Dedicated to Prof. D. N. Poenaru's 70th Anniversary.

NONLINEAR SPINOR FIELD IN BIANCHI TYPE-I COSMOLOGY: ACCELERATED REGIMES

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Abstract. A self-consistent system of interacting nonlinear spinor and scalar fields within the scope of a Bianchi type-I cosmological model filled with perfect fluid is considered. Exact self-consistent solutions to the corresponding field equations are obtained. The role of spinor field in the evolution of the Universe is studied. It is shown that the spinor field gives rise to an accelerated mode of expansion of the Universe. At the early stage of evolution the spinor field nonlinearity generates the acceleration while at the later stage it is done by the nonzero spinor mass.

Key words: Bianchi type I (BI) Universe, nonlinear spinor field, acceleration

1 INTRODUCTION

The accelerated mode of expansion of the present day Universe encourages many researchers to introduce different kind of sources that is able to explain this. Among them most popular is the dark energy given by a Λ term [1, 2, 3], quintessence [4, 5, 6, 7], Chaplygin gas [8, 9]. Recently cosmological models with spinor field have been extensively studied by a number of authors in a series of papers [11, 10, 12, 13, 14, 15]. The principal motive of the papers [11, 10, 12, 13, 14] was to find out the regular solutions of the corresponding field equations. In some special cases, namely with a cosmological constant (Λ term) that plays the role of an additional gravitation field, we indeed find singularity-free solutions. It was also found that the introduction of nonlinear spinor field results in a rapid growth of the Universe. This allows us to consider the spinor field as a possible candidate to explain the accelerated mode of expansion. Note that similar attempt is made in a recent paper by Kremer *et. al.* [16]. In this paper we study the role of a spinor field in generating an accelerated mode of expansion of the Universe. Since similar systems, though from different aspects were thoroughly studied in [13, 14], to avoid lengthy calculations regarding spinor and scalar fields, we mainly confine ourselves to the study of master equation describing the evolution of BI Universe. We here give the solutions to the spinor and scalar field equations, details of these solutions can be found in [13, 14].

2 BASIC EQUATIONS: A BRIEF JOURNEY

We consider a self consistent system of nonlinear spinor and scalar fields within the scope of a Bianchi type-I gravitational field filled with a perfect fluid. The spinor and the scalar field is given by the Lagrangian

$$\mathcal{L} = \frac{i}{2}\left[\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi\right] - m\bar{\psi}\psi + F + \frac{1}{2}(1+\lambda_1 F_1)\,\varphi_{,\alpha}\varphi^{,\alpha}, \tag{1}$$

where λ_1 is the coupling constant and F and F_1 are some arbitrary functions of invariants generated from the real bilinear forms of a spinor field. Here we assume $F = F(I,J)$ and $F_1 = F_1(I,J)$ with $I = S^2,\ S = \bar{\psi}\psi,\ J = P^2$ and $P = i\bar{\psi}\gamma^5\psi$.

The gravitational field is chosen in the form

$$ds^2 = dt^2 - a_1^2 dx_1^2 - a_2^2 dx_2^2 - a_3^2 dx_3^2, \tag{2}$$

where a_i are the functions of t only and the speed of light is taken to be unity. We also define

$$\tau = a_1 a_2 a_3. \tag{3}$$

We consider the spinor and scalar field to be space independent. In that case for the spinor and the scalar fields and metric functions we find the following expressions [14].

For $F = F(I)$ we find $S = C_0/\tau$ with C_0 being an integration constant. The components of the spinor field in this case read

$$\psi_{1,2}(t) = (C_{1,2}/\sqrt{\tau})e^{-i\beta}, \quad \psi_{3,4}(t) = (C_{3,4}/\sqrt{\tau})e^{i\beta}, \tag{4}$$

with the integration constants obeying C_0 as $C_0 = C_1^2 + C_2^2 - C_3^2 - C_4^2$. Here $\beta = \int(m - \mathcal{D})dt$ with $\mathcal{D} = dF/dS + (\lambda_1\dot{\varphi}^2/2)dF_1/dS$.

For $F = F(J)$ in case of massless spinor field we find $P = D_0/\tau$. The corresponding components of the spinor field in this case read: with

$$\begin{aligned} \psi_{1,2} &= (D_{1,2}e^{i\sigma} + iD_{3,4}e^{-i\sigma})/\sqrt{\tau}, \\ \psi_{3,4} &= (iD_{1,2}e^{i\sigma} + D_{3,4}e^{-i\sigma})/\sqrt{\tau}, \end{aligned} \tag{5}$$

with $D_0 = 2\,(D_1^2 + D_2^2 - D_3^2 - D_4^2)$. Here $\sigma = \int \mathcal{G}dt$ with $\mathcal{G} = dF/dP + (\lambda_1\dot{\varphi}^2/2)dF_1/dP$.

For the scalar field we find

$$\varphi = C \int \frac{dt}{\tau(1+\lambda_1 F_1)} + C_1, \tag{6}$$

where C and C_1 are the integration constants.

Solving the Einstein equation for the metric functions we find

$$a_i(t) = A_i[\tau(t)]^{1/3} \exp[X_i \int [\tau(t')]^{-1} dt'], \tag{7}$$

with the integration constants A_i and X_i obeying $A_1 A_2 A_3 = 1$ and $X_1 + X_2 + X_3 = 0$. Note that to evaluate the metric functions at any given time $\tilde{t}$ we should first integrate $\int \frac{dt}{\tau}$, and only then substitute t by $\tilde{t}$.

The theoretical arguments [17] and recent experimental data which support the existence of an anisotropic phase that approaches an isotropic one, led us to consider the models of Universe with anisotropic background. On the other hand the isotropy of the present-day Universe lead us to study how the initially anisotropic BI space-time can evolve into an isotropic Friedman-Robertson-Walker (FRW) one. Since for the FRW Universe $a_1(t) = a_2(t) = a_3(t)$, for the BI universe to evolve into a FRW one we should set $D_1 = D_2 = D_3 = 1$. Moreover, the isotropic nature of the present Universe leads to the fact that the three other constants X_i should be close to zero as well, i.e., $|X_i| << 1$, $(i = 1, 2, 3)$, so that $X_i \int [\tau(t)]^{-1} dt \to 0$ for $t < \infty$ (for $\tau(t) = t^n$ with $n > 1$ the integral tends to zero as $t \to \infty$ for any X_i). The rapid growth of the Universe due to the introduction of the nonlinear spinor field to the system results in the earlier isotropization.

As is seen from eqs. (4), (5), (6) and (7), the spinor, scalar and metric functions are in some functional dependence of τ. It should be noted that besides these, other physical quantities such as spin-current, charge etc. and invariant of space-time are too expressed via τ [13, 14]. It should be noted that at any space-time points where $\tau = 0$ the spinor, scalar and gravitational fields become infinity, hence the space-time becomes singular at this point [14]. So it is very important to study the equation for τ (which can be viewed as master equation) in details, exactly what we shall do in the section to follow. In doing so we analyze the role of spinor field in the character of evolution.

3 EVOLUTION OF BI UNIVERSE AND ROLE OF SPINOR FIELD

In this section we study the role of spinor field in the evolution of the Universe. But first of all let me qualitatively show the differences that occur at the later stage of expansion depending on how the sources of the gravitational field were introduced in the system. In doing so we write the Einstein equation in the following form:

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} = \kappa T_1^1 + \Lambda, \tag{8}$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = \kappa T_2^2 + \Lambda, \tag{9}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} = \kappa T_3^3 + \Lambda, \tag{10}$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = \kappa T_0^0 + \Lambda. \tag{11}$$

Here Λ is the cosmological constant, T_μ^ν is the energy-momentum tensor of the source field. The Eq. (11) is thoroughly studied in [13]. After a little manipulation from (11) one finds the equation for τ which is indeed the acceleration equation and has the following general form:

$$\frac{\ddot{\tau}}{\tau} = \frac{3}{2}\kappa\Big(T_1^1 + T_0^0\Big) + 3\Lambda, \tag{12}$$

Note also that here a positive Λ corresponds to the universal repulsive force which is often considered as a form of dark energy, while a negative one gives an additional gravitational force.

The Bianchi identity $G_{\mu;\nu}^\nu = 0$ in our case gives

$$\dot{T}_0^0 = -\frac{\dot{\tau}}{\tau}(T_0^0 - T_1^1). \tag{13}$$

After a little manipulation from (12) and (13) one finds the following expression for T_0^0:

$$\kappa T_0^0 = 3H^2 - \Lambda - C_{00}/\tau^2, \tag{14}$$

where the definition of the generalized Hubble constant H as

$$3H = \frac{\dot{\tau}}{\tau} = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} = H_1 + H_2 + H_3. \tag{15}$$

Let us analyze the relation (14) in details. Consider the case when $\Lambda = 0$. At the moment when the expansion rate is zero (it might be at a time prior to the "Big Bang", or sometimes in the far future when the universe cease to expand we have $H = 0$.) the nonnegativity of T_0^0 suggests that $C_{00} \le 0$. Before considering the case for large τ we should like to study the Eq. (13) in detail. For the spinor and scalar fields chosen in this paper they are identically fulfilled. If this is not the case, an additional equation, know as equation of state, is applied to connect pressure (T_1^1) with energy density (T_0^0). In the long run from (13) one finds something like $(T_0^0)^b\tau = $ const., where b is some constant (in case of perfect fluid $b = 1+\zeta$). Thus we see that the energy density of the source field introduced into the system as above decreases with the growth of τ. Now if we consider the case when τ is big enough for T_0^0 to be neglected, from (14) we find

$$3H^2 - \Lambda \to 0. \tag{16}$$

On account of (15) from (16) one finds

$$\tau \to \exp\,[\sqrt{3\Lambda}\,t]. \tag{17}$$

From (16) and (17) it follows that for τ to be infinitely large, $\Lambda \geq 0$. In case of $\Lambda = 0$ we find that beginning from some value of τ the rate of expansion of the Universe becomes trivial, that is the universe does not expand with time. Whereas, for $\Lambda > 0$ the expansion process continues forever. As far as negative Λ is concerned, its presence imposes some restriction on τ, namely, there exists some upper limit for τ (note that τ is essentially nonnegative, i.e. bound from below). Thus we see that a negative Λ, depending on the choice of parameters can give rise to an oscillatory mode of expansion [13]. Thus we can conclude the following :

Let T_μ^ν be the source of the Einstein field equation; T_0^0 is the energy density and T_1^1, T_2^2, T_3^3 are the principal pressure and $T_1^1 = T_2^2 = T_3^3$. An ever-expanding BI Universe may be obtained if and only if the Λ term is positive (describes a repulsive force and can be viewed as a form of dark energy) and is introduced into the system as in (11) or if the source field introduced as a part of energy-momentum tensor behaves like a Λ term as $\tau \to \infty$.

It should be noted that the sources of the gravitational field such as spinor, scalar and electromagnetic fields, perfect or imperfect fluids, as well as dark energy such as quintessence, Chaplygin gas are introduced into the system as parts of the total energy-momentum tensor T_μ^ν. It is also known that the dark energy was introduced into the system to explain the late time acceleration of the Universe. To show that though the dark energy is introduced into the system as a part of total energy-momentum tensor, it still behaves like a Λ term as $\tau \to \infty$, we write them explicitly. The quintessence and Chaplygin gas are given by the following equation of states:

$$p_q = w\varepsilon_q, \quad w \in [-1, 0], \tag{18}$$
$$p_c = -A/\varepsilon_c, \quad A > 0. \tag{19}$$

Note that the energy densities of the quintessence and Chaplygin gas are related to τ as [9]

$$\varepsilon_q = \varepsilon_{0q}/\tau^{1+w}, \quad w \in [-1, 0], \tag{20}$$
$$\varepsilon_c = \sqrt{\varepsilon_{0c}/\tau^2 + A}, \quad A > 0. \tag{21}$$

From (21) and (19) follows that $\varepsilon_c \to \sqrt{A}$ and $p_c \to -\sqrt{A}$ as $\tau \to \infty$. In case of a quintessence, for $w > -1$, both energy density and pressure tend to zero as τ tends to infinity. But for $w = -1$ (sometimes known as phantom matter) we have $\varepsilon_q \to \varepsilon_{0q}$ and $p_q \to -\varepsilon_{0q}$ as $\tau \to \infty$. It means a quintessence with $w = -1$ and Chaplygin gas behave like a λ term when $\tau \to \infty$ and hence can give rise to an ever expanding Universe.

Before solving the equation for τ we have to write the components of the energy-momentum tensor of the source fields in details:

$$\begin{aligned} T_0^0 &= mS - F + \frac{1}{2}(1 + \lambda_1 F_1)\dot{\varphi}^2 + \varepsilon_{pf}, \\ T_1^1 &= T_2^2 = T_3^3 = \mathcal{D}S + \mathcal{G}P - F - \frac{1}{2}(1 + \lambda_1 F_1)\dot{\varphi}^2 - p_{pf}, \end{aligned} \tag{22}$$

where, $\mathcal{D} = 2SdF/dI + \lambda_1 S\dot{\varphi}^2 dF_1/dI$ and $\mathcal{G} = 2PdF/dJ + \lambda_1 P\dot{\varphi}^2 dF_1/dJ$. In (22) ε_{pf} and p_{pf} are the energy density and pressure of the perfect fluid, respectively and related by the equation of state

$$p_{pf} = \zeta\varepsilon_{pf}, \quad \zeta \in [0,\, 1]. \tag{23}$$

Let us now study the equation for τ in details and clarify the role of material field in the evolution of the Universe. For simplicity we consider the case when both F and F_1 are the functions of $I\,(S)$ only. We also set $C = 1$ and $C_0 = 1$. Thanks to the spinor field equations and those for the invariants of the bilinear spinor form, the energy-momentum conservation law for the spinor field satisfied identically [13]. As a result the Eq. (13) now reads [13]

$$\dot{\varepsilon} + \frac{\dot{\tau}}{\tau}(\varepsilon + p) = 0. \tag{24}$$

In view of (23) from (24) for the energy density and pressure of the perfect fluid one finds

$$\varepsilon_{pf} = \frac{\varepsilon_0}{\tau^{1+\zeta}}, \quad p_{pf} = \frac{\zeta_0\varepsilon_0}{\tau^{1+\zeta}}.$$

Further we set $\varepsilon_0 = 1$. Assume that $F = \lambda S^q$ and $F_1 = S^r$ where λ is the self-coupling constant. As it was shown in [13], the spinor field equation, more precisely the equations for bilinear spinor forms, in this case gives $S = C_0/\tau$. Then setting $C_0 = 1$ for the energy density and the pressure from (22) we find

$$\begin{aligned} T_0^0 &= \frac{m}{\tau} - \frac{\lambda}{\tau^q} + \frac{\tau^{r-2}}{2(\lambda_1 + \tau^r)} + \frac{1}{\tau^{1+\zeta}} \equiv \varepsilon \\ T_1^1 &= \frac{(q-1)\lambda}{\tau^q} - \frac{[(1-r)\lambda_1 + \tau^r]\tau^{r-2}}{2(\lambda_1 + \tau^r)^2} - \frac{\zeta}{\tau^{1+\zeta}} \equiv p. \end{aligned} \tag{25}$$

Taking into account that T_0^0 and T_1^1 are the functions of τ only, the Eq. (12) can now be presented as

$$\ddot{\tau} = \mathcal{F}(q_1, \tau), \tag{26}$$

where we define

$$\mathcal{F}(q_1, \tau) = (3/2)\kappa\Big(m + \lambda(q-2)\tau^{1-q} + \lambda_1 r\tau^{r-1}/2(\lambda_1 + \tau^r)^2 + (1-\zeta)/\tau^{\zeta}\Big), \tag{27}$$

where $q_1 = \{\kappa, m, \lambda, \lambda_1, q, r, \zeta\}$ is the set of problem parameters. The En. (26) allows the following first integral:

$$\dot{\tau} = \sqrt{2[E - \mathcal{U}(q_1, \tau)]} \tag{28}$$

where we denote

$$\mathcal{U}(q_1, \tau) = -\frac{3}{2}\Big[\kappa\Big(m\tau - \lambda/\tau^{q-2} - \lambda_1/2(\lambda_1 + \tau^r) + \tau^{1-\zeta}\Big)\Big]. \tag{29}$$

From a mechanical point of view Eq. (26) can be interpreted as an equation of motion of a single particle with unit mass under the force $\mathcal{F}(q_1, \tau)$. In (28) E is the integration constant which can be treated as energy level, and $\mathcal{U}(q_1, \tau)$ is the potential of the force $\mathcal{F}(q_1, \tau)$. We solve the Eq. (26) numerically using Runge-Kutta method. The initial value of τ is taken to be a reasonably small one, while the corresponding first derivative $\dot{\tau}$ is evaluated from (28) for a given E.

Let us go back to the Eq. (26). In view of (27) one sees, $\ddot{\tau} \to (3/2)\kappa m > 0$ as $\tau \to \infty$, i.e., if $\ddot{\tau}$ is considered to be the acceleration of the BI Universe, then the massive spinor field essentially can be viewed as a source for ever lasting acceleration. Note that it does not contradicts our previous statement about the role of energy-momentum tensor on ever expanding Universe, since the spinor field satisfies the Bianchi identity identically.

Now a few words about considering $\ddot{\tau}$ as acceleration. The Einstein equations for the FRW model read

$$2\frac{\ddot{a}}{a} + \Big(\frac{\dot{a}}{a}\Big)^2 = \kappa T_1^1, \tag{30}$$

$$3\Big(\frac{\dot{a}}{a}\Big)^2 = \kappa T_0^0. \tag{31}$$

From (31) one finds

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(T_0^0 - 3T_1^1), \tag{32}$$

The equation (32) is known as the acceleration equation. In analogy for the BI Universe from (11) we can write

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} = -\frac{\kappa}{2}(T_0^0 - 3T_1^1), \tag{33}$$

and declare it as acceleration equation. Though setting $a_1 = a_2 = a_3$ we recover the original definition, hardly it will be helpful in our case. So in BI Universe we assume $\ddot{\tau}$ be the acceleration and Eq. (12) be the acceleration equation.

Let us now define the deceleration parameter. In FRW cosmology the deceleration parameter has the form

$$d_{\rm frw} = -\frac{a\ddot{a}}{\dot{a}^2} = -\Big[1 + \frac{\dot{H}_{\rm frw}}{H_{\rm frw}}\Big] = \frac{d}{dt}(\frac{1}{H_{\rm frw}}) - 1, \tag{34}$$

where $H_{\rm frw} = \dot{a}/a$ is the Hubble parameter for FRW model. In analogy we can define a deceleration parameter as well. If we define the generalized deceleration parameter in the following way:

$$d = -\Big[1 + \frac{\dot{H}_1 + \dot{H}_2 + \dot{H}_3}{H_1^2 + H_2^2 + H_3^2}\Big], \tag{35}$$

where $H_i = \dot{a}_i/a_i$, then the standard deceleration parameter is recovered at $a_1 = a_2 = a_3$. But is this case the definition for acceleration adopted here is no longer valid. So we switch to the second choice and following Belinchon and Harko *et. al.* [18, 19] define the generalized deceleration parameter as

$$d = \frac{d}{dt}(\frac{1}{3H}) - 1 = -\frac{\tau\ddot{\tau}}{\dot{\tau}^2}. \tag{36}$$

After a little manipulation in view of (11) and (14) the deceleration parameter can be presented as

$$d = -\frac{\kappa}{2}\frac{(T_1^1 + T_0^0)\tau^2}{\kappa T_0^0\tau^2 + C_{00}} \tag{37}$$

Let us now go back to the equations (26), (27), (28) and (29). As one sees, the positivity of the radical imposes some restriction on the value of τ, namely in case of $\lambda > 0$ and $q \geq 2$ the value of τ cannot be too close to zero at any space-time point. It is clearly seen from the graphical view of the potential [cf. Fig. 1]. Thus we can conclude that for some special choice of problem parameters the introduction of nonlinear spinor field given by a self-action provides singularity-free solutions. As it was shown in [13] the regular solution is obtained only at the expense of broken dominant-energy condition in the Hawking-Penrose theorem.

If, in an eigentetrad of $T_{\mu\nu}$, ε *denotes the energy density and* p_1, p_2, p_3 *denote the three principal pressure, then the dominant energy condition can be written as* [20]:

$$\varepsilon + \sum_\alpha p_\alpha \geq 0; \tag{38}$$

$$\varepsilon + p_\alpha \geq 0, \quad \alpha = 1,2,3. \tag{39}$$

The dominant energy condition for the BI metric can be written in the form:

$$T_0^0 \geq T_1^1 a_1^2 + T_2^2 a_2^2 + T_3^3 a_3^2, \tag{40}$$
$$T_0^0 \geq T_1^1 a_1^2, \tag{41}$$
$$T_0^0 \geq T_2^2 a_2^2, \tag{42}$$
$$T_0^0 \geq T_3^3 a_3^2. \tag{43}$$

In Fig. 2 we plot the potential for a negative λ. As one sees, in the vicinity of $\tau = 0$ there exists a bottomless potential hole. As one sees, if in case of a self-action the initial value of τ is too close to zero and the constant E is less than $\mathcal{U}_{\rm max}$ (the maximum value of the potential in presence of a self-action), the Universe will never come out of the hole.

For numerical solutions we set $\kappa = 1$, spinor mass $m = 1$, the power of nonlinearity we choose as $q = 4$, $r = 4$ and for perfect fluid we set $\zeta = 1/3$ that corresponds to a radiation. We also set $C_{00} = -0.001$ and $E = 10$. The initial value of τ is taken to be $\tau_0 = 0.4$. The coupling constant is chosen to be $\lambda_1 = 0.5$, while the self coupling constant is taken to be either $\lambda = 0.5$ or $\lambda = -0.5$. Here, in the figures we use the following notations:
1 corresponds to the case with self-action and interaction;
2 corresponds to the case with self-action only;
3 corresponds to the case with interaction only.

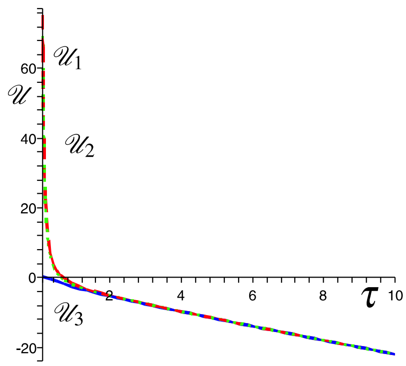

Figure 1: View of the potential $\mathcal{U}(\tau)$ for $\lambda > 0$.

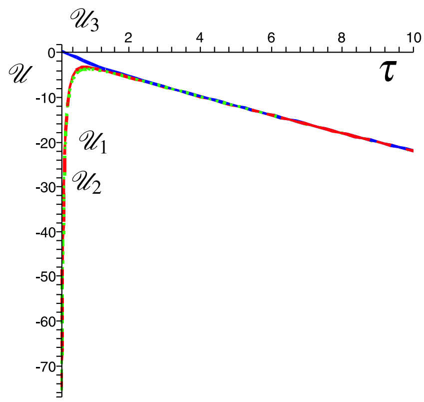

Figure 2: View of the potential $\mathcal{U}(\tau)$ for a negative λ.

As one sees from Fig. 1, in presence of a self-action of the spinor field with a positive λ, there occurs an infinitely high barrier as $\tau \to 0$, it means that in the case considered here τ cannot be trivial [if treated classically, the Universe cannot approach to a point unless it stays at an infinitely high energy level]. Thus, the nonlinearity of the spinor field provided by the self-action generates singularity-free evolution of the Universe. But, as was already mentioned, this regularity can be achieved only at the expense of dominant energy condition in Hawking-Penrose theorem. It is also clear that if the nonlinearity is induced by a scalar field, τ may be trivial as well, thus giving rise to space-time singularity. Note that cases in presence of a Λ term are thoroughly studied in [13, 14]. It was shown that introduction of a positive Λ just accelerates the speed of expansion, whereas, a negative Λ depending of the choice of E generates oscillatory or non-periodic mode of evolution. Note also that the regular solution obtained my means of a negative Λ in case of interaction does not result in broken dominant energy condition [14].

In the Figs. 3 and 4 we illustrate the acceleration of the Universe for positive and negative λ, respectively. As one sees, in both cases we have decreasing acceleration that tends to $(3/2)\kappa m$ as $\tau \to \infty$.

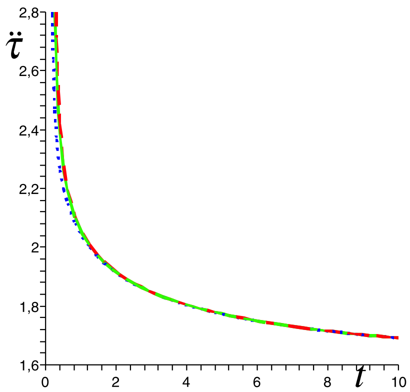

Figure 3: Acceleration of the Universe corresponding to a positive λ.

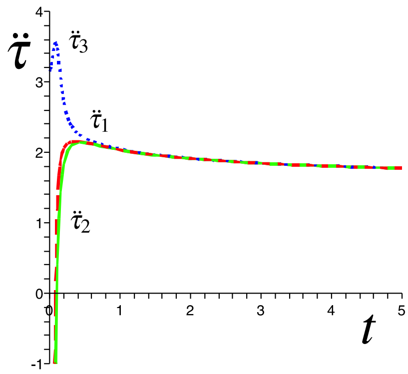

Figure 4: Acceleration of the Universe in case of a negative λ.

The Figs. 3 and 4 show the accelerated mode of expansion of the Universe. As one sees, the acceleration is decreasing with time. Depending of the choice of nonlinearity it undergoes an initial deceleration phase. It is also seen that the nonlinear term plays proactive role at the initial stage while at the later stage spinor mass is crucial for the accelerated mode of expansion.

4 CONCLUSION

We considered a system of interaction nonlinear spinor and scalar fields within the scope of a BI cosmological model filled with perfect fluid. The spinor field nonlinearity gives rise to an effective negative pressure in the course of evolution. Comparison of the effective pressure of the nonlinear spinor field with that of a dark energy given by a quintessence or Chaplygin gas leads us to conclude that the spinor field can be seen as an alternative to the dark energy able to explain the acceleration of the Universe. It was shown that the nonlinear spinor term is proactive at the early stage of the evolution and essentially accelerates the process of evolution, while at the later stage of evolution the spinor mass holds the key. Given the fact that *neutrino* is described by the spinor field equation and it too possesses mass (though too small but nonzero), the presence of huge number of neutrino in the Universe can be seen as one of the possible factor of the late time acceleration of the Universe. It was also shown that for some specific choice of parameters it is possible to construct singularity-free model of the Universe, but this regularity results in the broken dominant energy condition of the Hawking-Penrose theorem.

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