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ORIGINAL ARTICLE

Anisotropic Cosmological Models with a Perfect Fluid and a Λ Term

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Abstract We consider a self-consistent system of Bianchi type-I (BI) gravitational field and a binary mixture of perfect fluid and dark energy given by a cosmological constant. The perfect fluid is chosen to be the one obeying either the usual equation of state, i.e., $p = \zeta\varepsilon$, with $\zeta \in [0,\ 1]$ or a van der Waals equation of state. Role of the Λ term in the evolution of the BI Universe has been studied.

Keywords Bianchi type I (BI) model · Perfect fluid · Van der Waals fluid

1. Introduction

In view of its importance in explaining the observational cosmology many authors have considered cosmological models with dark energy. In a recent paper Kremer (2003) has modelled the Universe as a binary mixture whose constitutes are described by a van der Waals fluid and by a dark energy density. Zlatev et al. (1999) showed that "tracker field", a form of qiuntessence, may explain the coincidence, adding new motivation for the quintessence scenario. The fate of density perturbations in a Universe dominated by the Chaplygin gas, which exhibit negative pressure was studied by Fabris et al. (2002). Model with Chaplygin gas was also studied in the Dev et al. (2003) and Bento et al. (2002). In doing so the authors considered a spatially flat, homogeneous and isotropic Universe described by a Friedmann–Robertson–Walker (FRW) metric. Since the theoretical arguments and recent experimental data support the existence of an anisotropic phase that approaches an isotropic one, it makes sense to consider the models of Universe with anisotropic back-ground in presence of dark energy. The simplest of anisotropic models, which nevertheless rather completely describe the anisotropic effects, are Bianchi type-I (BI) homogeneous models whose spatial sections are flat but the expansion or contraction rate is direction-dependent. Given its importance for studying the possible effects of an anisotropy in the early Universe on present-day observations many researchers have investigated the BI model from different point of view. For example, BI Universe with viscous fluid was thoroughly studied several authors (Belinski and Khalatnikov, 1975; Banerjee et al., 1985; Huang, 1990; Chimento et al., 1997; van Elst et al., 1995; Gavrilov et al., 1997; Israel, 1976; Israel and Stewart, 1979a, 1979b; Coley et al., 1996; Coley and van den Hoogen, 1995a, 1995b; Grøn, 1990; Pradhan and Singh, 2004; Saha, 2005a). BI Universe in presence of a nonlinear spinor field was studied in Saha and Shikin (1997a, 1997b), Saha (2001a, 2005b), whereas in Saha (2001b), Saha and Todor (2004), we have studied the role of a Λ term in the evolution of a BI space-time in presence of spinor and/or scalar field with a perfect fluid satisfying the equation of state $p = \zeta\varepsilon$. In a recent paper (Khalatnikov and Kamenshchik, 2003) studied the Einstein equations for a BI Universe in the presence of dust, stiff matter and cosmological constant. The purpose of the present paper is to study the evolution of an anisotropic BI Universe in presence of a perfect fluid and Λ term. In addition to the previous study where the perfect fluid was chosen to be the one obeying barotropic equation of state, i.e., $p = \zeta\varepsilon$, here we consider also the Van der Waals gas. As it will be shown the Van der Waals gas can play crucial role in the initial stage of evolution by virtue of its specific form of equation of state.

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2. Basic equations

The Einstein field equation on account of the cosmological constant we write in the form

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = \kappa T_\mu^\nu + \delta_\mu^\nu \Lambda. \tag{2.1}$$

Here R_μ^ν is the Ricci tensor, R is the Ricci scalar and κ is the Einstein gravitational constant. As was mentioned earlier, Λ is the cosmological constant. To allow a steady state cosmological solution to the gravitational field equations Einstein (1917, 1919) introduced a fundamental constant, known as cosmological constant or Λ term, into the system. Soon after E. Hubble had experimentally established that the Universe is expanding, Einstein returned to the original form of his equations citing his temporary modification of them as the biggest blunder of his life. Λ term made a temporary comeback in the late 60's. Finally after the pioneer paper by Guth (1981) on inflationary cosmology researchers began to study the models with Λ term with growing interest. Note that in our previous papers (Saha, 2001b; Saha and Todor, 2004) we studied the Einstein field equations where the cosmological term appears with a negative sign. Here following the original paper by Einstein and one by Sahni (2004) we choose the sign to be positive. In this paper a positive Λ corresponds to the universal repulsive force, while a negative one gives an additional gravitational force. Note that a positive Λ is often considered to be a form of dark energy.

We study the gravitational field given by an anisotropic Bianchi type I (BI) cosmological model and choose it in the form:

$$ds^2 = dt^2 - a^2 dx^2 - b^2 dy^2 - c^2 dz^2, \tag{2.2}$$

with the metric functions $a,\ b,\ c$ being the functions of time t only.

The Einstein field equations (2.1) for the BI space-time in presence of the Λ term now we write in the form

$$\frac{\ddot b}{b} + \frac{\ddot c}{c} + \frac{\dot b}{b}\frac{\dot c}{c} = \kappa T_1^1 + \Lambda, \tag{2.3a}$$

$$\frac{\ddot c}{c} + \frac{\ddot a}{a} + \frac{\dot c}{c}\frac{\dot a}{a} = \kappa T_2^2 + \Lambda, \tag{2.3b}$$

$$\frac{\ddot a}{a} + \frac{\ddot b}{b} + \frac{\dot a}{a}\frac{\dot b}{b} = \kappa T_3^3 + \Lambda, \tag{2.3c}$$

$$\frac{\dot a}{a}\frac{\dot b}{b} + \frac{\dot b}{b}\frac{\dot c}{c} + \frac{\dot c}{c}\frac{\dot a}{a} = \kappa T_0^0 + \Lambda. \tag{2.3d}$$

Here over-dot means differentiation with respect to t. The energy-momentum tensor of the source is given by

$$T_\mu^\nu = (\varepsilon + p)u_\mu u^\nu - p\delta_\mu^\nu, \tag{2.4}$$

where u^μ is the flow vector satisfying

$$g_{\mu\nu}u^\mu u^\nu = 1. \tag{2.5}$$

Here ε is the total energy density of a perfect fluid, while p is the corresponding pressure. p and ε are related by an equation of state which will be studied below in detail. In a co-moving system of coordinates from (2.4) one finds

$$T_0^0 = \varepsilon, \quad T_1^1 = T_2^2 = T_3^3 = -p. \tag{2.6}$$

In view of (2.6) from (2.3) one immediately obtains (Saha, 2001b)

$$a(t) = D_1 \tau^{1/3} \exp\left[X_1 \int \frac{dt}{\tau(t)}\right], \tag{2.7a}$$

$$b(t) = D_2 \tau^{1/3} \exp\left[X_2 \int \frac{dt}{\tau(t)}\right], \tag{2.7b}$$

$$c(t) = D_3 \tau^{1/3} \exp\left[X_3 \int \frac{dt}{\tau(t)}\right]. \tag{2.7c}$$

Here D_i and X_i are some arbitrary constants obeying

$$D_1 D_2 D_3 = 1, \quad X_1 + X_2 + X_3 = 0.$$

Here τ is the volume scale of the BI Universe, i.e.,

$$\tau = \sqrt{-g} = abc. \tag{2.8}$$

From (2.3) for τ one finds

$$\frac{\ddot\tau}{\tau} = \frac{3\kappa}{2}(\varepsilon - p) + 3\Lambda. \tag{2.9}$$

On the other hand the conservation law for the energy-momentum tensor gives

$$\dot\varepsilon = -\frac{\dot\tau}{\tau}(\varepsilon + p). \tag{2.10}$$

After a little manipulations from (2.9) and (2.10) we find

$$\dot\tau^2 = 3(\kappa\varepsilon + \Lambda)\tau^2 + C_1, \tag{2.11}$$

with C_1 being an arbitrary constant. Let us now, in analogy with Hubble constant, define

$$\frac{\dot\tau}{\tau} = \frac{\dot a}{a} + \frac{\dot b}{b} + \frac{\dot c}{c} = 3H. \tag{2.12}$$

On account of (2.12) from (2.11) one derives

$$\kappa\varepsilon = 3H^2 - \Lambda - C_1/(3\tau^2). \tag{2.13}$$

It should be noted that the energy density of the Universe is a positive quantity. It is believed that at the early stage of evolution when the volume scale τ was close to zero, the energy density of the Universe was infinitely large. On the other hand with the expansion of the Universe, i.e., with the increase of τ, the energy density ε decreases and an infinitely large τ corresponds to a ε close to zero. Say at some stage of evolution ε is too small to be ignored. In that case from (2.13) follows

$$3H^2 - \Lambda \to 0. \tag{2.14}$$

From (2.14) one concludes the followings: (i) in this case Λ is essentially non-negative; (ii) in absence of a Λ term H becomes trivial, hence beginning from some value of τ the evolution of the Universe comes stand-still, i.e., τ becomes constant; (iii) in case of a positive Λ the process of evolution of the Universe never comes to a halt. Moreover it is believed that the presence of the dark energy (which can be given in the form of a positive Λ) results in the accelerated expansion of the Universe. As far as negative Λ is concerned, its presence imposes some restriction on ε, namely, ε can never be small enough to be ignored. In case of the perfect fluid given by $p = \zeta\varepsilon$ there exists some upper limit for τ as well (note that τ is essentially nonnegative, i.e. bound from below). In our previous papers we came to the same conclusion (Saha, 2001b; Saha and Todor, 2004) [with a positive Λ which in the present paper appears to be negative]. A suitable choice of parameters in this case may give rise to an oscillatory mode of expansion, whereas in case of a Van der Waals fluid the highly nonlinear equation of state may result in an exponential expansion as well.

Inserting (2.12) and (2.13) into (2.9) one now finds

$$\dot H = -\frac{1}{2}\left(3H^2 - \Lambda + \frac{C_1}{3\tau^2} + \kappa p\right) = -\frac{\kappa}{2}(\varepsilon + p) - \frac{C_1}{3\tau^2}. \tag{2.15}$$

In view of (2.13) from (2.15) follows that if the perfect fluid is given by a stiff matter where $p = \varepsilon$, the corresponding solution does not depend on the constant C_1.

Let us now go back to the Eq. (2.11). It is in fact the first integral of (2.9) and can be written as

$$\dot\tau = \pm\sqrt{C_1 + 3(\kappa\varepsilon + \Lambda)\tau^2} \tag{2.16}$$

On the other hand, rewriting (2.10) in the form

$$\frac{\dot\varepsilon}{\varepsilon + p} = \frac{\dot\tau}{\tau}, \tag{2.17}$$

and taking into account that p is a function of ε, one concludes that the right hand side of the Eq. (2.9) is a function of τ only, i.e.,

$$\ddot\tau = \frac{3\kappa}{2}(\varepsilon - p)\tau + 3\Lambda\tau = \mathcal{F}(\tau). \tag{2.18}$$

From a mechanical point of view Eq. (2.18) can be interpreted as an equation of motion of a single particle with unit mass under the force $\mathcal{F}(\tau)$. Then the following first integral exists (Saha and Todor, 2004):

$$\dot\tau = \sqrt{2[\mathcal{E} - \mathcal{U}(\tau)]}. \tag{2.19}$$

Here $\mathcal{E}$ can be viewed as energy and $\mathcal{U}(\tau)$ is the potential of the force $\mathcal{F}$. Comparing the Eqs. (2.16) and (2.19) one finds $\mathcal{E} = C_1/2$ and

$$\mathcal{U}(\tau) = -\frac{3}{2}(\kappa\varepsilon + \Lambda)\tau^2. \tag{2.20}$$

Let us finally write the solution to the Eq. (2.9) in quadrature:

$$\int \frac{d\tau}{\sqrt{C_1 + 3(\kappa\varepsilon + \Lambda)\tau^2}} = t + t_0, \tag{2.21}$$

where the integration constant t_0 can be taken to be zero, since it only gives a shift in time.

In what follows we study the Eqs. (2.9) and (2.10) for perfect fluid obeying different equations of state.

3. Universe filled with perfect fluid

In this section we consider the case when the source field is given by a perfect fluid. Here we study two possibilities: (i) the energy density and the pressure of the perfect fluid are connected by a linear equation of state; (ii) the equation of state is a nonlinear (Van der Waals) one.

3.1. Universe as a perfect fluid with $p_{\rm pf} = \zeta\varepsilon_{\rm pf}$

In this subsection we consider the case when the source field is given by a perfect fluid obeying the equation of state

$$p_{\rm pf} = \zeta\,\varepsilon_{\rm pf}, \tag{3.1}$$

where ζ is a constant and lies in the interval $\zeta \in [0, 1]$. Depending on its numerical value, ζ describes the following types of Universes (Jacobs, 1968)

$$\zeta = 0, \quad \text{(dust Universe)}, \tag{3.2a}$$

$$\zeta = 1/3, \quad \text{(radiation Universe)}, \tag{3.2b}$$

$$\zeta \in (1/3,\ 1), \quad \text{(hard Universes)}, \tag{3.2c}$$

$$\zeta = 1, \quad \text{(Zel}'\text{dovich Universe or stiff matter)}. \tag{3.2d}$$

In view of (3.1), from (2.10) for the energy density and pressure one obtains

$$\varepsilon_{\rm pf} = \varepsilon_0/\tau^{(1+\zeta)}, \quad p_{\rm pf} = \zeta\varepsilon_0/\tau^{(1+\zeta)}, \tag{3.3}$$

where ε_0 is the constant of integration. For τ from (2.21) one finds

$$\frac{d\tau}{\sqrt{C_1 + 3(\kappa\varepsilon_0\tau^{1-\zeta} + \Lambda\tau^2)}} = t. \tag{3.4}$$

As one sees, the positivity of the radical in (3.4) for a negative Λ imposes some restriction on the upper value of τ, i.e., τ should be bound from above as well. In Fig. 1 the graphical view of the potential $\mathcal{U}(\tau)$ is illustrated for a negative Λ. As it was mentioned earlier, $\mathcal{E}$ or C_1 in case of $\zeta = 1$ does not play any role. Universe in this case initially expands, reaches to the maximum and then begin to contract finally giving rise to a space-time singularity (cf Fig. 2). For the other cases depending on the choice of $\mathcal{E}$ expansion of the Universe is either non-periodic (Fig. 3) with a singularity at the end or oscillatory one without space-time singularity (Fig. 2). In Fig. 4 we demonstrate the evolution of the BI Universe with a positive Λ. In this case the Universe expands exponentially, the initial anisotropy quickly dies away and the BI Universe evolves into a isotropic FRW one. There does not any upper bound for τ in case of a positive Λ. Note that in the Figs. 1–4 **d**, **r**, **h** and **s** stand for dust, radiation, hard Universe and stiff matter, respectively.

In absence of the Λ term one immediately finds

$$\tau = At^{2/(1+\zeta)}, \tag{3.5}$$

with A being some integration constant. Note that the second constant of integration C_1 in this case is taken to be trivial. As one sees from (2.7), in absence of a Λ term, for $\zeta < 1$ the initially anisotropic Universe eventually evolves into an isotropic FRW one, whereas, for $\zeta = 1$, i.e., in case of stiff matter the isotropization does not take place. From (3.5) also follows that the smaller is the value of ζ, the quicker the anisotropy dies away. Note that, though in the real world the Universe was initially radiation dominated that was followed by the matter domination, here we consider the models with different types of perfect fluid to see the influence of particular type of perfect fluid in the evolution of the Universe.

3.2. Universe as a van der Waals fluid

Here we consider the case when the source field is given by a perfect fluid with a van der Waals equation of state in

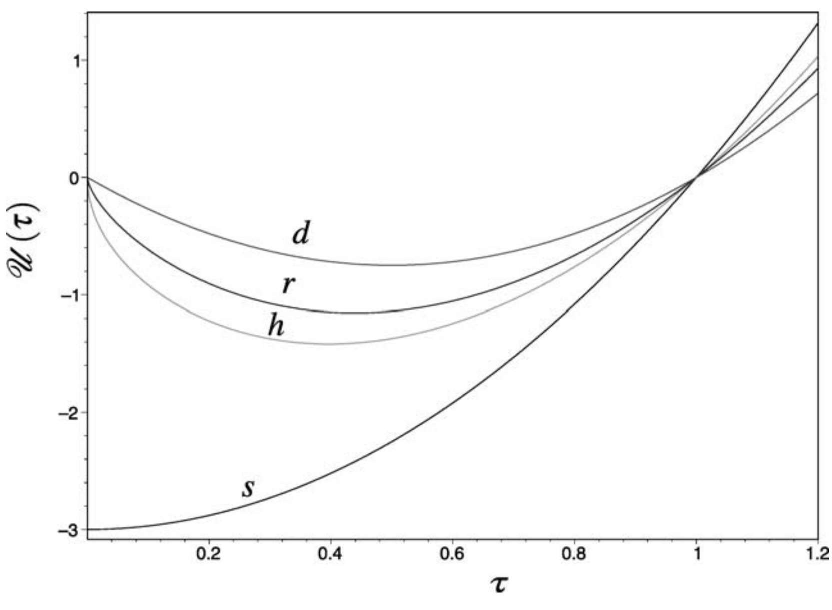

Fig. 1 View of the potential $\mathcal{U}(\tau)$. As one sees in case of stiff matter this potential allows only non-periodic solution

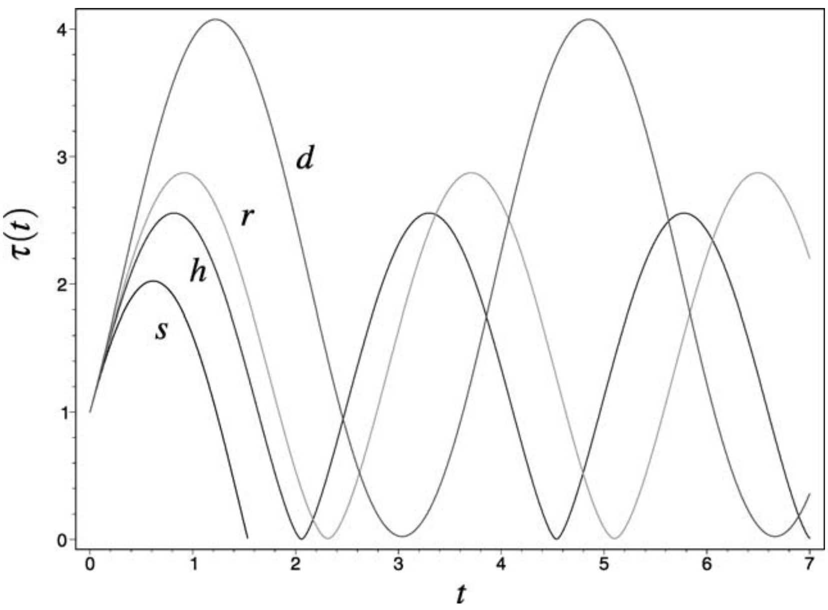

Fig. 2 Evolution of volume scale τ with a negative Λ and $C = -0.1$. As one sees, in this case the model with perfect fluid given by dust, radiation and hard Universe allow oscillation, whereas, stiff matter gives rise to a non-periodic solution

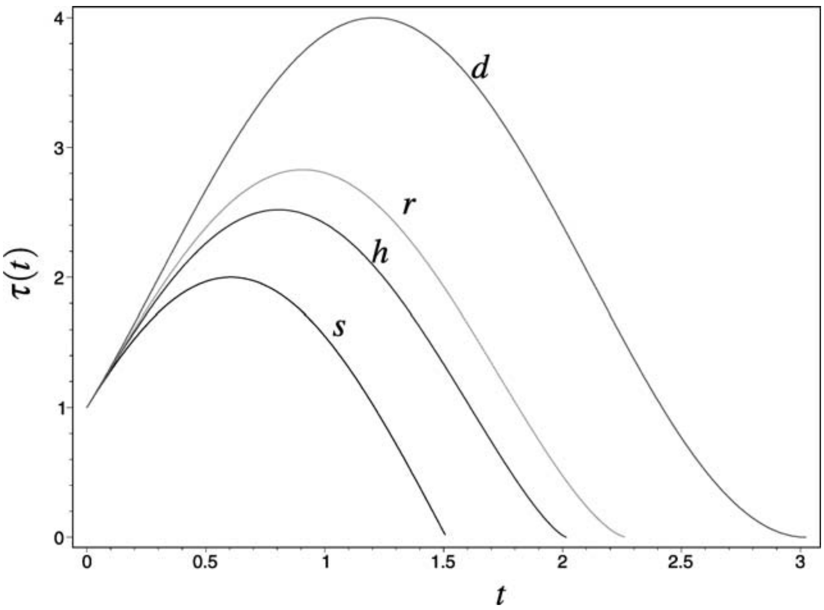

Fig. 3 Evolution of volume scale τ with a negative Λ and $C = 0$. In this case the model with perfect fluid given by dust, radiation, hard Universe and a stiff matter gives rise to a non-periodic solution

absence of dissipative process. The pressure of the van der Waals fluid p_{w} is related to its energy density ε_{w} by Kremer (2003)

$$p_{\mathrm{w}} = \frac{8W\varepsilon_{\mathrm{w}}}{3-\varepsilon_{\mathrm{w}}} - 3\varepsilon_{\mathrm{w}}^2. \tag{3.6}$$

In (3.6) the pressure and the energy density is written in terms of dimensionless reduced variables and W is a parameter connected with a reduced temperature. In the Figs. 5 and 6 the energy density and the pressure of the system are illustrated with a negative and a positive Λ term as well as in absence of it. As is seen from the Fig. 6, a positive Λ results in making the initial pressure less negative. It might be confusing, since a positive Λ itself is a negative pressure. Let us clarify this in details. First of all, here the energy density ε and pressure p are that of Van der Waals gas only, which, by virtue of self-consistency influenced by the space-time evolution as well. From (2.13), i.e.,

$$\varepsilon = [H^2 - \Lambda - C_1/(3\tau^2)]/\kappa$$

it follows that for a given H, C_1 and τ the value of ε in case of a positive Λ is less than that with $\Lambda = 0$. Hence, p is less negative as well. Moreover, as it was mentioned earlier, a positive Λ being a repulsive force causes the rapid growth of τ. It results in quick decrease of ε at the initial state of

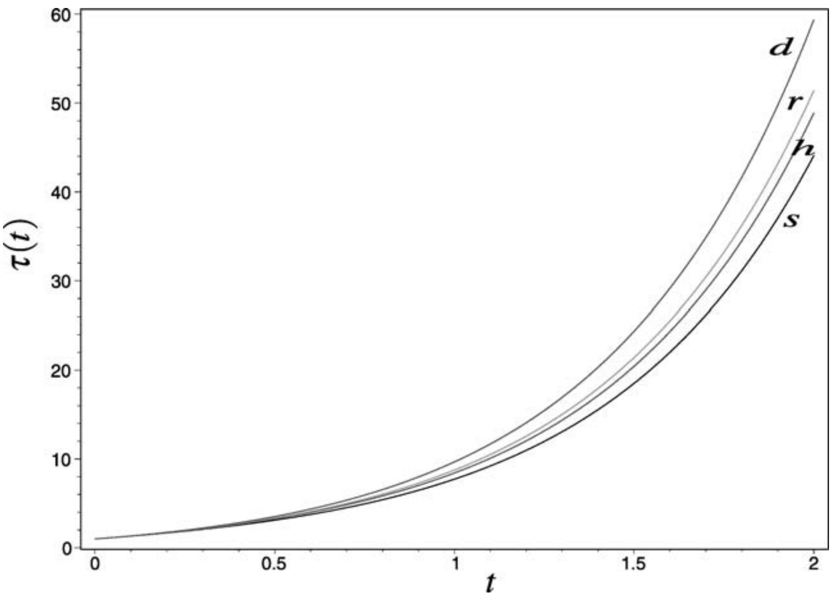

Fig. 4 Evolution of the Universe with a positive Λ. In this case independent to the choice of ζ we have ever expanding Universe. The anisotropy in this case dies away quicker than it does when $\Lambda = 0$

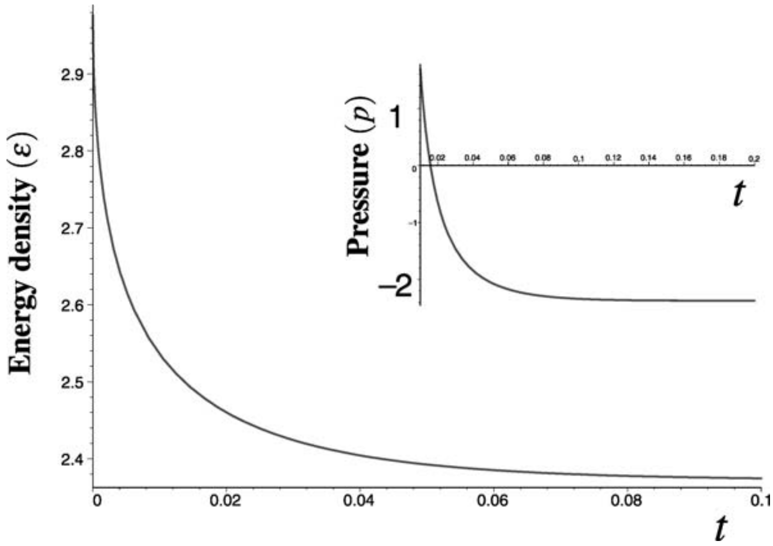

Fig. 5 View of energy density ε and pressure p in case of a Van der Waals fluid with a negative Λ

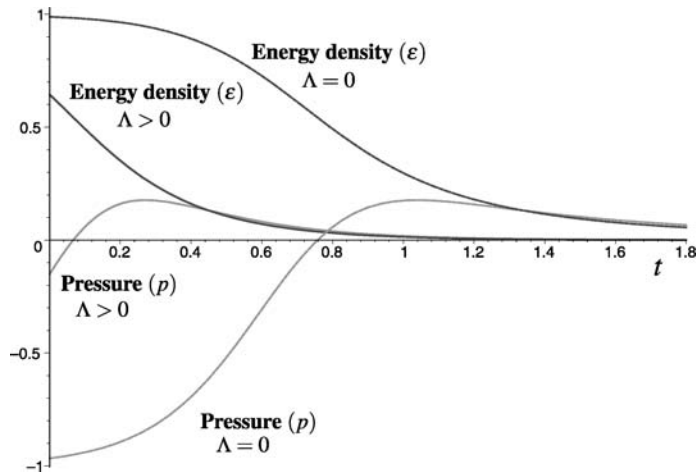

Fig. 6 View of energy density ε and pressure p in case of Van der Waals fluid with $\Lambda \geq 0$.

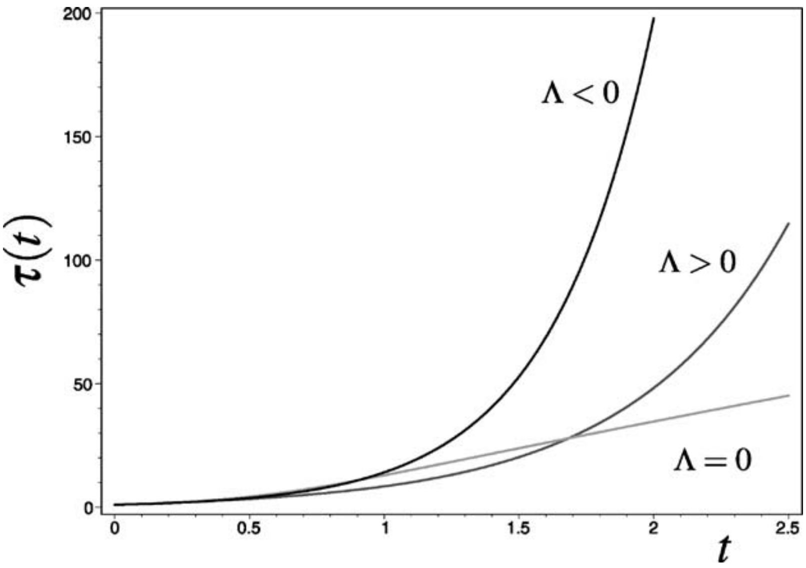

Fig. 7 Evolution of $\tau(t)$ with the BI Universe filled with Van der Waals fluid. Independent to the sign of Λ the model provides provides with rapidly expanding Universe

evolution, which on its part leads to p becoming positive quicker than the case with a trivial Λ. As far as a negative Λ is concerned, it makes the energy density ε more positive. It should be noted that the behavior of all the solutions such as ε, p, H, τ crucially depends on the choice of the initial value of H and τ, as well as Λ, C_1 and κ. As one sees from Fig. 7, a concrete choice of problem parameters may result in the more rapid growth of τ with a negative Λ. It is because the present choice of problem parameters with a negative Λ makes the pressure more negative (cf. Fig. 5).

Let us now go back to (2.15). Inserting (3.6) into (2.15) on account of (2.13) one finds

$$\dot{H} = -\frac{\{3H^2 - \Lambda - C_1/(3\tau^2)\}[(3+8W)\kappa - \{3H^2 - \Lambda - C_1/(3\tau^2)\}]}{2(3\kappa - \{3H^2 - \Lambda - C_1/(3\tau^2)\})} + \frac{3}{2\kappa}(\{3H^2 - \Lambda - C_1/(3\tau^2)\})^2. \tag{3.7}$$

It can be easily verified that the Eq. (3.7) in absence of Λ term and $C_1 = 0$ and $\kappa = 3$ coincides with that given in Kremer (2003):

$$\dot{H} = -\frac{3}{2}\left[H^2 + \frac{8WH^2}{3 - H^2} - 3H^4\right]. \tag{3.8}$$

The solution of the second-order differential equation (3.7) for $H(t)$ can be found by specifying the initial value for $H(t)$ at $t = 0$, for a given value of parameter W. Here we graphically present some results concerning the evolution of BI Universe with a Van der Waals fluid. In Fig. 7 we compare the evolution of τ with and without Λ term. As one sees, the character of evolution does not depend on the sign of Λ. In all cases we find expanding Universe, though the rapidity of growth depends on Λ. The Fig. 8 gives the comparison of

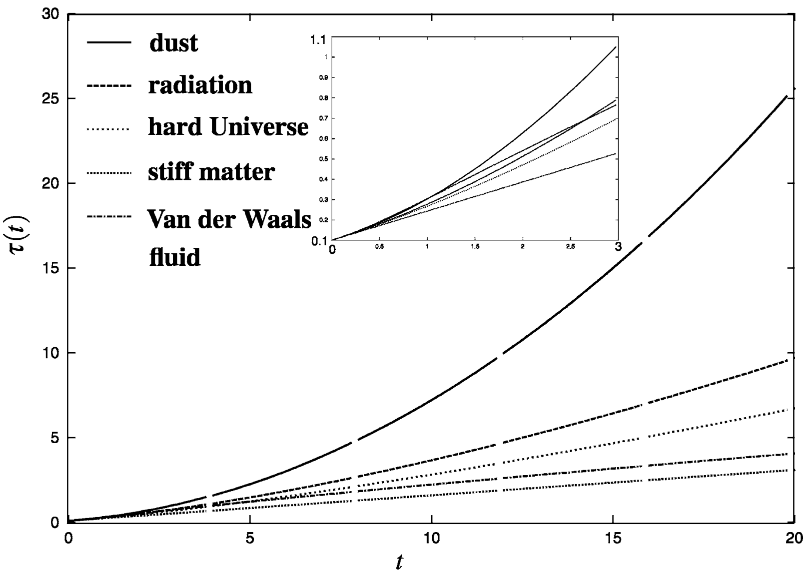

Fig. 8 Evolution of the BI Universe filled with different types of perfect fluid in absence of a Λ term. As one sees, in case of Van der Waals fluid $\tau(t)$ grows faster at the early stage, then slows down with time

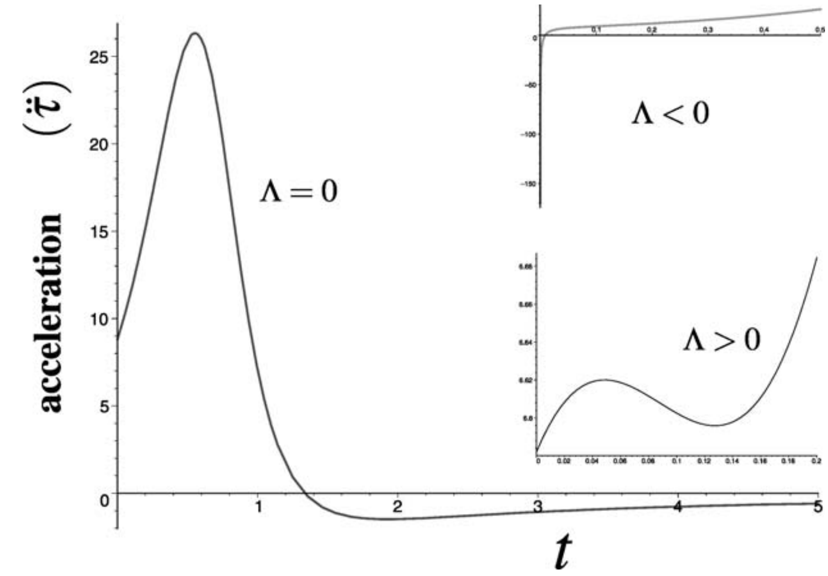

Fig. 9 Acceleration of a BI Universe filled with a Van der Waals fluid for different Λ. Here we choose $W = 0.5$. For Λ the following values are considered: $\Lambda = 0$, $\Lambda = 1$ and $\Lambda = -0.01$

the expansion of τ with perfect fluid obeying different equations of state. Finally, in Fig. 9 we plot the acceleration $\ddot{\tau}$ for different Λ.

4. Conclusion and discussions

The evolution of an anisotropic Universe given by a Bianchi type I cosmological model is studied in presence of a perfect fluid and a Λ term. It has been shown that in case of a perfect fluid obeying $p = \zeta\varepsilon$, where p and ε are the pressure and energy density of the fluid, respectively, a negative Λ may generate an oscillation in the system thus giving rise to a singularity-free mode of expansion. Introduction of a positive Λ in this case results in a rapid expansion of the Universe. If the Universe is filled with a Van der Waals fluid, no oscillatory or non-periodic mode of expansion occurs. Independent to the sign of Λ the Universe in this case expands exponentially.

It should be emphsized that in case of a Van der Waals gas the pressure is initially negative that becomes positive in the course of time (evolution). This negative pressure (repulsive force) can be viewed as a source of the initial inflation. In course of time the pressure becomes positive and the speed of expansion slows down. It is also known that the Universe was initially dominated by radiation that was followed by a matter dominated phase. So in our view for a complete picture of the evolution with initial inflation and present day acceleration we should consider the model with the source field being a mixture of a perfect fluid and dark energy and realize it with one of the compounds being dominated at a definite period of evolution as follows: van der Waals gas, radiation, dust, dark energy. This model seems to be more realistic and we plan to encounter it in near future.

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