TRANSMISSION OF CLUSTERS CONSISTING OF A FEW IDENTICAL PARTICLES THROUGH BARRIERS AND WELLS*

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The problem of quantum transmission through potential barriers and wells is studied for a composite system consisting of a few identical particles coupled by pair oscillator potentials in the new symmetrised-coordinate representation. A closed-channel method for solving the relevant boundary value problem is applied. We confirm the efficiency of the proposed approach by calculating the complex energy values of metastable states and analysing the shape and Feshbach resonances in systems of identical particles on a line, which give rise to quantum transparency of the repulsive barriers and the resonance reflection from the wells.

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1. Introduction

For a decade, the mechanism of quantum penetration of two bound particles through barriers and wells has attracted attention from both theoretical and experimental viewpoints in relation with such problems as near-surface quantum diffusion of molecules, fragmentation in producing neutron-rich light nuclei, heavy-ion collisions through multidimensional barriers, and the mechanism of ternary fission [1–4]. The generalisation of the two-particle model over quantum systems of n identical particles, considered in our earlier papers [5–7], is of great importance for the appropriate description of the above problems.

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In this paper, we apply the approach proposed by us earlier [5–7] to the calculations of complex energy values of metastable states, and to the analysis of the shape and Feshbach resonances of composite systems of a few identical particles on a line, giving rise to quantum transparency of the repulsive barriers and resonance reflection from the wells.

2. Statement of the problem

We consider the penetration of a cluster of n identical quantum particles, coupled by short-range oscillator-like interaction, through a potential barrier or well in the s-wave approximation, corresponding to one-dimensional Euclidian space. We assume that the spin part of the wave function is known, so that only the spatial part of the wave function is to be considered. This function may be symmetric or antisymmetric with respect to a permutation of n identical particles. The relevant Schrödinger equation in the oscillator units has the form of

$$\left[-\frac{\partial^2}{\partial \boldsymbol{x}^2}+\sum_{i,j=1;i<j}^{n}\frac{(x_{ij})^2}{n}+\sum_{i=1}^{n}V(x_i)-E\right]\Psi(\boldsymbol{x})=0\,. \tag{1}$$

Our goal is to find the solutions $\Psi(\boldsymbol{x})$ of Eq. (1), totally symmetric (or antisymmetric) with respect to the permutations of n particles that belong to the permutation group S_n. The permutation of particles is nothing but a permutation of their Cartesian coordinates $x_i \leftrightarrow x_j$, $i,j=1,\ldots,n$.

The above goal is achieved using the appropriately chosen new symmetrized coordinates rather than the conventional Jacobi ones [5–7],

$$\xi_0=\frac{1}{\sqrt{n}}\left(\sum_{t=1}^{n}x_t\right),\quad \xi_s=\frac{1}{\sqrt{n}}\left(x_1+\sum_{t=2}^{n}a_0x_t+\sqrt{n}x_{s+1}\right),\quad s=1,\ldots,n-1\,,$$

$$x_1=\frac{1}{\sqrt{n}}\left(\sum_{t=0}^{n-1}\xi_t\right),\quad x_s=\frac{1}{\sqrt{n}}\left(\xi_0+\sum_{t=1}^{n-1}a_0\xi_t+\sqrt{n}\xi_{s-1}\right),\quad s=2,\ldots,n\,,$$

where $a_0=1/(1-\sqrt{n}\,)<0$. The introduction of the symmetrized coordinates provides the invariance of the Hamiltonian with respect to permutations of n identical particles. In the symmetrized coordinates, Eq. (1) takes the form of

$$\left[-\frac{\partial^2}{\partial \xi_0^2}+\sum_{i=1}^{n-1}\left[-\frac{\partial^2}{\partial \xi_i^2}+\xi_i^2\right]+\sum_{i=1}^{n}V(x_i(\xi_0,\boldsymbol{\xi}))-E\right]\Psi(\xi_0,\boldsymbol{\xi};E)=0\,. \tag{2}$$

This equation *is invariant* under the permutations $\xi_i \leftrightarrow \xi_j$ for $i,j=1,\ldots,$ $n-1$, *i.e.*, the *invariance* of Eq. (1) under the permutations $x_i \leftrightarrow x_j$,

$i,j=1,\ldots,n$ is conserved, which significantly simplifies the construction of states, symmetric (or antisymmetric) with respect to the permutation of n particles [7,8], as compared to the Jacobi coordinates in the centre-of-mass reference frame [9]. However, the *invariance* of Eq. (2) under the permutations $\xi_i \leftrightarrow \xi_j$ *does not yield the invariance* of Eq. (1) with respect to the permutations $x_i \leftrightarrow x_j$, which is the essence of the problem of constructing translation-invariant models of light nuclei [9, 10].

To solve such a problem, the orthonormalised harmonic oscillator functions $\Phi_j^{\rm S(A)}(\boldsymbol{\xi})$, symmetric (S) (or antisymmetric (A)) under the permutations of n identical particles, and the corresponding eigenvalues $\epsilon_i^{\rm S(A)}$ were calculated using the two-step algorithm [7]. The solution of problem (2) in the symmetrised coordinates is sought in the form of expansion over the harmonic oscillator basis functions [5]

$$\Psi_{i_{\rm o}}^{\rm S(A)}(\xi_0,\boldsymbol{\xi})=\sum_{j=1}^{j_{\max}}\Phi_j^{\rm S(A)}(\boldsymbol{\xi})\chi_{ji_{\rm o}}^{\rm S(A)}(\xi_0)\,. \tag{3}$$

Thus, we arrive at the scattering problem for the set of coupled ordinary differential equations for the functions depending on the center-of-mass variable in the oscillator symmetrized-coordinate representation

$$\sum_{j=1}^{j_{\max}}\left[\left(-\frac{{\rm d}^2}{{\rm d}\xi_0^2}-\left(E-\epsilon_i^{\rm S(A)}\right)\right)\delta_{ij}+V_{ij}^{\rm S(A)}(\xi_0)\right]\chi_{ji_{\rm o}}^{\rm S(A)}(\xi_0)=0\,, \tag{4}$$

where $V_{ij}^{\rm S(A)}(\xi_0)=V_{ji}^{\rm S(A)}(\xi_0)$ are the elements of the symmetric matrix $\boldsymbol{V}^{\rm S(A)}(\xi_0)$ of effective potentials (see, for details, [5, 7]).

3. The analysis of shape and Feshbach resonances

The analysis of shape and Feshbach resonances is given for the transmission of two, three or four identical particles ($n=2,3,4$) coupled by the harmonic oscillator potential $V(x_t-x_{t'})=(x_t-x_{t'})^2$, $t',t=1,\ldots,n$ above the Gaussian barrier ($\alpha>0$) or well ($\alpha<0$): $V(x_t)=\alpha/(\sqrt{2\pi}\sigma)\exp(-x_t^2/\sigma^2)$.

The case of sub-barrier penetration was considered in [5–7]. Here, we consider the case of transmission above the barrier or well (*i.e.* for $E>2\alpha$). The probability of a transition from the ground state $i=1$ to any of $N_{\rm o}$ eigenstates $i_{\rm o}=1,\ldots,N_{\rm o}$ of the open channels $E>\epsilon_{i_{\rm o}}^{\rm S(A)}$ of the BVP for Eqs. (4) and the complex eigenvalues of metastable states are calculated using the program KANTBP [11]. The dependence of the probability upon the energy is non-monotonic, and the observed shape resonance peaks are manifestations of the quantum transparency effect (see Fig. 1).

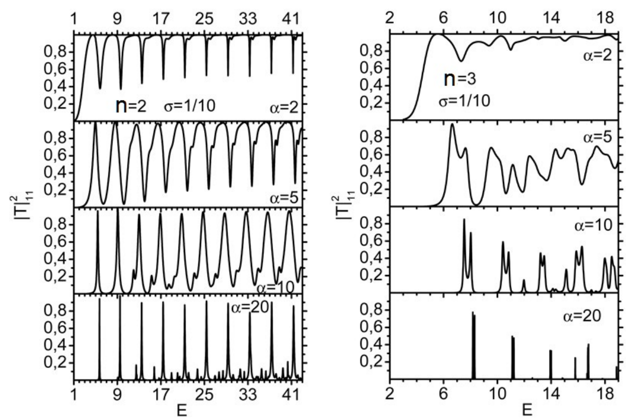

Fig. 1. The total probabilities *versus* the energy E (in oscillator units) for the transmission of a cluster of $n = 2, 3, 4$ particles, coupled by the oscillator potential and being in the ground state, through (or above) the Gaussian potential barriers.

Figure 2 presents the total transmission probability $|\boldsymbol{T}|^2_{11}$ *versus* the energy E (in oscillator units) for systems of the $n = 2, 3, 4$ particles, coupled by the oscillator potential, propagating above the Gaussian well with $\sigma = 0.1$ and $\alpha = -2$. One can see that the resonance structure becomes enriched with increasing the number of transmitted particles. So, in the case of $n = 2$, we see double-resonance structures, similar to the double-well case. In the case of $n = 3$ and 4, the double structure can appear with increasing the depth of wells $|\alpha|$.

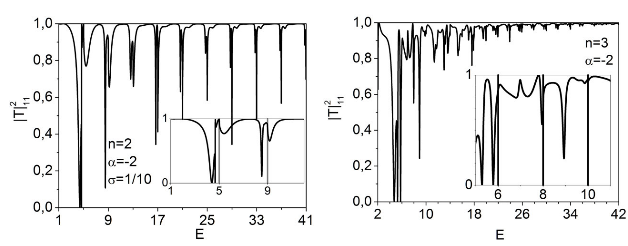

Fig. 2. The total transmission probability $|\boldsymbol{T}|^2_{11}$ *versus* the energy E in oscillator units). The cluster of $n = 2, 3, 4$ particles, coupled by the oscillator potential, propagates above the Gaussian well with $\sigma = 0.1$ and $\alpha = -2$. The system is initially in the ground state. The vertical lines in the pictures denote the threshold energies.

Figures 3 and 4 present the profiles of probability density $|\Psi(\xi_0, \xi_1)|^2$ for the symmetric states of two particles transmitted above the Gaussian barrier and well $\alpha = \pm 2$, $\sigma = 1/10$, revealing the resonance transmission and total reflection at resonance energies. One can see that the series of resonances in the transmission $|\boldsymbol{T}|^2_{11}$ from the ground state 1 are induced by Feshbach metastable states from the second and fifth closed channels, respectively, from left to right panels. In contrast to the case of barrier, in the vicinity of the well resonance, we see both the resonance reflection and the transmission (see two middle panels in Fig. 4).

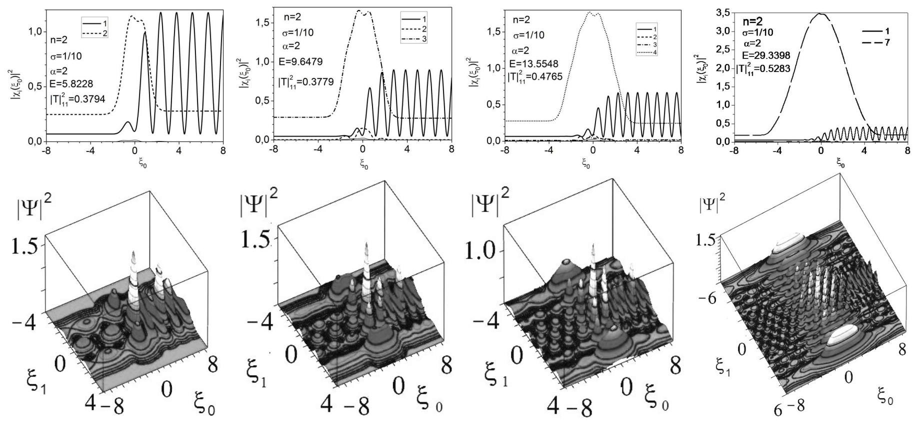

Fig. 3. The enhancement of probability densities of the coefficient functions $|\chi_i(\xi_0)|^2$ and the profiles of probability densities $|\Psi(\xi_0,\xi_1)|^2$ for symmetric states of two particles transmitted above the Gaussian barrier $\alpha = 2$, $\sigma = 1/10$, revealing the resonance reflection at the resonance energies ($E_1^M = 9.614 - \imath 0.217$, $E_2^M = 13.505 - \imath 0.144, \ldots$).

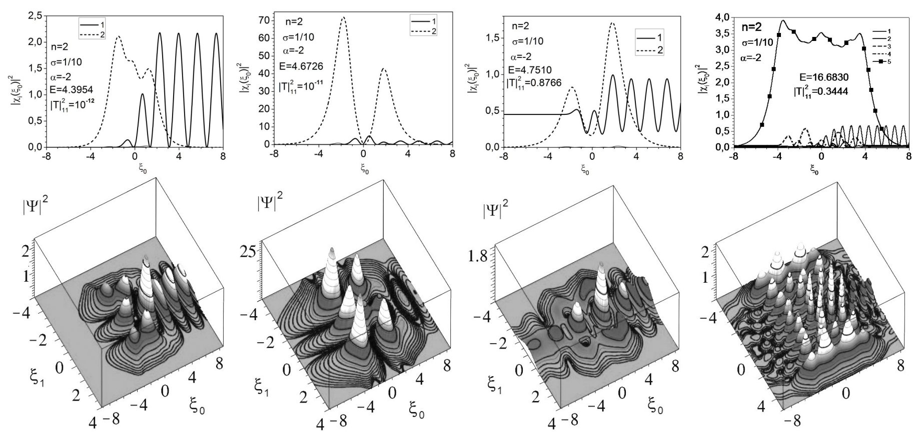

Fig. 4. The enhancement of probability densities of the coefficient functions $|\chi_i(\xi_0)|^2$ and the profiles of probability densities $|\Psi(\xi_0,\xi_1)|^2$ for symmetric states of two particles transmitted above the Gaussian well $\alpha = -2$, $\sigma = 1/10$, revealing the resonance reflection at the resonance energies ($E_1^M = 4.4348 - \imath 0.2572$, $E_2^M = 4.6764 - \imath 0.0058, \ldots$).

4. Conclusion

We formulated a model of n identical particles bound by the oscillator-type potential under the influence of the external field of a target (barrier or well) in the new symmetrized coordinates that was reduced to the scattering problem for the set coupled-channel equations using harmonic oscillator basis symmetric w.r.t. permutations of the particles. We proved that the effects of resonance transmission and reflections are due to the existence of metastable states with complex energy, embedded in the continuum, corresponding to shape and Feshbach resonances.

The proposed approach can be adapted and applied to the analysis of quantum transparency or total reflection effects, to the study of quantum diffusion of molecules, micro-clusters through surfaces, and the fragmentation mechanism in producing very neutron-rich light nuclei, heavy-ion collisions and the mechanism of ternary fission [1–4] as well as microscopic study of tetrahedral-symmetric nuclei.

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