Special theory of relativity and conventionality

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The general properties of the clock synchronization in Special Theory of Relativity with different "one way" velocities of light are discussed. It is argued that the customary irreducible element of conventionality of the synchronization problem may be eliminated.

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Physical theories should be based on firm experimental facts. In the case of Special Theory of Relativity (STR) it is commonly believed that it is based on the Michelson–Morley experiment [1]. As a matter of fact this famous experiment only checked the invariance of the so-called "both way" velocity of light. The problem of the equality of the "one way" velocities is still under extensive discussion [2, 3].

In all existing literature the discussion of conventionality in STR is concentrated around the choice of clock's synchronization. It is however easy to see that this is an enormous simplification of the problem. In fact, in order to perform any operational procedure to establish spacetime coordinates of elementary events we need to specify at least the following items [4]:

1. The class of observers (including their experimental equipment).
2. The class of elementary events.
3. The communication system between the observers and elementary events.
4. The interaction of agents with the elementary events.
5. The synchronization condition for clocks.
6. The definition of the distance from the observer to the events.

The standard discussions of conventional elements in STR disregard the fact that both the observers and the elementary events are treated only as classical objects (we loose therefore from the beginning any possibility to incorporate quantum physics in STR), the communication system is based only on lights impulses (also treated according to the principles of classical physics) and the interaction of lights with events is based on the laws of geometrical optics.

To see the narrowness of the existing discussion of clock synchronization let us remind that Einstein–Bondi synchronization is based on the equality

$$c\,(t - T_1) = c\,(T_2 - t)\,, \tag{1}$$

where t is the time shown by the synchronized clock located at the event while T_1 and T_2 are the emission and detection times fixed by the observer. Condition (1) simultaneously is the definition of the distance x between the event and the observer. Clearly, from (1) we get

$$x = \frac{c}{2}\,(T_2 - T_1) \tag{2}$$

and

$$t = \frac{T_1 + T_2}{2}\,. \tag{3}$$

H. Reichenbach [2] questioned condition (1) on the basis of the fact that it assumes the equality of light in two opposite directions, a fact which is impossible to check experimentally without a closed circle of reasoning. Instead of (3) he proposed to use more general synchronization condition of the form

$$t = (1 - \epsilon)\,T_1 + \epsilon T_2\,, \tag{4}$$

where ϵ is an arbitrary parameter such that

$$0 < \epsilon < 1\,. \tag{5}$$

Reichenbach's proposal (4) is equivalent to the replacement of (1) by the more general condition of the form

$$c_+\,(t - T_1) = c_-\,(T_2 - t)\,, \tag{6}$$

where in general $c_+ \neq c_-$. From (6) it follows that

$$x = \frac{c_+ c_-}{c_+ + c_-}\,(T_2 - T_1) \tag{7}$$

and

$$t = \frac{c_+ T_1 + c_- T_2}{c_+ + c_-} = (1 - \epsilon)\,T_1 + \epsilon T_2\,. \tag{8}$$

From this formulas we see that the synchronization of clocks both in Einstein and Reichenbach cases is tightly connected with the definition of the distance defined from the *a priori* assumption that light moves uniformly [5] as all material bodies do in classical mechanics for which the distance is proportional to the time of travelling. Clearly such an approach cannot be defended as being free from some elements of convention. Indeed, in a more general case we could equally well replace conditions (1) and (6) by a general condition of the form

$$f_1\left(t - T_1\right) = f_2\left(T_2 - t\right), \tag{9}$$

where f_1 and f_2 are two monotonically increasing functions. Einstein synchronization assumes that both functions f_1 and f_2 are linear and $f_1 = f_2$. Consequently we come to the condition (1) independently from the shape of the function f which defines the notion of the distance between events. Reichenbach also assumed that both functions f_1 and f_2 are linear but $f_1 \neq f_2$ what implies condition (6) independently from the shape of the functions f_1 and f_2. The choice of the functions f_1 and f_2 obviously defines the notion of the distance in (modified) STR. There is no doubt that the definition of distance, in addition to the choice of clock's synchronization, is also an element of conventionality in STR. Unfortunately, this aspect of STR was never discussed by the followers of Reichenbach.

It is interesting to note that it is also possible to construct a relativity theory based on the synchronization condition of the type

$$f_1\left(\frac{t}{T_1}\right) = f_2\left(\frac{T_2}{t}\right), \tag{10}$$

where again we may use two arbitrary monotonic functions. For example, for the choice

$$f_1\left(z\right) = f_2\left(z\right) \sim \ln z \tag{11}$$

we shall get a Galilean model of spacetime.

In the Reichenbach original approach the invariant properties of both velocities c_+ and c_- are not discussed. If this invariance takes place we certainly need to modify the standard Lorentz transformations in such a way that they will respect these two invariant velocities. Such generalized Lorentz transformations were derived in [6]. The derivation is as follows. We start from the general linear transformations of the type

$$x \rightarrow x' = \alpha\left(u\right)\left(x - ut\right) \tag{12}$$

and

$$t \rightarrow t' = \beta\left(u\right)t + \gamma\left(u\right)x, \tag{13}$$

where $\alpha\left(u\right) > 0$, $\beta\left(u\right) > 0$ and $\gamma\left(u\right)$ are three functions of u, the velocity of relative motion of two inertial reference frames. In addition we assume that

$$\alpha\left(0\right) = \beta\left(0\right) = 1, \quad \gamma\left(0\right) = 0. \tag{14}$$

From (12) and (13) we get the transformation rule for velocities of motion in two reference frames in the form

$$v\left(t\right) \rightarrow v'\left(t'\right) = \frac{\alpha\left(u\right)\left[v\left(t\right) - u\right]}{\beta\left(u\right) + \gamma\left(u\right)v\left(t\right)}. \tag{15}$$

Clearly, the invariant velocities c satisfy the condition $c' = c$. Relation (15) then gives the following equation for such velocities

$$c^2\gamma(u) + [\beta(u) - \alpha(u)]\, c + u\alpha(u) = 0\,. \tag{16}$$

Treating this relation as a quadratic equation for c and using the Vieta theorem we get the following transformations of spacetime coordinates

$$x' = \alpha(u)(x - ut)\,, \tag{17}$$

$$t' = \alpha(u)\left[\left(1 + \frac{c_+ - c_-}{c_+c_-}\right)t - \frac{u}{c_+c_-}\,x\right], \tag{18}$$

with the following transformation rule for velocities

$$v'(t') = \frac{v(t) - u}{1 + \dfrac{c_+ - c_-}{c_+c_-}\,u - \dfrac{uv(t)}{c_+c_-}}\,. \tag{19}$$

The group property of the transformations (17) and (18) leads to the following functional equation for the function $\alpha(u)$

$$\alpha(u_{12}) = \alpha(u_1)\,\alpha(u_2)\left(1 + \frac{u_1u_2}{c_+c_-}\right), \tag{20}$$

where the composition law for velocities is given by

$$u_{12} = \frac{u_1 + u_2 + \dfrac{c_+ - c_-}{c_+c_-}\,u_1u_2}{1 + \dfrac{u_1u_2}{c_+c_-}}. \tag{21}$$

It is easy to check that equation (20) has the general solution of the form

$$\alpha(u) = \frac{\left(1 - \dfrac{u}{c_+}\right)^{\lambda}}{\left(1 + \dfrac{u}{c_-}\right)^{\lambda+1}}\,, \tag{22}$$

with an arbitrary parameter λ. Clearly, for the standard Lorentz transformations $c_+ = c_-$ and $\lambda = -\frac{1}{2}$.

The composition law (21) determines the structure of the new relativity group which admits $c_+ \neq c_-$. It leads to the unusual expression for the opposite velocity u^- to a given velocity u:

$$u^- = -\frac{u}{1 + \dfrac{c_+ - c_-}{c_+c_-}\,u}\,. \tag{23}$$

The denominator arises from the different scaling of space and time coordinates in (17) and (18), respectively. It is easy to check that

$$(c_+)^- = -c_- \,, \quad (-c_-)^- = c_+ \,. \tag{24}$$

Transformation rules (17) and (18) take into account the conventional character of the choice of c_+ and c_-. Under the change of the choice of (c_+, c_-) to another choice $(\widetilde{c}_+, \widetilde{c}_-)$ we arrive to the Reichenbach transformations of the type

$$x \to \widetilde{x} = sx \,, \quad t \to \widetilde{t} + kx \,, \tag{25}$$

where

$$s = \frac{(c_+ + c_-)\, \widetilde{c}_+ \widetilde{c}_-}{(\widetilde{c}_+ + \widetilde{c}_-)\, c_+ c_-} \tag{26}$$

and

$$k = \frac{c_+ \widetilde{c}_- - \widetilde{c}_+ c_-}{(\widetilde{c}_+ + \widetilde{c}_-)\, c_+ c_-} \,. \tag{27}$$

These transformations also form a group which we shall call the Reichenbach group. The composition law for this group reads

$$s_{12} = s_1 s_2 \,, \quad k_{12} = k_1 + s_1 k_2 \,. \tag{28}$$

From the Michelson–Morley experiment it follows that

$$\frac{1}{c_+} + \frac{1}{c_-} = \frac{2}{c} \,, \tag{29}$$

where c is the "two way" velocity of light. In view of (27) it means that in our physical world only the representations of the Reichenbach group with $s = 1$ are realized.

The Reichenbach transformations (25) imply the following transformation rule for the velocities of motion

$$v\,(t) \to \widetilde{v}\,\left(\widetilde{t}\right) = \frac{sv\,(t)}{1 + kv\,(t)} \,. \tag{30}$$

The nonlinear character of this transformation rule means that Reichenbach transformations act between different inertial reference frames [7]. The class of all inertial reference frames splits threfore into subclasses inside which the generalized Lorentz transformations (17) and (18) act. Between different subclasses act the Reichenbach transformations. Denoting the Reichenbach transformations (25) by $R\,(\widetilde{c}_+, \widetilde{c}_-; c_+, c_-)$ and the generalized Lorentz transformations (17) and (18) by $L\,(c_+, c_-; u)$ we can prove the commutativity relation

$$R\,(\widetilde{c}_+, \widetilde{c}_-; c_+, c_-)\, L\,(c_+, c_-; u) = L\,(\widetilde{c}_+, \widetilde{c}_-; \widetilde{u})\, R\,(\widetilde{c}_+, \widetilde{c}_-; c_+, c_-) \,, \tag{31}$$

which is the main result of our report. From this commutativity property we see that Lorentz transformations with different choices of c_+ and c_- are similar.

Therefore the groups of Lorentz transformations with various choices of c_+ and c_- always are isomorphic groups. On the level of Lie algebras this was shown in [8]. Therefore, the particular Einstein case of $c_+ = c_-$ can be translated into general case with $c_+ \neq c_-$. In this way we see that Einstein synchronization has an universal meaning and does not lead to any conventionality in STR.

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