Research results and future plans of Bijan Saha

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Here is a detailed description of my scientific results up to date and the plans for my future study. The report is organized as follows: in the introductory section we give a brief note on the field in question which covers the motivation of research as well, then we give our main results obtained over the years, finally we sketch our future plan.

Introductory Remarks

Bianchi Type Cosmological Models

The nonlinear generalization of classical field theory remains one of the possible ways to overcome the difficulties of a theory which considers elementary particles as mathematical points. The gravitational field equation is nonlinear by nature and the field itself is universal and unscreenable. These properties lead to a definite physical interest in the gravitational field that goes with these matter fields.

The present-day cosmology is based largely on the Friedmann solutions of Einstein equations, which describe the completely uniform and isotropic Universe ("closed" and "open" models). The main feature of these solutions is their non-stationarity. The idea of an expanding Universe, following from this property, is confirmed by the astronomical observations and it is now safe to assume that the isotropic model provides, in its general features, an adequate description of the present state of the Universe. Although the Universe seems homogeneous and isotropic at present, the large scale matter distribution in the observable Universe, largely manifested in the form of discrete structures, does not exhibit a high degree of homogeneity. Recent space investigations detect anisotropy in the cosmic microwave background. In fact, the theoretical arguments and recent experimental data, which support the existence of an anisotropic phase that approaches an isotropic one, lead to consider the models of Universe with anisotropic back-ground.

Solitons in General Relativity

Since the early history of elementary particle physics, attempts to construct a divergence-free theory have been undertaken. Mie proposed a nonlinear modification of the Maxwell equations, with the nonlinear electric current of the form $j_\mu = (A_\nu A^\nu)^2 A_\mu$. Within the scope of this modification there exist regular solutions approximating the electron structure. Rosen considered a system of interacting electromagnetic and complex scalar fields that also admitted the existence of localized particle-like solutions. Nevertheless, these two models suffered the same defect: the mass of the localized object turned out to be negative. Recently it was shown that this defect of nonlinear electrodynamics can be corrected within the framework of general relativity. As is known, there do not exist regular static spherically or cylindrically symmetric configurations within the framework of gauge-invariant nonlinear electrodynamics. One possible way to overcome this difficulty is the nonlinear generalization of electrodynamics, with the use of a Lagrangian explicitly containing the 4-potential A_μ, $\mu = 0, 1, 2, 3$, thus breaking the gauge invariance inside a small critical sphere or cylinder. The introduction of terms depending explicitly on potentials in the electromagnetic equations presents the possibility to give an alternative explanation of such phenomena as inelastic photon-photon interactions, galactic red-shift anomalies, electric screening at low temperature in

the limit of indirect interaction of photons with the thermal neutrino background, the excess of high-energy photons in the isotropic flux, avoidance of the Big Bang singularity and the origin of self-focused beam in the effective nonlinear vector field theory. The corresponding terms appear in our scheme due to the interaction between the electromagnetic and scalar fields. This interaction being negligible at large distances, the Maxwellian structure of the electromagnetic equations (and therefore the gauge invariance) is reinstated far from the center of the system.

Quantum Mechanics - Extended Particle Formalism

From the history of quantum mechanics it is known that as early as 1927 in the framework of his ”theory of double solution” Louis de Broglie made an attempt to represent the electron as a source of waves obeying the Schrödinger equation. Later he modified his model showing that the electron should be described by regular solutions to some nonlinear equation coinciding with the Schrödinger one in the linear approximation. This scheme became famous as a causal nonlinear interpretation of quantum mechanics. Developing this concept, de Broglie remarked that it had much in common with Einstein's ideas about unified field theory according to which particles were to be considered as clot of some material fields obeying the nonlinear field equations. In recent years, these types of field configurations, known as soliton or particle-like solutions, came into active use to model extended elementary particles. The Einstein - de Broglie particle-soliton concept is applied to simulate stationary states of an electron in a hydrogen atom. According to this concept, the electron is described by the localized regular solutions to some nonlinear equations.

Toroidal Polarization

The history of electromagnetism is the history of the struggle of different rival concepts from the very early days of its existence. Though, after the historical observation by Hertz, all main investigations in electromagnetism were based on Maxwell equations, nevertheless this theory still suffers from some shortcomings inherent to its predecessors. Several attempts were made to remove the internal inconsistencies of the theory: introduction of magnetic charge in Maxwell equation, invocation of both the transverse and longitudinal (explicitly time independent) fields simultaneously, thus giving an equal footing to both the Maxwell-Hertz and Maxwell-Lorentz equations etc.

In early 1950s, while solving the problem of the multipole radiation of a spatially bounded source, Franz and Wallace found a contribution to the electric part of radiation at the expense of magnetization. Further Zel'dovich pointed out the noncorrespondence between the existence of two known multipole sets, Coulomb and magnetic, and the number of form factors for a spin-$\frac{1}{2}$ charged particles. Following the parity nonconservation law in weak interactions Zel'dovich suggested a third form factor in the parametrization of the Dirac spinor particle current. As a classical counterpart of this form factor he introduced the *anapole* in connection with the global electromagnetic properties of a toroid coil that are impossible to describe within the charge or magnetic dipole moments in spite of explicit axial symmetry of the toroid coil. In 1963 Shirokov and Cheshkov constructed the parametrization for relativistic matrix elements of currents of charged and spinning particles, which contain the third set of form factors. Finally, in 1974 Dubovik and Cheskov determined the toroid moment in the framework of classical electrodynamics. Note that *anapole* and *toroid dipole* are not the different names of one and the same thing. They are indeed quite different in nature. For example, the anapole cannot radiate at all while the toroid coil and its pointlike model, toroid dipole, can. The matter is that the anapole is some composition of electric dipole and actual toroid dipole giving destructive interference of their radiation. Thus it comes out that the toroid moment corresponds to the pointlike toroidal solenoid, whereas the anapole contains, in

addition to the toroid moment, a linear element of direct current centered in it. Toroid polarization is made evident in different condensed matter by a large number of investigations. Moreover, a principally new type of magnetism known as *aromagnetism* was observed in a class of organic substances, suspended either in water or in other liquids. Later, it was shown that this phenomena of aromagnetism cannot be explained in a standard way, e.g., by ferromagnetism, since the organic molecules do not possess magnetic moments of either orbital or spin origin. It was also shown that the origin of aromagnetism is the interaction of a vortex electric field induced by an alternative magnetic one with the axial toroid moments of the fragment C_6 in aromatic elements. The latest theoretical and experimental development forces the introduction of toroid moments in the framework of conventional classical electrodynamics that in its part inevitably leads to the modification of the equations of electromagnetism and the equations of motion of particles in an external electromagnetic field.

Symmetry in Physics

Attention has been recently drawn to the problem of defining the response of gauge potentials to coordinate transformations and characterizing those configurations that are invariant under specific coordinate changes. The reason that this classical problem retains interest in the presence of gauge-invariance.

According to the modern standpoint, space-time theory is the one that possesses a mathematical representation whose elements are a smooth four-dimensional manifold $\mathcal{M}$ and geometric objects defined on this manifold. The geometry on the manifold is defined by metric and linear connection. In general, the linear connection is in no way related to the metric since these concepts define on the manifold $\mathcal{M}$ different geometric operations. The metric on the manifold defines the length of a curve while the linear connection defines parallel transport (displacement) of vectors along arbitrary path on $\mathcal{M}$. It should be emphasized that soon after the creation of General Relativity A. Eddington put forward the idea to derive all theory on the basis of parallel displacement only. Here the metric and the linear connection as a totally independent geometric objects by structure will be considered as fundamental fields. According to the fundamental idea of Einstein, metric corresponds to gravitational field while all other fields, being the source of gravitational one, carry energy. Hence and from the above made assumption it follows that, like the electromagnetic field, the field of parallel displacement carries energy and appears to be the source of gravitational field, possessing geometric meaning.

Super-energy Tensors and Spinning Particles in Taub-NUT Geometry

The Taub-NUT geometry is involved in many modern studies in physics: Kaluza-Klein monopole of Gross, Perry, Sorkin; gravitational instantons; relative motion of slow Bogomolny-Prasad-Sommerfield monopoles; etc...

From the mathematical point of view, the Taub-NUT geometry is very interesting. Besides the usual 4 Killing vectors there are 3 Staeckel-Killing (symmetric tensors) and 4 Killing-Yano (antisymmetric) tensors. The presence of the symmetric tensors is an indication for hidden symmetries. Indeed in the Taub-NUT space there is a sort of Runge-Lenz vector. Moreover the symmetric Killing tensor can be decomposed in a product of antisymmetric Killing-Yano tensor. To each Killing-Yano tensor it is possible to associate a Dirac-type operator.

Summary of the Results Obtained

Bianchi Type Cosmological; Models

We consider a self-consistent system of interacting spinor and/or scalar fields within the framework of a Bianchi type-I (BI) cosmological model filled with perfect fluid. The interacting term in the Lagrangian is chosen in the form of derivative coupling, i.e., $\mathcal{L}_{\rm int} = \frac{\lambda}{2}\varphi_{,\alpha}\varphi^{,\alpha}F$. Here F is a power or trigonometric function of the invariants I and/or J constructed from bilinear spinor forms $S = \bar{\psi}\psi$ and $P = i\bar{\psi}\gamma^5\psi$. Self-consistent solutions to the spinor, scalar and BI gravitational field equations are obtained. The problems of initial singularity and asymptotically isotropization process of the initially anisotropic space-time are studied. The role of the Cosmological constant (Λ term) in the evolution of a BI universe is studied. It is shown that a positive Λ generates an oscillatory mode of expansion of the BI model, whereas, if F in $\mathcal{L}_{\rm int}$ is chosen to be a trigonometric function of its arguments, there exists a non-exponential mode of evolution even with a negative Λ. It is shown also that for a suitable choice of problem parameters the present model allows regular solutions without broken dominant energy condition.

Within the scope of the Bianchi type VI (BVI) models the self-consistent system of nonlinear spinor and gravitational fields is considered. Exact self-consistent solutions to the spinor and gravitational field equations are obtained for a special choice of spatial inhomogeneity and nonlinear spinor term. The role of inhomogeneity in the evolution of spinor and gravitational field is studied. Some solutions allow an oscillating behavior of the the universe's volume. It should be emphasized that for a suitable choice of spinor field nonlinearity some of these solutions are non-singular at all space-time point.

In a series of recent papers we studied the evolution of a BI Universe filled with viscous fluid. A qualitative analysis of the system of nonlinear differential equations has been performed. The role of Λ term in the evolution of the system has been studied. A self-consistent system of nonlinear spinor, scalar and BI gravitational fields filled with viscous fluid and Λ term is under active consideration. We have also studied the system of nonlinear spinor spinor fields in a Bianchi type VI anisotropic and inhomogeneous cosmological model and plane-symmetric cosmological model. We have also studied the system when the BI cosmological model is filled with magneto-fluid thus considering all the four known fields in a single system.

BI Universe as a binary mixture of perfect fluid given by either usual equation of state $p = \zeta\varepsilon$ or a Van der Waals equation of state and dark energy has been studied. As a dark energy both quintessence and Chaplygin gas have been considered. I have also proposed an alternative model of quintessence that allows a oscillatory mode of expansion. Recently I have should that the spinor field can be viewed as a source for initial inflation (the spinor field nonlinearity gives rise to a rapid growth of the Universe at the early stage of evolution and helps the isotropization process to take place at an early time) while the spinor mass can give rise to the late time acceleration of the expansion of the Universe.

Solitons in General Relativity

A self-consistent system of fields to obtain particle-like configurations in the framework of general relativity has been considered. Solitons with spherical and/or cylindrical symmetry to the equations governing the interacting system of scalar, electromagnetic, and gravitational fields have been obtained. As a particular case it is shown that the equations of motion admit a special kind of solution with a sharp boundary, known as droplets. For these solutions, the physical fields vanish and the space-time is flat outside of the critical sphere or cylinder. Therefore, the mass and the electric charge of these configurations are zero. It is noteworthy that the effective potentials in this case possess confining property, i.e., create a strong repulsion on certain surfaces in configuration space.

Scalar fields with induced non-linearity has been studied in external Friedmann-Robertson-Walker and G*ö*del Universes. It has been shown that in F-R-W Universe with $k = +1$ all the solutions are localized in the region $0 \leq r \leq 1$. Beside the droplets few other special field configurations (anti-droplets, hats) have been obtained and their stability has been studied in details.

Recently we consider a system of nonlinear spinor and scalar fields with minimal coupling in general relativity. The nonlinearity in the spinor field Lagrangian is given by an arbitrary function of the invariants generated from the bilinear spinor forms $S = \bar{\psi}\psi$ and $P = i\bar{\psi}\gamma^5\psi$; the scalar Lagrangian is chosen as an arbitrary function of the scalar invariant $\Omega = \varphi_{,\alpha}\varphi^{,\alpha}$, that becomes linear at $\Omega \to 0$. The spinor and the scalar fields in question interact with each other by means of a gravitational field which is given by a plane-symmetric metric. Exact plane-symmetric solutions to the gravitational, spinor and scalar field equations have been obtained. Role of gravitational field in the formation of the field configurations with limited total energy, spin and charge has been investigated. Influence of the change of the sign of energy density of the spinor and scalar fields on the properties of the configurations obtained has been examined. It has been established that under the change of the sign of the scalar field energy density the system in question can be realized physically iff the scalar charge does not exceed some critical value. In case of spinor field no such restriction on its parameter occurs. In general it has been shown that the choice of spinor field nonlinearity can lead to the elimination of scalar field contribution to the metric functions, but leaving its contribution to the total energy unaltered.

Quantum Mechanics - Extended Particle Formalism

In the framework of Synge model for interacting scalar and electromagnetic fields a system of integral equations has been obtained, which describes the interaction between charged 3D soliton and Coulomb center. The asymptotic expressions for physical fields, describing soliton moving around the fixed Coulomb center, have been obtained with the help of integral equations. It is shown that the electron-soliton center travels along some stationary orbit around the Coulomb center. The electromagnetic radiation is absent as the Poynting vector has non-wave asymptote $O(r^{-3})$ after averaging over angles, i.e. the existence of spherical surface corresponding to null Poynting vector stream, has been proved. Vector lines for Poynting vector are constructed in asymptotic area. Moreover, some first principles that could serve as the foundation for quantum theory of extended particles are formulated.

Toroidal Polarization

With regard to the toroid contributions, a modified system of equations of electrodynamics of moving continuous media has been obtained. Alternative formalisms (Lagrangian, Hamiltonian and Gauge-like one) to introduce the toroid moment contributions in the equations of electromagnetism has been worked out. The two four-potential formalism has been further developed for the equations obtained. It has been shown that the modified system is Lorentz covariant. Lorentz transformation laws for the toroid polarizations has been given. Covariant form of equations of electrodynamics of continuous media with toroid polarizations has been written. It should be emphasized that the ambiguous transformation of toroid polarizations under motion makes the thing more puzzling, leaving a lot of questions behind it.

Symmetry in Physics

In a recent paper the gauge symmetry inherent in the concept of manifold has been discussed. Within the scope of this symmetry the linear connection or displacement field can be considered as a natural gauge field on the manifold. The gauge invariant equations for the displacement field have been derived. It has been shown that the energy-momentum tensor of this field conserves

and hence the displacement field can be treated as one that transports energy and gravitates. To show the existence of the solutions of the field equations we have derived the general form of the displacement field in Minkowski space-time which is invariant under rotation and space and time inversion. With this anzats we found spherically-symmetric solutions of the equations in question.

Future Plan

Spinors in Bianchi Universes

The investigation of relativistic cosmological models usually has the energy momentum tensor of matter generated by a perfect fluid. To consider more realistic models one must take into account the viscosity mechanisms, which have already attracted the attention of many researchers. It is suggested that strong dissipative due to the neutrino viscosity may considerably reduce the anisotropy of the blackbody radiation. Viscosity mechanism in cosmology can explain the anomalously high entropy per baryon in the present universe. Bulk viscosity associated with the grand-unified-theory phase transition may lead to an inflationary scenario. In view of what has been mentioned above, it would be interesting to study the influence of viscous fluid to the system of material (say spinor and/or scalar) and BI gravitational fields in presence of a Λ-term as well.

Dark matter and dark energy are considered essential missing pieces in the cosmic jigsaw puzzle. Although the nature of neither dark matter (DM) nor dark energy (DE) is currently known, it is felt that both DM and DE are non-baryonic in origin, and that DM is distinguished from DE by the fact that the former clusters on sub-Megaparsec scales (in order to explain galatic rotation curves) whereas the later has large negative pressure (and can make the universe accelerate). Along with real fluid we also plan to investigate the role of dark energy in the evolution of an initially anisotropic space-time.

Solitons in General Relativity

In near future we plan to study the nonlinear spinor field together with the scalar and/or electromagnetic one within the framework of a spherically and/or cylindrically symmetric gravitational field. We also plan to investigate the existence of droplet-like configurations for the nonlinear spinor field. As the next step to this study we plan to investigate interaction between droplets and other physical objects.

Quantum Mechanics - Extended Particle Formalism

Further we plan to construct the probability amplitudes analogical to those in quantum mechanics on the basis of perturbed solutions. The principal aim of future investigation is to construct quantum theory for extended objects with possible justifications of quantization.

Symmetry in Physics

As the next step of this investigation we like to extend our study for more general cases establishing relation between diffeomorphism and gauge transformation. We also plan to study the role of temporal field (when “time” is viewed as some field) in the evolution of the Universe.

Super-energy Tensors and Spinning Particles in Taub-NUT Geometry

We would like to investigate the motion of a relativistic spinning (spin 1/2) particle in Taub-NUT space. Our aim is to see if additional symmetries (hidden symmetries) could influence (obstruct ?) the appearance of chaotic behavior.

The study of super-energy tensors in general relativity is an interesting and open problem. The papers of Senovilla represent an extension of the famous Bel-Robinson tensor. They investigated the super-energy tensor mainly for scalar and electromagnetic fields. It is worth studying the super-

tensors starting with Killing tensors and Killing-Yano tensors. That means to construct super-tensors and to investigate new (?) conservation laws for hidden symmetries. The prototype space is again the Euclidean Taub-NUT space.

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