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Soliton Model of Extended Quantum Particles

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ABSTRACT. Some first principles that, we believe, could serve as foundation for quantum theory of extended particles are formulated. It is also shown that in the point-like particles limit the non-relativistic quantum mechanics can be restored. Bohm problem of nonlinear resonance has been considered and its possible solution has been given. Within the frame-work of the Einstein-de Broglie soliton model a hydrogen atom has been simulated.

RÉSUMÉ. Nous formulons quelques principes fondamentaux qui pourraient, à notre avis, servir de base de la théorie quantique des particules étendues. Nous montrons aussi que dans la limite des particules ponctuelles la mécanique quantique non relativiste se rétablit. Nous considérons le problème posé par D. Bohm, de la résonance non linéaire, dont nous proposons la solution. Dans les cadres du schéma non linéaire d'Einstein - de Broglie nous simulons le modèle solitonien de l'atome à type hydrogène.

Preface

"The questions of causal interpretation of Quantum Mechanics had always been among the fields of interests of Professor Georges Lochak who actively popularized de Broglie's views on Quantum Mechanics as a linear limit of some essentially nonlinear field theory. Dedicating present work to Professor Georges Lochak, we express the hope that the ideas of the soliton interpretation of Quantum Mechanics lay also in the field of his interests."

1 Introduction

The de Broglie-Bohm causal interpretation of quantum mechanics [1, 2, 3, 4, 5] represents a consistent counterexample to the historically dominant view that in quantum mechanics observational events cannot be causally connected by a continuous sequence of unique and well defined intermediate events. As early as 1927 in the framework of his "theory of double solution" Louis de Broglie made an attempt to represent the electron as a source of waves obeying the Schrödinger equation [6]. Later he modified his model showing that the electron should be described by regular solutions to some nonlinear equation coinciding with the Schrödinger one in the linear approximation. This scheme became famous as a causal nonlinear interpretation of quantum mechanics [1]. Developing this concept, de Broglie remarked that it had much in common with Einstein's ideas about unified field theory according to which particles were to be considered as clots of some material fields obeying the nonlinear field equations [7]. In recent years, these types of field configurations, known as soliton or particle-like solutions, came into active use to model extended elementary particles [8].

2 Basic Principles and Bohm Problem of Nonlinear Resonance

To begin with let us formulate the first principles for possible quantum theory of extended particles:

- Following A. Einstein and L. de Broglie we describe the extended particles by the stable soliton-like solutions to non-linear field equations.
- Along the line of D. Bohm's thought we accept that the wave properties of particles have the origin in non-linear resonance effect.
- We assume that the statistical properties of particles can be deduced in the point-like limit from an analog of the wave function describing the Blokhintsev quantum statistical ensemble of extended particles.

To illustrate these principles we consider the simplest scalar field model given by the Lagrangian in the Minkowski space-time

$$L_0 = \partial_i \phi^* \partial_j \phi \eta^{ij} - (mc/\hbar)^2 \phi^* \phi + F(S), \quad S = \phi^* \phi, \tag{1}$$

with $F(S)$ behaving as S^n, $n > 1$, for $S \to 0$. This model admits, for many choices of F, e.g., $F = kS^n,\ k > 0,\ 1 < n < 5/3$, stable soliton-like solution of stationary type

$$\phi_0 = u(r)e^{-i\omega_0 t}, \quad r = |\mathbf{r}|, \quad u^\star = u, \tag{2}$$

with the energy

$$E = \int d^3x T^{00}(\phi_0), \quad T^{00} = u'^2 + u^2\left[(mc/\hbar)^2 + \omega_0^2\right] - F(u^2), \tag{3}$$

and the electric charge

$$Q = e\omega_0 \int d^3xu^2. \tag{4}$$

D. Bohm in his book "Causality and Chance in Modern Physics" discussed the following problem. Let $\phi = \phi_0 + \xi(t, \mathbf{r})$ describes the perturbed soliton-like solution. D. Bohm [9] put the following question: "Does there exist any nonlinear model for which the spatial asymptote of $\xi(r \to \infty)$ represents oscillations with characteristic frequency

$$\omega = E/\hbar?'' \tag{5}$$

As is clear from the structure of the Lagrangian (1), at spatial infinity the field equation reduces to the linear Klein-Gordon one

$$[\,\Box - (mc/\hbar)^2]\phi = 0, \tag{6}$$

and therefore the principle of non-linear resonance by Bohm (5) holds only for solitons with the energy $E = mc^2$. It shows that the universality of the Planck-de Broglie relation (5) fails. To reinstate the universality of the relation (5) we modify the model (1) including gravity:

$$L = c^4R/16\pi G + \partial_i\phi^*\partial_j\phi\eta^{ij} - I(g_{ij})\phi^*\phi + F(\phi^*\phi). \tag{7}$$

The crucial point of the model is to choose the invariant $I(g_{ij})$ with the asymptotic property

$$\lim_{r\to\infty} I(g_{ij}) = (mc/\hbar)^2, \tag{8}$$

where m stands for the Schwarzschild mass of the soliton. It can be verified that the relation (8) holds if one chooses

$$I = (I_1^4/I_2^3)c^6\hbar^{-2}G^{-2}, \tag{9}$$

where $I_1 = R_{ijkl}R^{ijkl}/48, \quad I_2 = -R_{ijkl;n}R^{ijkl;n}/432$. Estimating R^{ijkl} at large distance one finds $I_1 = G^2m^2/(c^4r^6), \quad I_2 = G^2m^2/(c^4r^8)$. Thus we conclude that the principle of wave-particle duality has gravitational origin in our model [10, 12].

3 Fundamental Equations and their Solutions

Let us now construct the analog of the wave function. Suppose that the field ϕ describes n particles and has the form

$$\phi(t, \mathbf{r}) = \sum_{k=1}^{n} \phi^{(k)}(t, \mathbf{r}), \tag{10}$$

where

$$\mathrm{supp}\, \phi^{(k)} \cap \ \mathrm{supp}\, \phi^{(k')} = 0, \quad k \neq k',$$

and the same for the conjugate momenta

$$\pi(t, \mathbf{r}) = \partial L/\partial\phi_t = \sum_{k=1}^{n} \pi^{(k)}(t, \mathbf{r}), \quad \phi_t = \partial\phi/\partial t.$$

Let us define the auxiliary functions

$$\varphi^{(k)}(t, \mathbf{r}) = \frac{1}{\sqrt{2}}(\nu_k\phi^{(k)} + i\pi^{(k)}/\nu_k) \tag{11}$$

with the constants ν_k satisfying the normalization condition

$$\hbar = \int d^3x|\varphi^{(k)}|^2. \tag{12}$$

Now we define the analog of the wave function in the configurational space $\{\mathbf{r}_1, \cdots \mathbf{r}_n\} \in \Re^{3n}$ as

$$\Psi_N(t, \mathbf{r}_1, \cdots \mathbf{r}_n) = (\hbar^n N)^{-1/2} \sum_{i=1}^{N} \prod_{k=1}^{n} \varphi_i^{(k)}(t, \mathbf{r}_k), \tag{13}$$

where $N \gg 1$ stands for the number of trials (observations) and $\varphi_i^{(k)}$ is the one-particle function (11) for the i -th trial. It can be shown [1] that the quantity

$$\rho_N = \frac{1}{(\triangle\vee)^n} \int\limits_{(\triangle\vee)^n \subset \Re^{3n}} d^{3n}x|\Psi_N|^2,$$

where $\triangle\vee$ is the elementary volume which is supposed to be much greater than the proper volume of the particle $\vee_0 \ll \triangle\vee$, plays the role of coordinate probability density. If we choose the classical observable A with the generator $\hat{M}_A$, one can represent it in the form

$$A_j = \int d^3x \pi_j i\hat{M}_A \phi_j = \sum_{k=1}^{n} \int d^3x \varphi_j^{*(k)} \hat{M}_A^{(k)} \varphi_j^{(k)},$$

for the j - th trial. The corresponding mean value is

$$\begin{aligned} < A > &= \frac{1}{N} \sum_{j=1}^{N} A_j = \frac{1}{N} \sum_{j=1}^{N} \sum_{k=1}^{n} \int d^3x \varphi_j^{*(k)} \hat{M}_A^{(k)} \varphi_j^{(k)} \\ &= \int d^3x \Psi_N^* \hat{A} \Psi_N + O(\frac{\vee_0}{\triangle\vee}) \end{aligned} \tag{14}$$

where the hermitian operator $\hat{A}$ reads

$$\hat{A} = \sum_{k=1}^{n} \hbar \hat{M}_A^{(k)}. \tag{15}$$

Thus, up to the terms of the order $\vee_0/\triangle\vee \ll 1$, we obtain the standard quantum mechanical rule for the calculation of mean values [12]. It is interesting to underline that the solitonian scheme contains also the well-known spin - statistic correlation [10]. Namely, if $\varphi_i^{(k)}$ is transformed under the group rotation by irreducible representation $D^{(J)}$ of $SO(3)$, then the transposition of two identical extended particles is equivalent to the relative 2π rotation of $\varphi_i^{(k)}$ that gives the multiplication factor $(-1)^{2J}$ in Ψ_N. It can be also proved that Ψ_N upto the terms of order $\vee_0/\triangle\vee$ satisfies the standard Schrödinger equation [11].

Now we apply the solitonian scheme to the hydrogen atom [13, 14]. Let us introduce the nucleus Coulomb field $A_i^{\text{ext}} = \delta_i^0 Ze/r$ and consider the scalar field Lagrangian density

$$\mathcal{L} = -\frac{1}{16\pi}(F_{ik})^2 + |[\partial_k - i\varepsilon(A_k + A_k^{\text{ext}})]\phi|^2 - (mc/\hbar)^2\phi^*\phi + F(\phi^*\phi), \tag{16}$$

where $\varepsilon = e/\hbar c$. Suppose that for $A_k^{\text{ext}} = 0$ the field equations admit stable stationary soliton-like solution of type (2) describing configurations with mass m and electric charge e. For simplicity we omit the gravitational field supposing that it has been taken into account due to the non-linear resonance condition (5). Then, in the non-relativistic approximation we may put

$$\phi = \psi\,\exp\left(-imc^2t/\hbar\right). \tag{17}$$

Therefore, the corresponding field equations read

$$\begin{aligned} i\hbar\,\partial_t\psi + (\hbar^2/2m)\triangle\psi + (Ze^2/r)\psi &= -(\hbar^2/2m)\hat{f}(\mathbf{A}, A_0, \psi^*\psi)\psi \\ &\equiv -(\hbar^2/2m)\Big[2i\varepsilon(\mathbf{A}\nabla)\psi + 2(\varepsilon mc/\hbar)A_0\psi \\ &\qquad + i\varepsilon\psi\,\mathrm{div}\mathbf{A} + F'(\psi^*\psi)\psi\Big], \end{aligned} \tag{18}$$

$$\Box A_0 = (8\pi me/\hbar^2)|\psi|^2 \equiv -4\pi\varrho, \tag{19}$$

$$\Box\mathbf{A} = 4\pi[2\varepsilon^2\mathbf{A}|\psi|^2 - i\varepsilon(\psi^*\nabla\psi - \psi\nabla\psi^*)] \equiv -(4\pi/c)\,\mathbf{j}, \tag{20}$$

$$\partial_t A_0 + c\,\mathrm{div}\mathbf{A} = 0 \tag{21}$$

We will seek for the solutions to these equations describing a stationary state of an atom when the electron - soliton center moves along a circular orbit of radius a_0 with some angular velocity Ω. We have two characteristic lengths in this problem: the size of the soliton $\ell_0 = \hbar/mc$ and the Bohr radius $a = \hbar^2/mZe^2 \gg \ell_0$. Near the soliton center, where $r - a_0 \leq \ell_0$, we get in non-relativistic approximation

$$\psi{=}u(\mathbf{R})e^{iS/\hbar} = \psi_-, \quad A_0 = A_0(\mathbf{R}), \quad \mathbf{A} = \frac{1}{c}\dot{\boldsymbol{\zeta}}(t)A_0(\mathbf{R})$$

with

$$S{\approx}m\dot{\boldsymbol{\zeta}}\cdot\mathbf{R} + C_0t + \chi(t), \quad m\ddot{\boldsymbol{\zeta}} = -Ze^2\boldsymbol{\zeta}/\zeta^3,$$

where

$$\chi(t){=}\int\limits_0^t \left(\frac{m}{2}\dot{\boldsymbol{\zeta}}^2 + \frac{Ze^2}{\zeta}\right)dt$$

is the Hamiltonian action.

The function $u(\mathbf{R})$, where $\mathbf{R} = \mathbf{r} - \boldsymbol{\zeta}(t)$ satisfies the following soliton-like equation $\hbar^2(\hat{f} + \triangle u/u) = 2mC_0$. For ψ we have the integral equation

$$\begin{aligned} \psi(t,\mathbf{r}){=}&C_n\psi_n(\mathbf{r})\,\exp\left(-i\omega_nt\right) \\ &+\frac{1}{2\pi}\int d\omega\int dt'\int d^3x'\,\exp\left[-i\omega(t-t')\right]G(\mathbf{r},\mathbf{r}';\omega+i0)\hat{f}\psi(t',\mathbf{r}'), \end{aligned} \tag{22}$$

with G being the Coulomb resolvent, $E_n = \hbar\omega_n$ is the eigenvalue of the Coulomb Hamiltonian. For $R \gg \ell_0$ we may put in (22)

$$\hat{f}\,\psi(t,\mathbf{r}) = g\,\exp\left(-i\omega_n t\right)\delta(\mathbf{r}-\boldsymbol{\zeta}(t)), \quad g = const.$$

Calculating the integral (22) by stationary phase method we get

$$\psi = \psi_+ \approx C_n\psi_n(\mathbf{r})\,e^{-i\omega_n t} - \frac{g|\omega_n|ma}{8\pi^2\hbar\sqrt{a_0\cos^2(\vartheta/2)}}e^{-i\omega_n t}R^{-3/2}e^{-R\sqrt{2m|\omega_n|/\hbar}},$$

where $\cos\vartheta = \sin\theta\cos(\alpha-\Omega t)$. Now to find the constants C_0, C_n, a_0, Ω, g we must match the functions ψ_+ and ψ_- at $R = \ell_0$. That gives the following results

$$\begin{aligned} a_0 &= an, \quad \Omega^2 = Ze^2/ma_0^3, \quad C_0 = -m\Omega^2 a_0^2, \\ C_n\psi_n(a_0) &= \frac{g|\omega_n|ma}{8\pi^2\hbar\sqrt{a_0}}\ell_0^{-3/2}e^{-\ell_0\sqrt{2m|\omega_n|/\hbar}} + u(\ell_0), \\ g &= \int\limits_{\vee_0} d^3x\hat{f}u, \quad \vee_0 = \frac{4}{3}\pi\ell_0^3. \end{aligned} \tag{23}$$

Note that, the system (23) is close and sufficient to determine the constants mentioned previously. This system gives first approximation for the solution of the problem since it does not take into account the smooth matching of the functions ψ_+ and ψ_-, i.e., the equality of their normal derivatives.

The last step is the calculation of the electromagnetic field for $R \gg \ell_0$ and for large time $t \gg 1/|\omega_n|$, that gives the semi-sum of the retarded and advanced potentials: $A_\mu = \frac{1}{2}\left(A_\mu^{\text{adv}} + A_\mu^{\text{ret}}\right)$. It is interesting to write down the components of the Poynting vector $\mathbf{S}$:

$$\begin{aligned} S_r &= \frac{e^2\,a_0^2\,\Omega^4}{16\pi c^3\,r^2}\sin^2\vartheta\sin 2(\alpha-\Omega t)\sin(2\Omega r/c), \\ S_\vartheta &= \frac{e^2\,a_0\,\Omega^2}{4\pi c\,r^3}\cos\vartheta\sin(\alpha-\Omega t)\sin(\Omega r/c), \\ S_\alpha &= \frac{e^2\,a_0\,\Omega^2}{4\pi c\,r^3}\cos(\alpha-\Omega t)\sin(\Omega r/c). \end{aligned}$$

Thus we conclude that the radiation is absent. The various aspects of the solitonian scheme were discussed in details in [10, 11, 12, 13, 14, 15].

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