Gravitation & Cosmology, Vol. 12 (2006), No. 2-3 (46-47), pp. 215–218
Proceedings of the International Conference on Gravitation, Cosmology, Astrophysics and Nonstationary Gas Dynamics, Dedicated to Prof. K.P. Staniukovich's 90th birthday, Moscow, 2–6 March 2006

SPINOR FIELD AND ACCELERATED REGIMES IN COSMOLOGY

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Received 25 April 2006

A self-consistent system of interacting nonlinear spinor and scalar fields is considered in a Bianchi type-I universe filled with a perfect fluid. The role of spinor field in the evolution of the Universe is studied. It is shown that for some special choice of nonlinearity the spinor field can explain the late-time accelerated expansion of the Universe.

Спинорное поле и ускоренные фазы в космологии
Биджан Саха

Рассмотрена самосогласованная система взаимодействующих нелинейных спинорного и скалярного полей во Вселенной I типа Бианки, заполненной идеальной жидкостью. Исследуется роль спинорного поля в эволюции Вселенной. Показано, что при некотором специальном выборе нелинейности спинорное поле может объяснить позднее ускоренное расширение Вселенной.

1. Introduction

The accelerated expansion of the present-day Universe encourages many researchers to introduce different kinds of sources able to explain this. Among them most popular is the dark energy given by a Λ term [1–3], quintessence [4–7] and Chaplygin gas [8, 9]. Recently, cosmological models with spinor field have been extensively studied by a number of authors in a series of papers [10–15]. The principal motivation of Refs. [10–14] was to find regular solutions of the corresponding field equations. In some special cases, namely, with a cosmological constant (Λ term), which plays the role of an additional gravitation field, we indeed find singularity-free solutions. It was also found that introduction of a nonlinear spinor field results in a rapid growth of the Universe. This allows us to consider the spinor field as a possible candidate for explaining the accelerated expansion. Note that a similar attempt was made in a recent paper by Kremer *et al.* [16].

In this paper, we study the role of a spinor field in generating an accelerated mode of expansion of the Universe. Since similar systems, though from different aspects, were thoroughly studied in [13, 14], to avoid lengthy calculations regarding spinor and scalar fields, we mainly confine ourselves to studying the master equation describing the evolution of BI Universe. We here give solutions to the spinor and scalar field equations, whose details can be found in [13, 14].

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2. Basic equations and their solutions

We consider a self-consistent system of nonlinear spinor and scalar fields in a Bianchi type-I universe filled with a perfect fluid. The spinor and the scalar field are given by the Lagrangian

$$\mathcal{L} = (i/2)\Big[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\Big] - m\bar\psi\psi + F + (1/2)(1+\lambda_1 F_1)\,\varphi_{,\alpha}\varphi^{,\alpha}, \tag{1}$$

where λ_1 is a coupling constant and F and F_1 are arbitrary functions of invariants generated by real bilinear forms of a spinor field. We assume $F = F(I, J)$ and $F_1 = F_1(I, J)$ with $I = S^2$, $S = \bar\psi\psi$, $J = P^2$ and $P = i\bar\psi\gamma^5\psi$.

The gravitational field is chosen in the form

$$ds^2 = dt^2 - a_1^2 dx_1^2 - a_2^2 dx_2^2 - a_3^2 dx_3^2, \tag{2}$$

where a_i are functions of t only and the speed of light is taken to be unity. We also define

$$\tau = a_1 a_2 a_3. \tag{3}$$

We assume the spinor and scalar field to be space-independent. Then, for the spinor and scalar fields and metric functions we find the following expressions [14].

For $F = F(I)$, we find $S = C_0/\tau$, C_0 being an integration constant, and the spinor field components read

$$\begin{aligned} \psi_{1,2}(t) &= (C_{1,2}/\sqrt{\tau})\,\mathrm{e}^{-i\beta}, \\ \psi_{3,4}(t) &= (C_{3,4}/\sqrt{\tau})\,\mathrm{e}^{i\beta}, \end{aligned} \tag{4}$$

with the integration constants obeying

$$C_0 = C_1^2 + C_2^2 - C_3^2 - C_4^2.$$

Here $\beta = \int (m - \mathcal{D})dt$ with

$$\mathcal{D} = dF/dS + (\lambda_1 \dot\varphi^2/2)dF_1/dS.$$

For $F = F(J)$ and a massless spinor field we find $P = D_0/\tau$. The corresponding components of the spinor field in this case read

$$\begin{aligned} \psi_{1,2} &= \big(D_{1,2}\,\mathrm{e}^{i\sigma} + iD_{3,4}\,\mathrm{e}^{-i\sigma}\big)/\sqrt{\tau}, \\ \psi_{3,4} &= \big(iD_{1,2}\,\mathrm{e}^{i\sigma} + D_{3,4}\,\mathrm{e}^{-i\sigma}\big)/\sqrt{\tau}, \end{aligned} \tag{5}$$

where

$$D_0 = 2\,(D_1^2 + D_2^2 - D_3^2 - D_4^2).$$

Here $\sigma = \int \mathcal{G} dt$ with

$$\mathcal{G} = dF/dP + (\lambda_1 \dot\varphi^2/2)dF_1/dP.$$

For the scalar field we find

$$\varphi = C\int \frac{dt}{\tau(1+\lambda_1 F_1)} + C_1, \tag{6}$$

where C and C_1 are integration constants.

Solving the Einstein equations, we find

$$a_i(t) = A_i\Big[\tau(t)]^{1/3}\exp\big[X_i\int [\tau(t')]^{-1}dt'\Big], \tag{7}$$

with the integration constants A_i and X_i obeying $A_1A_2A_3 = 1$ and $X_1 + X_2 + X_3 = 0$. Note that to evaluate the metric functions at any given time $\tilde t$ we should first integrate $\int dt/\tau$, and only then substitute t by $\tilde t$.

Theoretical arguments [17] and the recent experimental data which support the existence of an anisotropic phase before an isotropic one, led us to consider models of the Universe with an anisotropic background. On the other hand, the isotropy of the present-day Universe lead us to study how the initially anisotropic BI space-time can evolve into an isotropic Friedmann-Robertson-Walker (FRW) one. Since for the FRW Universe $a_1(t) = a_2(t) = a_3(t)$, for the BI universe to evolve into a FRW one we should set $D_1 = D_2 = D_3 = 1$. Moreover, the isotropic nature of the present Universe implies that the three other constants X_i should be close to zero as well, i.e., $|X_i| \ll 1$ $(i = 1,2,3)$, so that $X_i \int [\tau(t)]^{-1}dt \to 0$ for $t < \infty$ (for $\tau(t) = t^n$ with $n > 1$ the integral tends to zero as $t \to \infty$ for any X_i). A rapid growth of the Universe due to introduction of the nonlinear spinor field to the system results in earlier isotropization.

The spinor, scalar and metric functions functionally depend on τ, as well as other physical quantities such as the spin current, charge etc. and the space-time invariants [13, 14]. It should be noted that at any space-time point where $\tau = 0$ the spinor, scalar and gravitational fields become infinite, hence the space-time becomes singular [14]. So it is very important to study the equation for τ (which can be viewed as the master equation) in detail, precisely what we will do in the next section. In doing so we analyze the role of the spinor field in the character of the evolution.

3. Evolution of a BI universe and the role of spinor field

In this section we study the role of the spinor field in the evolution of the Universe. From the Einstein equations one finds an equation for τ which is indeed the acceleration equation and has the following general form:

$$\frac{\ddot\tau}{\tau} = \frac{3}{2}\kappa\Big(T_1^1 + T_0^0\Big). \tag{8}$$

The Bianchi identity $G_{\mu;\nu}^{\nu} = 0$ in our case gives

$$\dot T_0^0 = -\frac{\dot\tau}{\tau}\big(T_0^0 - T_1^1\big). \tag{9}$$

The spinor field, as well as any other source, affects the evolution of the Universe by its energy-momentum tensor only. With $p_{\rm pf} = \zeta\varepsilon_{\rm pf}$ and $\zeta \in [0,1]$, where $\varepsilon_{\rm pf}$ and $p_{\rm pf}$ are the energy density and pressure of a perfect fluid, for this fluid Eq. (9) gives

$$\varepsilon_{\rm pf} = \frac{\varepsilon_0}{\tau^{1+\zeta}}, \qquad p_{\rm pf} = \frac{\zeta_0\varepsilon_0}{\tau^{1+\zeta}}.$$

Further we set $\varepsilon_0 = 1$. Assume that $F = \lambda S^q$ and $F_1 = S^r$ where λ is a self-coupling constant. As was shown in [13], the spinor field equation, more precisely, the equations for bilinear spinor forms, in this case gives $S = C_0/\tau$. Then, setting $C_0 = 1$, for the energy density and the pressure we find

$$\begin{aligned} T_0^0 &= \frac{m}{\tau} - \frac{\lambda}{\tau^q} + \frac{\tau^{r-2}}{2(\lambda_1 + \tau^r)} + \frac{1}{\tau^{1+\zeta}} \equiv \varepsilon, \\ T_1^1 &= \frac{(q-1)\lambda}{\tau^q} - \frac{[(1-r)\lambda_1 + \tau^r]\tau^{r-2}}{2(\lambda_1 + \tau^r)^2} - \frac{\zeta}{\tau^{1+\zeta}} \equiv -p. \end{aligned} \tag{10}$$

Taking into account that ε and p are functions of τ only, Eq. (8) can now be presented as

$$\ddot\tau = \mathcal{F}(q_1, \tau), \tag{11}$$

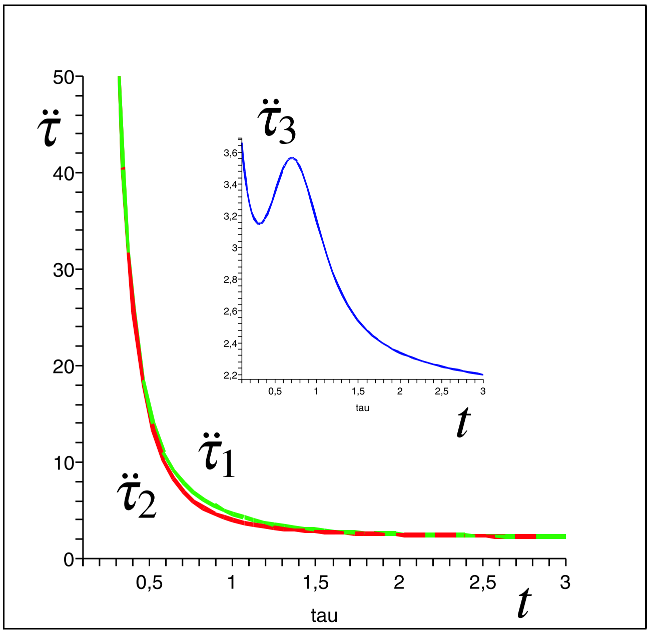

Figure 1: Acceleration of BI universe caused by spinor and scalar fields

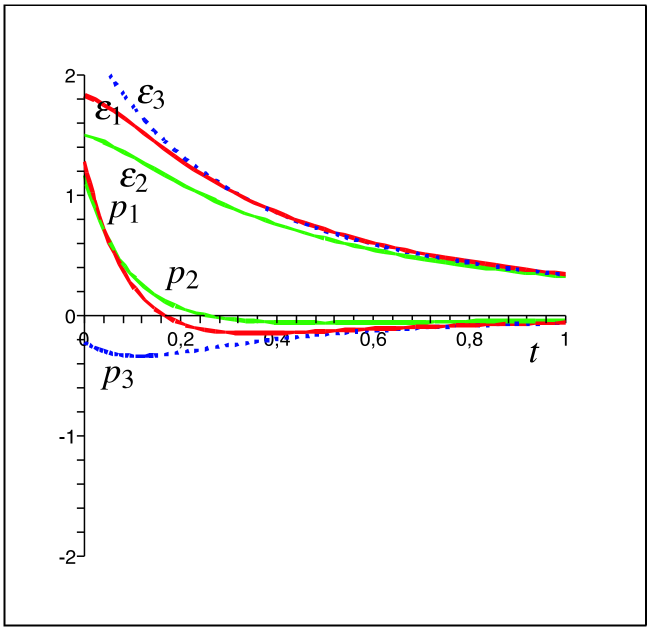

Figure 2: Energy density and pressure in three different cases

where we define

$$\mathcal{F}(q_1,\tau) = \frac{3\kappa}{2}\Bigl(m + \frac{\lambda(q-2)}{\tau^{q-1}} + \frac{\lambda_1 r \tau^{r-1}}{2(\lambda_1+\tau^r)^2} + \frac{1-\zeta}{\tau^\zeta}\Bigr). \tag{12}$$

Here $q_1 = \{\kappa, m, \lambda, \lambda_1, q, r, \zeta\}$ is the set of problem parameters. Eq. (11) allows the first integral

$$\dot{\tau} = \sqrt{2[E - \mathcal{U}(q_1,\tau)]} \tag{13}$$

where we denote

$$\mathcal{U}(q_1,\tau) = -\frac{3\kappa}{2}\Bigl(m\tau - \frac{\lambda}{\tau^{q-2}} - \frac{\lambda_1}{2(\lambda_1+\tau^r)} + \tau^{1-\zeta}\Bigr). \tag{14}$$

From a mechanical point of view, Eq. (11) can be interpreted as an equation of motion of a single particle with unit mass under the force $\mathcal{F}(q_1,\tau)$. In (13), E is an integration constant which can be treated as an energy level, and $\mathcal{U}(q_1,\tau)$ is the potential of the force $\mathcal{F}(q_1,\tau)$. We solve Eq. (11) numerically using the Runge-Kutta method. The initial value of τ is taken to be reasonably small, while the corresponding first derivative $\dot{\tau}$ is evaluated from (13) for given E. As is seen, the positivity of the radical imposes some restriction on the value of τ, namely, in case $\lambda > 0$ and $q \geq 2$ the value of τ cannot be too close to zero at any point. As can be easily seen from Eq. (14), there is a potential wall of infinite height in the vicinity of $\tau \to 0$. Thus we can see that, for some special choice of the problem parameters, introduction of a nonlinear spinor field provides singularity-free solutions. As was shown in [13], a regular solution is obtained only at the expense of a broken dominant-energy condition in the Hawking-Penrose theorem [18]. It is also shown that the spinor field nonlinearity gives rise to an accelerated mode of expansion at an early stage of evolution. To show that, let us go back to Eq. (11). The left-hand side of this equation describes acceleration. From (12) one sees that, as $\tau \to \infty$, $\ddot{\tau} \to (3/2)\kappa m > 0$, i.e., a massive spinor field provides a nontrivial acceleration at large τ, while the spinor field nonlinearity generated by the self-action gives rise to an exponential mode of expansion at the early stage of evolution. As one sees from Fig. 1, with self-action, the acceleration at the early stage is very large, whereas in the case of induced nonlinearity it is not the case. With expansion, the acceleration becomes small and finally tends to $(3/2)\kappa m$.

For numerical solutions we set $\kappa = 1$, the spinor mass $m = 1$, the power of nonlinearity $q = 4$, $r = 4$ and, for the perfect fluid, we set $\zeta = 1/3$ which corresponds to radiation. We also set $E = 1$. The initial value of τ is taken to be $\tau_0 = 0.4$. In the figures, we use the following notations: "1" corresponds to the case with self-action and interaction, $\lambda = 0.5$, $\lambda_1 = 0.5$; "2" to self-action only, $\lambda = 0.5$; "3" to interaction only, $\lambda = 0$, $\lambda_1 = 0.5$.

Fig. 2 shows the energy density and pressure of the source field. The purpose is to show that the energy density of the source field indeed decreases with the Universe expansion, whereas the pressure becomes negative, which lets us conclude that the spinor field can be viewed as an alternative to dark energy, since the latter (e.g., quintessence or Chaplygin gas), invoked to explain the late-time acceleration of the Universe, has negative pressure.

4. Conclusion

We have considered a system of interacting nonlinear spinor and scalar fields in a BI universe filled with perfect fluid. It is shown that the spinor field nonlinearity generated by a self-action gives rise to a singularity-free evolution. But this regular solution violates the dominant energy condition which is essential in the Hawking-Penrose theorem. It is also shown that, at the early stage of evolution, the nonlinear spinor field generates an accelerated mode of evolution. Comparing the effective pressure of the nonlinear spinor field with that of quintessence we notice some similarities in their behaviour. It gives us a reason to consider a nonlinear spinor field as an alternative to dark energy.

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