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Interaction of a charged 3D soliton with a Coulomb center

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Abstract

The Einstein-de Broglie particle-soliton concept is applied to simulate stationary states of an electron in a hydrogen atom. According to this concept, the electron is described by the localized regular solutions to some nonlinear equations. In the framework of the Synge model for interacting scalar and electromagnetic fields a system of integral equations has been obtained, which describes the interaction between a charged 3D soliton and a Coulomb center. The asymptotic expressions for physical fields, describing a soliton moving around the fixed Coulomb center, have been obtained with the help of integral equations. It is shown that the electron-soliton center travels along some stationary orbit around the Coulomb center. The electromagnetic radiation is absent as the Poynting vector has a nonwave asymptote $O(r^{-3})$ after averaging over angles, i.e. the existence of a spherical surface corresponding to a null Poynting vector stream has been proved. The vector lines for the Poynting vector are constructed in an asymptotical area.

Keywords: Soliton; Nonlinear resonance; Wave-particle dualism; Theory of double solution; Electromagnetic radiation; Stationary orbit

1. Introduction

From the history of quantum mechanics it is known that as early as 1927 in the framework of his "theory of double solution" Louis de Broglie made an attempt to represent the electron as a source of waves obeying the Schrödinger equation [1]. Later he modified his model, showing that the electron should be described by regular solutions to some nonlinear equation coinciding with the Schrödinger one in the linear approximation. This scheme became famous as a causal nonlinear interpretation of quantum mechanics [2]. Developing this concept, de Broglie remarked that it had much in common with Einstein's ideas about unified field theory according to which particles were to be considered as clots of some material fields obeying the nonlinear field equations [3]. In recent years, these types of field configurations, known as soliton or particle-like solutions, came into active use to model extended elementary particles [4].

In this paper the Einstein-de Broglie soliton concept is employed to model stationary states of the electron in a hydrogen atom.

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2. Bohm problem of nonlinear resonance and its possible solution

As a starting point, we will consider an interesting problem posed by Bohm. Long ago, in Ref. [5] Bohm discussed the possible connection between the wave-particle dualism in quantum mechanics and the hypothetical nonlinear origin of fundamental equations in a future theory of elementary particles. To illustrate the line of Bohm's argument we will consider a simple scalar model in the Minkowski space-time given by the Lagrangian density

$$\mathcal{L} = \partial_i\phi^* \, \partial_j\phi \, \eta^{ij} - (mc/\hbar)^2\phi^*\phi + F(\phi^*\phi). \quad (2.1)$$

Here $i, j = 0, 1, 2, 3$, $\eta^{ij} = \mathrm{diag}(1, -1, -1, -1)$, the nonlinear function $F(s)$ behaves at $s \to 0$ as s^n, $n > 1$, and is assumed such that the corresponding field equations allow the existence of particle-like (soliton) solutions, i.e. regular configurations localized in space and endowed with finite energy. Let us note the *ununiqueness* of the choice of the function F in (2.1). The existence of particle-like solutions for the model in question with various F was proved by several authors [6–9]. In particular, it can be shown that if one chooses $F(s) = ks^{3/2}$, $k > 0$, model (2.1), known as the Synge model [10], admits the following stationary solutions,

$$\phi_0 = u(r) \exp(-\mathrm{i}\omega_0 t), \quad r = |\boldsymbol{r}|. \quad (2.2)$$

Here, the real radial function $u(r)$ is regular everywhere and exponentially decreases as $r \to \infty$, which provides the finiteness of the energy of the configuration

$$E = \int \mathrm{d}^3x \, T^{00}(\phi_0), \quad (2.3)$$

where T^{ij} is the corresponding energy-momentum tensor.

Moreover, the model mentioned is intriguing due to the fact that nodeless solitons turn out to be stable in the Lyapunov sense provided their charge is fixed [11]. So there exist perturbed solitons slightly differing from the stationary solitons (2.2),

$$\phi = \phi_0 + \xi(t, \boldsymbol{r}). \quad (2.4)$$

Note that the perturbation ξ in (2.4) is small as compared with ϕ_0 only in the area of localization of the soliton, where ϕ_0 significantly differs from zero. Nonetheless, far from the soliton center, where ϕ_0 is negligibly small, one can put $\phi \approx \xi$, i.e. the *tail* of the soliton is completely defined by the perturbation ξ.

Bohm put the following question: Does there exist any nonlinear model for which the spatial asymptote (as $r \to \infty$) of a perturbed soliton-like solution represents oscillations with characteristic frequency $\omega = E/\hbar$? In other words, for the model in question the principal Fourier amplitude in the expansion of the field $\phi \approx \xi$ as $r \to \infty$ should correspond to the frequency ω connected with the soliton energy (2.3) by the Planck–de Broglie formula

$$E = \hbar\omega. \quad (2.5)$$

Note that for model (2.1) at spatial infinity, where $\phi \to 0$, the field equation reduces to the linear Klein–Gordon one

$$[\Box - (mc/\hbar)^2]\phi = 0, \quad (2.6)$$

and therefore relation (2.5) holds only for solitons with unique energy $E = mc^2$ defined by the mass m fixed in (2.1). Thus the universality of relation (2.5) breaks down in model (2.1), so forcing its modification. In the light of the above universality, the frequency ω in (2.5) being defined by the mass of the system, it seems natural that in the new, modified model one should use the gravitational field, whose spatial asymptote is also defined by the mass of the considered localized system. Thus, to solve the Bohm problem, the possibility to invoke the gravitational field comes to reality [12].

So we will describe the new model with the Lagrangian density $\mathcal{L} = \mathcal{L}_{\mathrm{m}} + \mathcal{L}_{\mathrm{g}}$, where

$$\mathcal{L}_{\mathrm{g}} = c^4R/16\pi G$$

corresponds to Einstein's theory of gravity, and $\mathcal{L}_{\mathrm{m}}$ is chosen as

$$\mathcal{L}_{\mathrm{m}} = \partial_i\phi^* \, \partial_j\phi \, g^{ij} - I(g_{ij})\phi^*\phi + F(\phi^*\phi). \quad (2.7)$$

The crucial point of this scheme is to build up the invariant $I(g_{ij})$ depending on the metric tensor g_{ij} of the Riemannian space-time and its derivatives. This invariant should possess such properties that in the vicinity of the soliton with mass m, the relation

$$\lim_{r\to\infty} I(g_{ij}) = (mc/\hbar)^2 \quad (2.8)$$

should hold. It is easy to see that on the basis of (2.8) one can asymptotically deduce Eq. (2.6) from Lagrangian (2.7).

We argue that the invariant I can be built from the curvature tensor R_{ijkl} and its covariant derivatives $R_{ijkl;n}$,

$$I = (I_1^4/I_2^3)c^6\hbar^{-2}G^{-2}, \tag{2.9}$$

where G is Newton's gravitational constant and the invariants I_1 and I_2 take the form

$$I_1 = R_{ijkl}R^{ijkl}/48, \quad I_2 = -R_{ijkl;n}R^{ijkl;n}/432.$$

Estimating R^{ijkl} at a large distance r with the help of the Schwarzschild metric, one can find

$$I_1 = G^2m^2/c^4r^6, \quad I_2 = G^2m^2/c^4r^8.$$

So from (2.9) there immediately follows (2.8). Thus, within the modified model (2.7) for all massive particles the Planck–de Broglie relation (2.5) is automatically fulfilled. This means that in the framework of the scheme mentioned the principle of wave–particle dualism is valid, according to which the relation (2.5) is realized as a condition of the nonlinear resonance. We should mention the ambiguity of the choice of invariant (2.9). We note that the form of the invariant (2.9) agrees with the dimensional behavior of the invariants I_1 and I_2 and with the assumption of the localized character of the system (e.g. exponential fall-off of material fields). This property of the system allows one to exploit the Schwarzschild solution for an asymptotic estimate.

To verify the fact that solitons can really possess wave properties, a gedanken diffraction experiment with individual electron-solitons similar to the numerical one of Biberman et al. [13] was realized. Solitons with some velocity were dropped into a rectilinear slit, cut in the impermeable screen, and the transverse momentum was calculated which they gained while passing the slit whose width significantly exceeded the size of the soliton. As a result, the picture of the distribution of the centers of scattered solitons was restored on the registration screen, by considering their initial distribution to be uniform over the transverse coordinate. It was clarified that though the center of each soliton fell into a definite place of the registration screen (depending on the point of crossing of the slit and the initial soliton profile), the statistical picture in many ways was similar to the well-known diffraction distribution in optics, i.e. Fresnel's picture at short distances from the slit and Fraunhofer's picture at large distances [14,15].

The fulfillment of the quantum mechanics correspondence principle for the Einstein–de Broglie soliton model was discussed in Refs. [16–20]. In these papers it was shown that in the framework of the soliton model all quantum postulates were regained at the limit of point particles so that from the physical fields one can build the amplitude of probability and the average can be calculated as a scalar product in the Hilbert space by introducing the corresponding quantum operators. In this paper, we will show that in the framework of the Einstein–de Broglie soliton model a hydrogen atom can be simulated.

3. Fundamental equations and structure of solutions

Let us consider a hydrogen atom with the electron replaced by a localized object "soliton" that is moving around the nucleus (Fig. 1). For the soliton-like solution to exist one has to consider a nonlinear model. As physical fields we choose the complex scalar field ϕ interacting with the electromagnetic one, $F_{ik} = \partial_i A_k - \partial_k A_i$. The nucleus field is assumed to be the Coulomb one: $A_i^{\rm ext} = \delta_i^0 Ze/r$. The Lagrangian density is taken in the following form,

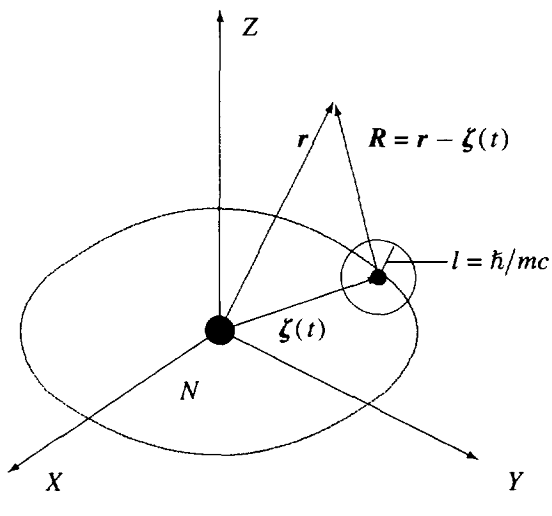

Fig. 1.

$$\mathcal{L} = -\frac{1}{16\pi}(F_{ik})^2 + |[\partial_k - i\epsilon(A_k + A_k^{ext})]\phi|^2 - (mc/\hbar)^2\phi^*\phi + F(\phi^*\phi), \qquad (3.1)$$

where $\epsilon = e/\hbar c$ is the coupling constant, $F(\phi^*\phi)$ is some nonlinear function, decreasing faster than $|\phi|^2$ as $\phi \to 0$ and is chosen so that the field equation at $A_i^{ext} = 0$ allows the existence of stable stationary soliton-like solutions of the type (2.2), describing configurations with mass m and charge e.

Note that for simplicity we do not write down the terms corresponding to the gravitational field that will be taken into account implicitly with the help of the nonlinear resonance condition (2.5).

Let us consider the nonrelativistic approximation assuming that

$$\phi = \psi \exp(-imc^2t/\hbar), \qquad (3.2)$$

neglecting in the equations of motion higher derivatives of ψ with respect to time and retaining only linear terms in A_i. As a result, taking (3.2) into account we get the following system of equations,

$$i\hbar\partial_t\psi + (\hbar^2/2m)\triangle\psi + (Ze^2/r)\psi = -(\hbar^2/2m)\hat{f}(A, A_0, \psi^*\psi)\psi, \qquad (3.3)$$

$$\Box A_0 = (8\pi me/\hbar^2)|\psi|^2 \equiv -4\pi\rho, \qquad (3.4)$$

$$\Box A = 4\pi[2\epsilon^2 A|\psi|^2 - i\epsilon(\psi^*\nabla\psi - \psi\nabla\psi^*)] \equiv -(4\pi/c)\,j, \qquad (3.5)$$

where

$$\hat{f}(A, A_0, \psi^*\psi)\psi \equiv 2i\epsilon(A\cdot\nabla)\psi + 2(\epsilon mc/\hbar)A_0\psi + i\epsilon\psi\,\mathrm{div}\,A + F'(\psi^*\psi)\psi.$$

Moreover in Eqs. (3.3)–(3.5) it is supposed that the four-potential A_i of the proper electromagnetic field of the soliton obeys the Lorentz condition

$$\partial_t A_0 + c\,\mathrm{div}A = 0,$$

which is consistent with Eqs. (3.3)–(3.5) owing to the conservation of electric charge.

We will seek for the solutions to Eqs. (3.3)–(3.5) describing the stationary state of an atom when the electron-soliton center is assumed to be moving along a circular orbit of the radius a_0 with some angular velocity Ω. In this problem there arise two characteristic

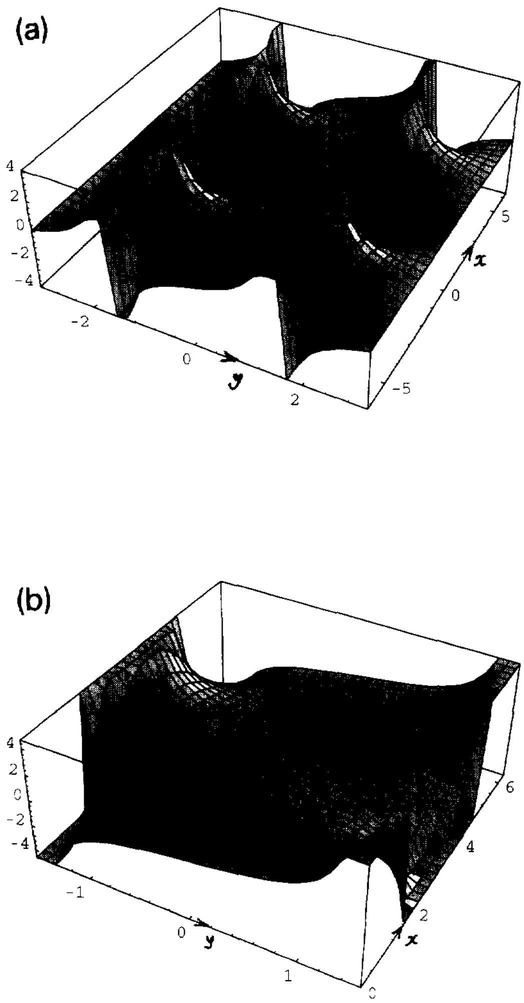

Fig. 2. The overall picture of the distribution of Poynting vector lines for uniform circular motion of the soliton center (a) and details (b).

lengths: the size of the soliton $l = \hbar/mc$ and the Bohr radius $a = \hbar^2/mZe^2$. It is obvious that $a_0 \sim a \gg l$.

Let us first consider the area near the soliton center where $r - a_0 \sim l$. Suppose the soliton center trajectory to be $r = \zeta(t)$. Putting into (3.3) the configuration

$$\psi = u(r - \zeta(t))\exp(iS/\hbar),$$

neglecting the contribution of the proper electromagnetic field and separating the real and imaginary parts, we get

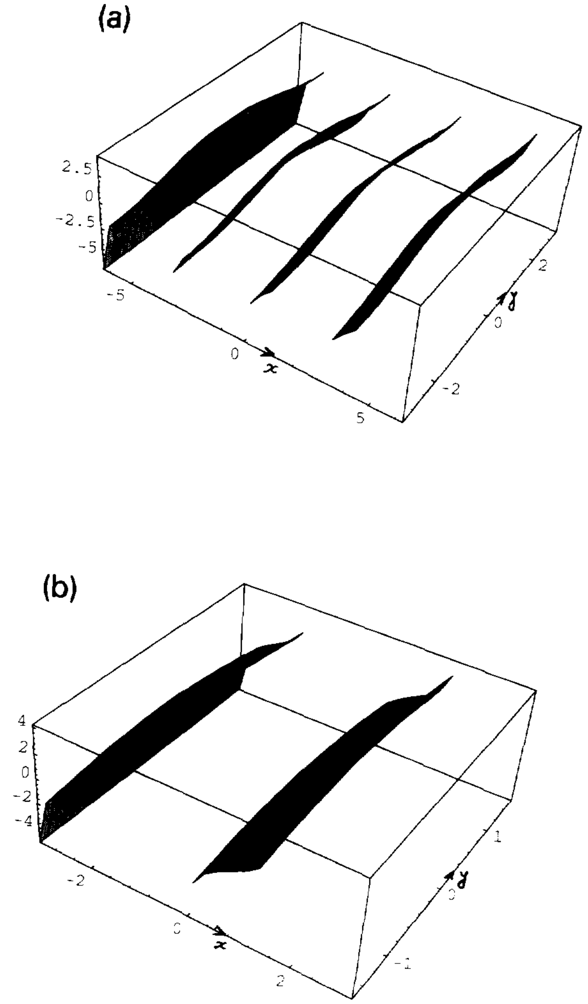

Fig. 3. The radial distribution of integral surfaces of Poynting vector lines for uniform circular motion of the soliton center (a) and details (b).

$$\partial_t S - \frac{Ze^2}{r} + \frac{1}{2m}(\nabla S)^2 - \frac{\hbar^2}{2m}\left(\hat{f} + \frac{\triangle u}{u}\right) = 0, \tag{3.6}$$

$$\triangle S + 2(\nabla S - m\dot{\boldsymbol{\zeta}}) \cdot \nabla u/u = 0. \tag{3.7}$$

Assuming S to be a slowly varying function of a point in the vicinity of the soliton center, from (3.7) we deduce

$$S \approx m\dot{\boldsymbol{\zeta}} \cdot (\boldsymbol{r} - \boldsymbol{\zeta}) + C_0 t + \chi(t), \quad C_0 = \text{const.} \tag{3.8}$$

Taking into account the classical equations of motion of a charged particle in the Coulomb field

$$m\ddot{\boldsymbol{\zeta}} = -Ze^2\boldsymbol{\zeta}/\zeta^3$$

and using the expansion

$$\frac{1}{r} \approx \frac{1}{\zeta} - \frac{\boldsymbol{\zeta} \cdot (\boldsymbol{r} - \boldsymbol{\zeta})}{\zeta^3},$$

from (3.6) and (3.8) we derive

$$\partial_t \chi = \frac{m}{2}\dot{\zeta}^2 + \frac{Ze^2}{\zeta} \equiv \mathcal{L}(t),$$

where $\mathcal{L}(t)$ is the Lagrangian of a particle in the Coulomb field. Thus, the function χ is the classical action on the trajectory,

$$\chi(t) = \int_0^t \mathcal{L}(t)\, \mathrm{d}t, \tag{3.9}$$

and the function u is the soliton-like solution to the quasi-stationary problem

$$\hbar^2(\hat{f} + \triangle u/u) = 2mC_0. \tag{3.10}$$

In this case according to (3.4) and (3.5)

$$\rho = -(2me/\hbar^2)u^2, \quad \boldsymbol{j} = -2\epsilon c u^2(\epsilon \boldsymbol{A} + m\dot{\boldsymbol{\zeta}}/\hbar),$$

which makes it possible, using the common solutions to Eqs. (3.4), (3.5) and (3.10), to calculate the potentials A_i of the electromagnetic field in the vicinity of the soliton center,

$$A_0 = A_0(\boldsymbol{r} - \boldsymbol{\zeta}(t)), \quad c\boldsymbol{A} = \dot{\boldsymbol{\zeta}}(t) A_0(\boldsymbol{r} - \boldsymbol{\zeta}(t)),$$

where the terms $\dot{\zeta}^2/c^2$ are neglected.

Let us now study the asymptotic behavior of a soliton at a large distance, i.e. we will study its "*tail*". We will use a successive approximation method. Thus, we need to rewrite differential equations (3.3)–(3.5) in integral form. Note that we are not solving a Cauchy evolution problem, choosing a definite initial condition. Were it a question of a Cauchy problem, we would be bound to use retarded Green's functions. In this case it has been assumed that the object under consideration (atom) already exists infinitely long. Thus we consider the problem of a corresponding steady state, which eliminates the possibility of using a retarded electromagnetic Green's function. The problem mentioned, in our view, can be satisfied by a half-sum of retarded and advanced solutions. The above mentioned selection can be justified by the assumption that during the evolution process the radiation of the independent i.e. half-difference of the retarded and advanced electromagnetic fields occurred.

To find the field ψ far from the soliton center, we rewrite Eq. (3.3) in the integral form

$$\psi(t,\boldsymbol{r}) = C_n\psi_n(\boldsymbol{r})\exp(-\mathrm{i}\omega_n t) + \frac{1}{2\pi}\int \mathrm{d}\omega \int \mathrm{d}t' \int \mathrm{d}^3x' \exp[-\mathrm{i}\omega(t-t')] \times G(\boldsymbol{r},\boldsymbol{r}';\omega+\mathrm{i}0)\hat{f}\psi(t',\boldsymbol{r}'), \tag{3.11}$$

where $\psi_n(\boldsymbol{r})$ is the eigenfunction of the Hamiltonian of a hydrogen atom for a stationary state of number n with energy $E_n = \hbar\omega_n$, C_n = const and $G(\boldsymbol{r},\boldsymbol{r}';\omega)$ is the Hamiltonian's resolvent having the form [21]

$$G(\boldsymbol{r},\boldsymbol{r}';\omega) = \frac{\Gamma(1-\mathrm{i}\nu)}{4\pi R}\begin{vmatrix} W_{i\nu,1/2}(-\mathrm{i}kr_+) & M_{i\nu,1/2}(-\mathrm{i}kr_-) \\ \dot{W}_{i\nu,1/2}(-\mathrm{i}kr_+) & \dot{M}_{i\nu,1/2}(-\mathrm{i}kr_-) \end{vmatrix}. \tag{3.12}$$

Here the following notation is used,

$$k = (2m\omega/\hbar)^{1/2}, \quad \mathrm{Im}\, k > 0, \quad \nu = (ka)^{-1},$$

$$r_\pm = r + r' \pm |\boldsymbol{r} - \boldsymbol{r}'|$$

and the Whittaker functions $W_{i\nu,1/2}, M_{i\nu,1/2}$ and their derivatives $\dot{W}_{i\nu,1/2}$, $\dot{M}_{i\nu,1/2}$ are introduced. To find the field ψ at large distances from the electron-soliton center, i.e. at $|r - a_0| \gg l$, it is sufficient to put in (3.11)

$$\hat{f}\psi(t,\boldsymbol{r}) = g\exp(-\mathrm{i}\omega_n t)\delta(\boldsymbol{r} - \boldsymbol{\zeta}(t)), \quad g = \mathrm{const}, \tag{3.13}$$

where relation (2.5) is taken into account. As a result, we get

$$\psi(t,\boldsymbol{r}) = C_n\psi_n(\boldsymbol{r})\exp(-\mathrm{i}\omega_n t) + \frac{1}{2\pi}\int \mathrm{d}\omega \int \mathrm{d}t' \exp[-\mathrm{i}\omega t + \mathrm{i}t'(\omega-\omega_n)] \times G(\boldsymbol{r},\boldsymbol{r}';\omega+\mathrm{i}0). \tag{3.14}$$

It is easy to verify that the field (3.14) decreases exponentially at large distances. With the help of (3.14) and Eqs. (3.4) and (3.5), one can evaluate the electromagnetic field outside the soliton. In (3.14) C_n is an unknown constant and $\psi_n(\boldsymbol{r})$ is the wave function of the electron in the stationary state. In each iteration step one obtains (3.14), where the stationary "tail" of the soliton is marked out and its center moves along some effective orbit. The orbital parameter and constant C_n may be defined in an arbitrary approximation of the minimization $||\psi_{(k)} - \psi_{(k+1)}||$ [22]. The constants C_n and g have not been found explicitly and we hope to obtain them in forthcoming papers.

Let us now solve Eqs. (3.4) and (3.5). Considering that the nonlinear source is a rather weak one, we will replace the right hand sides of Eqs. (3.4) and (3.5) by δ-functions. Let us also notice that

$$\boldsymbol{E}_- = (\boldsymbol{E}_- + \boldsymbol{E}_+)/2 + (\boldsymbol{E}_- - \boldsymbol{E}_+)/2,$$

where $\boldsymbol{E}_- = \boldsymbol{E}^{\mathrm{ret}}$, $\boldsymbol{E}_+ = \boldsymbol{E}^{\mathrm{adv}}$. It is well known that the half-difference of retarded and advanced fields radiates. So it will be sufficient to consider, as was discussed earlier, the half-sum of retarded and advanced solutions, describing the stationary state. This means that we will seek the strength of the electromagnetic field as the half-sum of those for retarded and advanced fields. This means that for large times $|\omega_n|t \gg 1$ the four-potential A_k will contain only a stationary part $A_k = (A_k^{\mathrm{ret}} + A_k^{\mathrm{adv}})/2$.

Let us find the expression for $\boldsymbol{E}_-$. The radius of the soliton l is rather small in comparison to the Bohr radius a, i.e. $a \gg l$. So the source can be considered as a proper one. Let the point-like charge e move along the given trajectory $\boldsymbol{r} = \boldsymbol{\zeta}(t)$ with velocity $\boldsymbol{v}(t) = \dot{\boldsymbol{\zeta}}(t)$. Then, to describe the electromagnetic field, generated by the charge, one can write the charge density and the current density as

$$\rho(t,\boldsymbol{r}) = e\delta[\boldsymbol{r} - \boldsymbol{\zeta}(t)], \quad \boldsymbol{j}(t,\boldsymbol{r}) = e\boldsymbol{v}(t)\delta[\boldsymbol{r} - \boldsymbol{\zeta}(t)]. \tag{3.15}$$

Then Eqs. (3.4) and (3.5) take the form

$$\Box A_0 = -4\pi e\delta[\boldsymbol{r} - \boldsymbol{\zeta}(t)],$$

$$\Box \boldsymbol{A} = -\frac{4\pi e\boldsymbol{v}}{c}\delta[\boldsymbol{r} - \boldsymbol{\zeta}(t)].$$

which leads to the well-known Lienard–Wiechert potentials. As we know, the retarded time is writen as

$$t_- = t - R(t_-,\boldsymbol{r})/c,$$

which leads to

$$\frac{\partial t_-}{\partial t} = \frac{1}{1 - \boldsymbol{n}_- \cdot \boldsymbol{v}/c}, \qquad \frac{\partial \boldsymbol{R}(t_-, \boldsymbol{r})}{\partial t_-} = -\boldsymbol{n}_- \cdot \boldsymbol{v},$$
$$\boldsymbol{\nabla} t_- = -\frac{\boldsymbol{n}_-}{c - \boldsymbol{n}_- \cdot \boldsymbol{v}}.$$

where $\boldsymbol{n} = \boldsymbol{R}(t,\boldsymbol{r})/R(t,\boldsymbol{r})$. Using the following expressions for $A_{0-}(t,\boldsymbol{r})$ and $\boldsymbol{A}_-(t,\boldsymbol{r})$ [23,24],

$$A_{0-}(t,\boldsymbol{r}) = \frac{ec}{(cR - \boldsymbol{v}\cdot\boldsymbol{R})_{t_-}},$$
$$\boldsymbol{A}_-(t,\boldsymbol{r}) = \left(\frac{e\boldsymbol{v}}{cR - \boldsymbol{v}\cdot\boldsymbol{R}}\right)_{t_-},$$

one finds the expression for the retarded field

$$\boldsymbol{E}_- = e\left(\frac{(c^2 - v^2)(c\boldsymbol{n}_- - \boldsymbol{v})}{R^2(c - \boldsymbol{n}_- \cdot \boldsymbol{v})^3} + \frac{\boldsymbol{n}_- \times [(c\boldsymbol{n}_- - \boldsymbol{v}) \times \dot{\boldsymbol{v}}]}{R(c - \boldsymbol{n}_- \cdot \boldsymbol{v})^3}\right)_{t_-}, \tag{3.16}$$

and

$$\boldsymbol{B}_- = (\boldsymbol{n}_- \times \boldsymbol{E}_-)_{t_-}. \tag{3.17}$$

Analogously, writing the advanced time as

$$t_+ = t + R(t_+, \boldsymbol{r})/c,$$

for the advanced field we find

$$\boldsymbol{E}_+ = e\left(\frac{(c^2 - v^2)(c\boldsymbol{n}_- + \boldsymbol{v})}{R^2(c + \boldsymbol{n}_- \cdot \boldsymbol{v})^3} + \frac{\boldsymbol{n}_- \times [(c\boldsymbol{n}_- + \boldsymbol{v}) \times \dot{\boldsymbol{v}}]}{R(c + \boldsymbol{n}_- \cdot \boldsymbol{v})^3}\right)_{t_-}, \tag{3.18}$$

and

$$\boldsymbol{B}_- = -(\boldsymbol{n}_+ \times \boldsymbol{E}_+)_{t_+}. \tag{3.19}$$

To calculate the power, lost by the charge due to radiation, one has to compose the Poynting vector $\boldsymbol{S}$ and retain the terms of the order $1/R^2$ as the integration will take place along an infinitely distant surface. As we know, the Poynting vector is expressed by the relation

$$\boldsymbol{S} = \frac{c}{4\pi}\boldsymbol{E} \times \boldsymbol{B}. \tag{3.20}$$

In this case the field strengths are

$$\boldsymbol{E} = \tfrac{1}{2}(\boldsymbol{E}_+ + \boldsymbol{E}_-),$$
$$\boldsymbol{B} = \tfrac{1}{2}(\boldsymbol{n}_- \times \boldsymbol{E}_- - \boldsymbol{n}_+ \times \boldsymbol{E}_+), \tag{3.21}$$

where $\boldsymbol{E}_- = \boldsymbol{E}^{\mathrm{ret}}$, $\boldsymbol{E}_+ = \boldsymbol{E}^{\mathrm{adv}}$, $\boldsymbol{n}_\pm = \boldsymbol{R}_\pm/R_\pm$, $\boldsymbol{R}_\pm = \boldsymbol{r} - \boldsymbol{\zeta}(t_\pm)$ and $t_\pm$ are the roots of the equations $t_\pm = t \pm R_\pm/c$.

Using some manipulations from vector analysis we rewrite the Poynting vector as

$$\boldsymbol{S} = \frac{c}{16\pi}\{\boldsymbol{n}_-\boldsymbol{E}_-^2 - \boldsymbol{n}_+\boldsymbol{E}_+^2 + (\boldsymbol{n}_- - \boldsymbol{n}_+)(\boldsymbol{E}_- \cdot \boldsymbol{E}_+) - \boldsymbol{E}_-[(\boldsymbol{E}_- + \boldsymbol{E}_+)\cdot\boldsymbol{n}_-] + \boldsymbol{E}_+[(\boldsymbol{E}_- + \boldsymbol{E}_+)\cdot\boldsymbol{n}_+]\}. \tag{3.22}$$

Let us now rewrite all these in a spherical system of coordinates. In this case

$$\boldsymbol{R}(\boldsymbol{r}, t_\pm) = \{r - a_0 \sin\theta\cos\alpha_\pm, - a_0\cos\theta\cos\alpha_\pm, a_0\sin\alpha_\pm\},$$
$$\boldsymbol{v}(t_\pm) = \{a_0\Omega\sin\theta\sin\alpha_\pm, a_0\Omega\cos\theta\sin\alpha_\pm, a_0\Omega\cos\alpha_\pm\},$$
$$\dot{\boldsymbol{v}}(t_\pm) = \{-a_0\Omega^2\sin\theta\cos\alpha_\pm, - a_0\Omega^2\cos\theta\cos\alpha_\pm, a_0\Omega^2\sin\alpha_\pm\},$$
$$R(\boldsymbol{r}, t_\pm) = \sqrt{r^2 + a_0^2 - 2a_0 r\sin\theta\cos\alpha_\pm},$$

where $\alpha_\pm = \alpha - \Omega t_\pm$. Retaining the terms of the order a_0/r for $\boldsymbol{n}_-$ and $\boldsymbol{n}_+$ one can obtain

$$\boldsymbol{n}_\pm = \{1, -(a_0/r)\cos\theta\cos(\alpha - \Omega t_\pm), (a_0/r)\sin(\alpha - \Omega t_\pm)\}.$$

Using the first nonvanishing approximation of the order v/c, from (3.16) and (3.18) we will get

$$\boldsymbol{E}_- \approx e\left(\frac{\boldsymbol{n}_-}{R^2} + \frac{\boldsymbol{n}_-(\boldsymbol{n}_- \cdot \dot{\boldsymbol{v}}) - \dot{\boldsymbol{v}}}{c^2 R}\right)_{t_-}, \tag{3.23}$$

$$\boldsymbol{E}_+ \approx e\left(\frac{\boldsymbol{n}_+}{R^2} + \frac{\boldsymbol{n}_+(\boldsymbol{n}_+ \cdot \dot{\boldsymbol{v}}) - \dot{\boldsymbol{v}}}{c^2 R}\right)_{t_+}. \tag{3.24}$$

Taking into account that

$$1/R \approx 1/r - (a_0/r^2)\sin\theta\cos\alpha_-, \qquad 1/R^2 \approx 1/r^2,$$
$$(\boldsymbol{n}_- \cdot \dot{\boldsymbol{v}}) \approx -a_0\Omega^2\sin\theta\cos\alpha_- + (a_0^2\Omega^2/r)(1 - \sin^2\theta\cos^2\alpha_-),$$

where $\alpha_- = \alpha - \Omega t_-$, for $\boldsymbol{E}_-$ one gets

$$\boldsymbol{E}_- \approx e\left\{ \frac{1}{r^2}[1 + (a_0^2\Omega^2/c^2)(1 - \sin^2\theta\cos^2\alpha_-)], \right.$$
$$(a_0\Omega^2/c^2 r)\cos\theta\cos\alpha_-,$$
$$\left. -(a_0\Omega^2/c^2 r)\sin\alpha_- \right\}.$$

Analogously one finds

$$\boldsymbol{E}_+ \approx e\left\{ \frac{1}{r^2}[1 + (a_0^2\Omega^2/c^2)(1 - \sin^2\theta\cos^2\alpha_+)], \right.$$
$$(a_0\Omega^2/c^2 r)\cos\theta\cos\alpha_+,$$
$$\left. -(a_0\Omega^2/c^2 r)\sin\alpha_+ \right\}.$$

Putting the above expressions for $\boldsymbol{n}_\pm, \boldsymbol{E}_\pm$ into (3.22) one can find the expressions for the Poynting vector. In doing so we will take into account that the normals $\boldsymbol{n}_-$ and $\boldsymbol{n}_+$ coincide as $r \to \infty$ with $\boldsymbol{n} = \boldsymbol{r}/r$. Retaining the terms $(a_0/r)^3$ and also taking into account that $a_0\Omega = v \ll c$, for the circular motion in the spherical coordinates r, θ and α we have the following components of the Poynting vector $\boldsymbol{S}$,

$$S_r = \frac{e^2 a_0^2 \Omega^4}{16\pi c^3 r^2}\sin^2\theta\sin 2(\alpha - \Omega t)\sin(2\Omega r/c),$$
$$S_\theta = \frac{e^2 a_0 \Omega^2}{4\pi c r^3}\cos\theta\sin(\alpha - \Omega t)\sin(\Omega r/c),$$
$$S_\alpha = \frac{e^2 a_0 \Omega^2}{4\pi c r^3}\cos(\alpha - \Omega t)\sin(\Omega r/c). \tag{3.25}$$

It is obvious that for $r_k = ck\pi/\Omega$ with $k = 0, 1, 2, \ldots$ all the components of the Poynting vector turn to zero, i.e. $\boldsymbol{S} = 0$.

All the calculations made above can be summed up as follows. From (3.21) it follows that the projection of the Poynting vector $\boldsymbol{S}$ in the direction of the vector $\boldsymbol{N} = (\boldsymbol{n}_+ + \boldsymbol{n}_-)/2$, coinciding as $r \to \infty$ with $\boldsymbol{n} = \boldsymbol{r}/r$, takes the form

$$S_N = \frac{c}{16\pi\sqrt{2}}(E_-^2 - E_+^2)(1 + \boldsymbol{n}_+ \cdot \boldsymbol{n}_-)^{1/2}. \tag{3.26}$$

Since $\boldsymbol{n}_\pm = \boldsymbol{n} + \mathrm{O}(r^{-1})$, after averaging expression (3.26) over the sphere, we find

$$\langle S_N \rangle = \frac{c}{16\pi}(\langle E_-^2\rangle - \langle E_+^2\rangle) = \mathrm{O}(r^{-3}). \tag{3.27}$$

Thus according to (3.27) the electromagnetic radiation from the system is absent. In particular, for the circular motion in the spherical coordinates r, θ and α we have the following structure of the Poynting vector $\boldsymbol{S}$,

$$S_r = \frac{\kappa}{r^2}\sin^2\theta\sin 2(\alpha - \Omega t)\sin(2\Omega r/c),$$
$$S_\theta = \sin(\Omega r/c)\mathrm{O}(r^{-3}),$$
$$S_\alpha = \sin(\Omega r/c)\mathrm{O}(r^{-3}), \tag{3.28}$$

where $\kappa = e^2 a_0^2 \Omega^4/16\pi c^3$. From (3.28) as well as from (3.25) it is obvious that there exist spherical surfaces where either $S_r = 0$ or $\boldsymbol{S} = 0$, thus once again confirming the fact that in the stationary states described, radiation is absent [25].

Let us describe the vector lines for the Poynting vector. In spherical system of coordinates we have

$$\frac{\mathrm{d}r}{S_r} = r\frac{\mathrm{d}\theta}{S_\theta} = r\sin\theta\frac{\mathrm{d}\alpha}{S_\alpha}. \tag{3.29}$$

The last two fractions form an integrable combination. Putting S_θ and S_α, from this equality we obtain

$$\frac{\mathrm{d}\theta}{\sin\theta\cos\theta} = \tan\alpha\,\mathrm{d}\alpha, \tag{3.30}$$

which leads to the first integral

$$|\tan\theta\cos(\alpha - \Omega t)| = P_1, \quad P_1 = \mathrm{const}. \tag{3.31}$$

Now we consider the first two fractions. Putting $\cos(\alpha - \Omega t) = P_1/\tan\theta$ we find

$$\frac{c^2}{\Omega^2}\frac{\mathrm{d}r}{r^2\cos(\Omega r/c)} = P_1\sin\theta\,\mathrm{d}\theta. \tag{3.32}$$

As we consider the region where $r \to \infty$ it is possible to factor out the term $1/r^2$ from the integrand. Then we obtain approximately

$$\frac{c^2}{\Omega^2 r^2}\int\frac{\mathrm{d}r}{\cos(\Omega r/c)} = P_1\int\sin\theta\,\mathrm{d}\theta, \tag{3.33}$$

which leads to

$$\frac{c^2}{\Omega^2 r^2}\ln|\tan(\pi/4 + \Omega r/2c)| = -P_1\cos\theta + P_2,$$
$$P_2 = \mathrm{const}. \tag{3.34}$$

Thus, we built vector lines for the Poynting vector in the asymptotic region.

4. Conclusion

In the considered soliton model of a hydrogen atom the stability condition of spatial stationary motions of electrons in the field of the Coulomb center is fulfilled. The existence of this kind of motion had also been anticipated by Boguslavsky [26] and Chetaev [27]. In particular, due to the fulfillment of the nonlinear resonance condition (2.5) the energy spectrum of these stationary states coincides with that of a hydrogen atom. This fact indicates the role of nonlinearity in the formation of extended micro-objects, whose laws of evolution agree with quantum mechanics.

We would also like to cite a number of interesting papers by Enz exploiting the de Broglie idea [28,29]. Within the sine-Gordon model the author investigates the wave property of the breather type solitons.

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