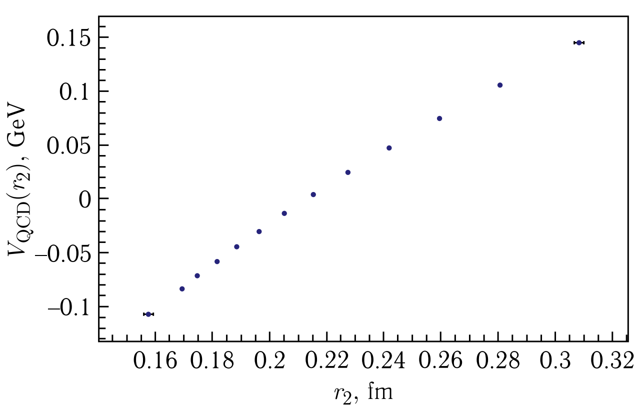

Fig. 5. Dependence of the QCD potential V_{QCD} on the parameter r_2

It is interesting to note that the positive and negative values of the potential $V_{\text{QCD}}(r_2)$ are separated by $r_2 = 0.21\,\text{fm}$ which is equal to the Compton wavelength of proton, λ_C^p. In other words, this point separates protons produced at short distances ($r_2 \leqslant 0.21\,\text{fm}$) from protons produced at long distances ($r_2 > 0.21\,\text{fm}$).

Using the QCD potential V_{QCD}, we can determine the force acting between quarks of hadrons by the following formula (Fig. 6):

$$F(r_2) = -\frac{dV_{\text{QCD}}(r_2)}{dr_2}. \tag{8}$$

Taking the first derivative from formula (8) of the potential, we obtain

$$F(r_2) = \left(-\frac{4a_0}{3}\frac{1}{r_2^2}\frac{1}{\ln\left(\frac{a_1}{r_2^2} - a_2\right)} - k\right) + \frac{4a_0}{3}\frac{1}{r_2^2}\frac{2a_1}{a_1 - a_2 r_2^2}\frac{1}{\left(\ln\left(\frac{a_1}{r_2^2} - a_2\right)\right)^2}, \tag{9}$$

where

$$a_0 = \frac{4\pi}{\beta_0}, \quad \beta_0 = 11 - \frac{2}{3}N_f,$$

$$a_1 = \frac{2E_a}{m_p\Lambda^2} = 2132.208\frac{1}{\text{GeV}^2} = 2132.208 \cdot 0.038809\,\text{fm}^2 = 82.7488\,\text{fm}^2,$$

$$a_2 = \frac{m_a^2 + m_c^2}{\Lambda^2} = \frac{0.899165\,\text{GeV}^2}{0.04\,\text{GeV}^2} = 22.4791.$$

In the calculation of the force using the formula for q^2 not depending on the explicit form from the distance, we do not need to take the derivative

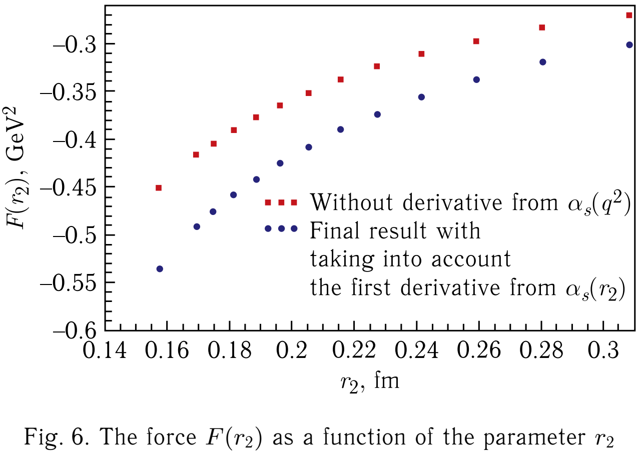

Fig. 6. The force $F(r_2)$ as a function of the parameter r_2

from q^2 (or $\alpha_s(q^2)$), see formula (3). But once we have expressed the formula for q^2 as a function of the parameter r_2 (see formula (5″)), we must take an additional derivative from the formula for q^2 by the parameter r_2.

This procedure gives us an additional second term with a positive sign in formula (9). In other words, this term gives the correction related to the additional derivative from the formula for q^2 (see formula (5″)) by the parameter r_2.

This procedure gives us the corrections from 10 to 15% to the force in comparison with the case without a derivative from q^2. The correction slightly increases as r_2 decreases.

Table 1

No.	n_c	$\langle n_c \rangle$	N_p	r_1, fm	r_2, fm
1	< 0.5	0.448 ± 0.0095	57	0.4534 ± 0.0032	0.3085 ± 0.001
2	0.5–0.6	0.560 ± 0.0016	255	0.3754 ± 0.0011	0.2807 ± 0.0004
3	0.6–0.7	0.656 ± 0.0010	705	0.3206 ± 0.0032	0.2594 ± 0.0002
4	0.7–0.8	0.755 ± 0.0007	1384	0.2785 ± 0.0005	0.2418 ± 0.0001
5	0.8–0.9	0.853 ± 0.0005	2328	0.2463 ± 0.0002	0.2274 ± 0.0001
6	0.9–1.0	0.949 ± 0.0005	2737	0.2214 ± 0.0001	0.2156 ± 0.0001
7	1.0–1.1	1.047 ± 0.0006	2239	0.2007 ± 0.0001	0.2053 ± 0.0001
8	1.1–1.2	1.145 ± 0.0007	1523	0.1834 ± 0.0001	0.1963 ± 0.0001
9	1.2–1.3	1.242 ± 0.0010	669	0.1692 ± 0.0001	0.1885 ± 0.0001
10	1.3–1.4	1.342 ± 0.0016	308	0.1565 ± 0.0001	0.1813 ± 0.0001
11	1.4–1.5	1.446 ± 0.0026	118	0.1453 ± 0.0002	0.1747 ± 0.0001
12	1.5–1.6	1.540 ± 0.0034	68	0.1364 ± 0.0003	0.1693 ± 0.0001
13	> 1.6	1.760 ± 0.0181	50	0.1186 ± 0.0017	0.1575 ± 0.0013

Table 2

No.	n_c	$\alpha_s(r_2)$	$V_{\rm QCD}(r_2)$, GeV	$F(r_2)$, GeV^2	$\Delta F(r_2)$, GeV^2
1	< 0.5	0.1928 ± 0.0002	$+0.1446 \pm 0.0015$	-0.2700 ± 0.0005	0.03195 ± 0.0001
2	0.5–0.6	0.1874 ± 0.0001	$+0.1057 \pm 0.0006$	-0.2839 ± 0.0002	0.03629 ± 0.0001
3	0.6–0.7	0.1832 ± 0.0001	$+0.0743 \pm 0.0003$	-0.2975 ± 0.0002	0.04044 ± 0.0001
4	0.7–0.8	0.1796 ± 0.0001	$+0.0471 \pm 0.0002$	-0.3114 ± 0.0001	0.04462 ± 0.0001
5	0.8–0.9	0.1765 ± 0.0001	$+0.0239 \pm 0.0001$	-0.3250 ± 0.0001	0.04866 ± 0.0001
6	0.9–1.0	0.1740 ± 0.0001	$+0.0041 \pm 0.0001$	-0.3381 ± 0.0001	0.05249 ± 0.0001
7	1.0–1.1	0.1717 ± 0.0001	-0.0139 ± 0.0001	-0.3515 ± 0.0001	0.05633 ± 0.0001
8	1.1–1.2	0.1697 ± 0.0001	-0.0302 ± 0.0001	-0.3647 ± 0.0001	0.06010 ± 0.0001
9	1.2–1.3	0.1679 ± 0.0001	-0.0448 ± 0.0002	-0.3776 ± 0.0001	0.06373 ± 0.0001
10	1.3–1.4	0.1662 ± 0.0001	-0.0588 ± 0.0002	-0.3910 ± 0.0002	0.06746 ± 0.0001
11	1.4–1.5	0.1646 ± 0.0001	-0.0722 ± 0.0003	-0.4048 ± 0.0003	0.07125 ± 0.0001
12	1.5–1.6	0.1633 ± 0.0001	-0.0835 ± 0.0004	-0.4172 ± 0.0004	0.07465 ± 0.0001
13	> 1.6	0.1606 ± 0.0002	-0.1070 ± 0.0017	-0.4516 ± 0.0062	0.08385 ± 0.0015

Note that in the case of not taking the derivative from the formula for q^2 by the parameter r_2, we obtain only the first term with a negative sign in the parentheses for the force formula (9).

The main characteristics of the parameters r_1 and r_2 and values of the potential and force are given in Tables 1 and 2.

CONCLUSIONS

• We have expressed the formula for q^2 as a function of the parameters r_1 and r_2 in the explicit form. This necessitates taking into account an additional derivative by the parameter r_2 (or r_1) from the running coupling constant $\alpha_s(r_2)$ in formula (7). As a result of this procedure, an additional term has appeared in the formula for the force, which gives the correction to the force from 10.5 to 15.5% in comparison with the case without derivative from $\alpha_s(r_2)$.

• On the basis of the experimental distributions of the parameters r_1 and r_2 we have assumed that the parameter r_1 may correspond to the expansion distance and the parameter r_2 corresponds to the particle formation length.

• The QCD potential $V_{\rm QCD}$ is presented for the first time using the formula for the particle formation length $r_2 = 1/m_p\sqrt{n_c} = 0.21\,{\rm fm}/\sqrt{n_c}$.

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