SUMMARY OF RESULTS OBTAINED by Bijan Saha

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My active scientific carrier began in the early nineties after getting admission as a Ph.D. student in 1990. As a graduate student, I made first steps in the world of theoretical physics studying the Einstein - de Broglie particle-soliton concept and further applying this concept to simulate stationary states of an electron in a hydrogen atom. In 1991 I became interested in gravitation and cosmology which results intensive study of soliton-like configurations in general relativity and evolution of nonlinear spinor feild in anisotropic back-ground. I got my Ph.D. in theoretical physics in 1993 and in 1994 when I moved to JINR, Dubna. I refocused my interest to electrodynamics with toroid polarization. Finally, in 1998 I got involved in some projects where as a starting point inherent symmetry of the manifold has been expounded. Though the fields of my interest are scattered, there is a common string - it is the *nonlinearity*. Nonlinear differential equations and group theory are also among the fields of my interest. Here is a short account of the fields of my interest and results obtained for last ten years. It should be emphasized that the items (i), (ii) and (iii) represent the summary of my Ph.D. thesis though they have been developed further even after defence.

- **(i) Interacting fields in anisotropic cosmological background**

  The quantum field theory in curved space-time has been a matter of great interest in recent years because of its applications to cosmology and astrophysics. The evidence of existence of strong gravitational fields in our Universe led to the study of quantum effects of matter fields in an external classical gravitational field. Although the Universe seems homogeneous and isotropic at present, there is no observational data that guaranties the isotropy in the era prior to the recombination. In fact, there are theoretical arguments that sustain the existence of an anisotropic phase that approaches an isotropic one. Interest in studying Klein-Gordon and Dirac equations in anisotropic models has increased since it has been shown that the creation of scalar particles in anisotropic backgrounds can dissipate the anisotropy as the Universe expands.

  A Bianchi type-I (B-I) Universe, being the straightforward generalization of the flat Robertson-Walker (RW) Universe, is one of the simplest models of an anisotropic Universe that describes a homogenous and spatially flat Universe. Unlike the RW Universe which has the same scale factor for each of the three spatial directions, a B-I Universe has a different scale factor in each direction, thereby introducing an anisotropy to the system. It moreover has the agreeable property that near the singularity it behaves like a Kasner Universe even in the presence of matter and consequently falls within the general analysis of the singularity. And in a Universe filled with matter for $p = \gamma\varepsilon, \quad \gamma < 1$, any initial anisotropy in a B-I Universe quickly dies away and a B-I Universe eventually evolves into a RW Universe. Since the present-day Universe is surprisingly isotropic, this feature of the B-I Universe makes it a prime candidate for studying the possible effects of an anisotropy in the early Universe on present-day observations.

  In light of the importance of what has been mentioned above, we studied various aspects of nonlinear spinor and system of interacting spinor and scalar fields in B-I background. It has

been shown that the solutions obtained are initially singular and the space-time is basically asymptotically isotropic independent of the choice of interacting term in the Lagrangian. The existence of initially regular are possible only by virtue of breaking energy-dominant condition. Moreover, in this case it is possible to construct the cosmological model, which begins to expand at t_0, acquires its maximum at t_{max} and then collapses into a point at t_1.

It has been shown that a suitable choice of nonlinearity (i) provides with a singularity free Universe; (ii) accelerates isotropization process; (iii) explains the late time accelerated expansion of the Universe.

It has been shown that (i) in absence of a Λ term beginning from some value the evolution of the Universe comes stand-still; (ii) in case of a positive Λ the process of evolution of the Universe never comes to a halt; (iii) in case of a negative Λ we have oscillatory regime of evolution.

Further I consider the self-consistent system of nonlinear spinor and scalar fields in a BI gravitational filed filled with viscous fluid and cosmological constant. It was shown that in this case the character of evolution does not depend on the sign of Λ term. It was found that a viscous fluid together with a nonlinear spinor filed gives rise to a Big Rip solution.

Another direction of this study was to investigate the system in presence of Van der Waals gas, dark energy given by a Λ term, quintessence or Chapligyn gas. Some alternative model of quintessence able to realize a periodic mode of evolution is suggested. I have also worked out a model with nonlinear spinor and scalar fields that in one hand can explain the early inflation and late time acceleration in the expansion of the Universe.

Recently we have found analytical expression for the spinor field nonlinearity that can describe a perfect fluid from phantom matter to ekpyrotic matter.

The evolution of the Universe in presence of a magnetic field and cosmic string has also been thoroughly studied.

Another aspect of our study was the construction of Bel-Robinson super energy tensor for the anisotropic cosmological models.

All the results are published in internationally recognized journals.

- **(ii) Quantum Mechanics - Extended Particle Formalism**

From the history of quantum mechanics it is known that as early as 1927 in the framework of his "theory of double solution" Louis de Broglie made an attempt to represent the electron as a source of waves obeying the Schrödinger equation. Later he modified his model showing that the electron should be described by regular solutions to some nonlinear equation coinciding with the Schrödinger one in the linear approximation. This scheme became famous as a causal nonlinear interpretation of quantum mechanics. Developing this concept, de Broglie remarked that it had much in common with Einstein's ideas about unified field theory according to which particles were to be considered as clot of some material fields obeying the nonlinear field equations. In recent years, these types of field configurations, known as soliton or particle-like solutions, came into active use to model extended elementary particles. The Einstein - de Broglie particle-soliton concept is applied to simulate

stationary states of an electron in a hydrogen atom. According to this concept, the electron is described by the localized regular solutions to some nonlinear equations. In the framework of Synge model for interacting scalar and electromagnetic fields a system of integral equations has been obtained, which describes the interaction between charged 3D soliton and Coulomb center. The asymptotic expressions for physical fields, describing soliton moving around the fixed Coulomb center, have been obtained with the help of integral equations. It is shown that the electron-soliton center travels along some stationary orbit around the Coulomb center. The electromagnetic radiation is absent as the Poynting vector has non-wave asymptote $O(r^{-3})$ after averaging over angles, i.e. the existence of spherical surface corresponding to null Poynting vector stream, has been proved. Vector lines for Poynting vector are constructed in asymptotic area. Moreover, some first principles that could serve as the foundation for quantum theory of extended particles are formulated. Further we plan to construct the probability amplitudes analogical to those in quantum mechanics on the basis of perturbed solutions. The principal aim of future investigation is to construct quantum theory for extended objects with possible justifications of quantization.

- **(iii) Solitons in General Relativity**

Since the early history of elementary particle physics, attempts to construct a divergence-free theory have been undertaken. Mie proposed a nonlinear modification of the Maxwell equations, with the nonlinear electric current of the form $j_\mu = (A_\nu A^\nu)^2 A_\mu$. Within the scope of this modification there exist regular solutions approximating the electron structure. Rosen considered a system of interacting electromagnetic and complex scalar fields that also admitted the existence of localized particle-like solutions. Nevertheless, these two models suffered the same defect: the mass of the localized object turned out to be negative. Recently it was shown that this defect of nonlinear electrodynamics can be corrected within the framework of general relativity. As is known, there do not exist regular static spherically or cylindrically symmetric configurations within the framework of gauge-invariant nonlinear electrodynamics. One possible way to overcome this difficulty is the nonlinear generalization of electrodynamics, with the use of a Lagrangian e explicitly containing the 4-potential $A_\mu, \quad \mu = 0,\ 1,\ 2,\ 3$, thus breaking the gauge invariance inside a small critical sphere or cylinder. The introduction of terms depending explicitly on potentials in the electromagnetic equations presents the possibility to give an alternative explanation of such phenomena as inelastic photon-photon interactions, galactic red-shift anomalies, electric screening at low temperature in the limit of indirect interaction of photons with the thermal neutrino background, the excess of high-energy photons in the isotropic flux, avoidance of the Big Bang singularity and the origin of self-focused beam in the effective nonlinear vector field theory. The corresponding terms appear in our scheme due to the interaction between the electromagnetic and scalar fields. This interaction being negligible at large distances, the Maxwellian structure of the electromagnetic equations (and therefore the gauge invariance) is reinstated far from the center of the system.

In the papers a self-consistent system of fields to obtain particle-like configurations in the framework of general relativity has been considered. Solitons with spherical and/or cylindrical symmetry to the equations governing the interacting system of scalar, electromagnetic,

and gravitational fields have been obtained. As a particular case it is shown that the equations of motion admit a special kind of solution with a sharp boundary, known as droplets. For these solutions, the physical fields vanish and the space-time is flat outside of the critical sphere or cylinder. Therefore, the mass and the electric charge of these configurations are zero. It is noteworthy that the effective potentials in this case possess confining property, i.e., create a strong repulsion on certain surfaces in configuration space. Scalar fields with induced non-linearity has been studied in external Friedmann-Robertson-Walker and G*ö*del Universes. It has been shown that in F-R-W Universe with $k = +1$ all the solutions are localized in the region $0 \leq r \leq 1$. Beside the droplets few other special field configurations (anti-droplets, hats) have been obtained and their stability has been studied in details. As the next step to this study we plan to investigate interaction between droplets and other physical objects.

Within the framework of plane-symmetric cosmological model it has been shown that the spinor field nonlinearity in account of its proper gravity gives rise to a configuration with finite energy density and limited total energy. It has been shown that both spinor field nonlinearity and gravity are essential for particle-like solutions.

- **(iv) Symmetry methods in physics**

According to the modern standpoint, space-time theory is the one that possesses a mathematical representation whose elements are a smooth four-dimensional manifold $\mathcal{M}$ and geometric objects defined on this manifold. The geometry on the manifold is defined by metric and linear connection. In general, the linear connection is in no way related to the metric since these concepts define on the manifold $\mathcal{M}$ different geometric operations. The metric on the manifold defines the length of a curve while the linear connection defines parallel transport (displacement) of vectors along arbitrary path on $\mathcal{M}$. It should be emphasized that soon after the creation of General Relativity A. Eddington put forward the idea to derive all theory on the basis of parallel displacement only. Here the metric and the linear connection as a totally independent geometric objects by structure will be considered as fundamental fields. According to the fundamental idea of Einstein, metric corresponds to gravitational field while all other fields, being the source of gravitational one, carry energy. Hence and from the above made assumption it follows that, like the electromagnetic field, the field of parallel displacement carries energy and appears to be the source of gravitational field, possessing geometric meaning. The gauge symmetry inherent in the concept of manifold has been discussed. Within the scope of this symmetry the linear connection or displacement field can be considered as a natural gauge field on the manifold. The gauge invariant equations for the displacement field have been derived. It has been shown that the energy-momentum tensor of this field conserves and hence the displacement field can be treated as one that transports energy and gravitates. To show the existence of the solutions of the field equations we have derived the general form of the displacement field in Minkowski space-time which is invariant under rotation and space and time inversion. With this anzats we found spherically-symmetric solutions of the equations in question.

- **(v) Electrodynamics with toroid polarizations**

The history of electromagnetism is the history of the struggle of different rival concepts from the very early days of its existence. Though, after the historical observation by Hertz, all main investigations in electromagnetism were based on Maxwell equations, nevertheless this theory still suffers from some shortcomings inherent to its predecessors. Several attempts were made to remove the internal inconsistencies of the theory: introduction of magnetic charge in Maxwell equation, invocation of both the transverse and longitudinal (explicitly time independent) fields simultaneously, thus giving an equal footing to both the Maxwell-Hertz and Maxwell-Lorentz equations etc.

In early 1950s, while solving the problem of the multipole radiation of a spatially bounded source, Franz and Wallace found a contribution to the electric part of radiation at the expense of magnetization. Further Zel'dovich pointed out the noncorrespondence between the existence of two known multipole sets, Coulomb and magnetic, and the number of form factors for a spin-$\frac{1}{2}$ charged particles. Following the parity nonconservation law in weak interactions Zel'dovich suggested a third form factor in the parametrization of the Dirac spinor particle current. As a classical counterpart of this form factor he introduced the *anapole* in connection with the global electromagnetic properties of a toroid coil that are impossible to describe within the charge or magnetic dipole moments in spite of explicit axial symmetry of the toroid coil. In 1963 Shirokov and Cheshkov constructed the parametrization for relativistic matrix elements of currents of charged and spinning particles, which contain the third set of form factors. Finally, in 1974 Dubovik and Cheskov determined the toroid moment in the framework of classical electrodynamics. Note that *anapole* and *toroid dipole* are not the different names of one and the same thing. They are indeed quite different in nature. For example, the anapole cannot radiate at all while the toroid coil and its pointlike model, toroid dipole, can. The matter is that the anapole is some composition of electric dipole and actual toroid dipole giving destructive interference of their radiation. Thus it comes out that the toroid moment corresponds to the pointlike toroidal solenoid, whereas the anapole contains, in addition to the toroid moment, a linear element of direct current centered in it. Toroid polarization is made evident in different condensed matter by a large number of investigations. Moreover, a principally new type of magnetism known as *aromagnetism* was observed in a class of organic substances, suspended either in water or in other liquids. Later, it was shown that this phenomena of aromagnetism cannot be explained in a standard way, e.g., by ferromagnetism, since the organic molecules do not possess magnetic moments of either orbital or spin origin. It was also shown that the origin of aromagnetism is the interaction of a vortex electric field induced by an alternative magnetic one with the axial toroid moments of the fragment C_6 in aromatic elements. The latest theoretical and experimental development forces the introduction of toroid moments in the framework of conventional classical electrodynamics that in its part inevitably leads to the modification of the equations of electromagnetism and the equations of motion of particles in an external electromagnetic field.

With regard to the toroid contributions, a modified system of equations of electrodynamics moving continuous media has been obtained. Alternative formalisms (Lagrangian, Hamiltonian and Gauge-like one) to introduce the toroid moment contributions in the equations of

electromagnetism has been worked out. The two four-potential formalism has been further developed for the equations obtained. It has been shown that the modified system is Lorentz covariant. Lorentz transformation laws for the toroid polarizations has been given. Covariant form of equations of electrodynamics of continuous media with toroid polarizations has been written. It should be emphasized that the ambiguous transformation of toroid polarizations under motion makes the thing more puzzling, leaving a lot of questions behind it. We plan to come back to these questions and answer some of them in our forthcoming works.