Laser-stimulated radiative recombination of antihydrogen in a magnetic field in the presence of Doppler broadening

V.V.Serov, V.L.Derbov, V.I.Lobanov[a], O.Chuluunbaatar, A.A.Gusev, S.I.Vinitsky[b]

[a] Saratov State University, Astrakhanskaya 83, 410012 Saratov, Russia
[b] Joint Institute for Nuclear Research, Dubna 141980, Russia

ABSTRACT

To enhance the laser-stimulated recombination of antihydrogen from cold antiproton-positron plasma in a trap we propose to use a new resonance mechanism involving the quasi-stationary states of the positron that arise from the joint action of the Coulomb field of the antiproton and the strong magnetic field of the trap. The recombination rate is expressed via the cross-section of laser ionization of the atom that has strongly non-monotonic frequency dependence due to the presence of quasi-stationary states merged into the continuum background. The estimates using previously calculated ionization cross-section show the possibility to enhance the laser-stimulated recombination by means of the optimal laser frequency choice.

Keywords: antihydrogen, recombination, ionization, bound states, continuous spectrum, laser stimulation, magnetic field, quasi-stationary states

1. INTRODUCTION

At present intense experimental studies are carried out in CERN aimed at formation and storage of antihydrogen atoms in traps.[1–3] In the ATHENA experiment[1] the antihydrogen atoms were obtained in the process of diffusion of antiprotons in the positron plasma, while in the ATRAP experiment[2] the recombination took place during multiple flights of antiprotons through the positron plasma. In this case the three-body recombination is the dominant mechanism. Another mechanism implemented in the ATRAP experiment is the recombination due to the charge exchange that takes place when a highly-excited positronium atom is passed through the positron plasma.[3] In all these methods the antihydrogen atoms are obtained in highly excited states with $n \sim 50$.[2,3] An alternative method is the laser-induced recombination allowing one to get the atoms in the states that are relatively low and, what is even more important, may be specified. The main problem here is that the increase of laser power leads to plasma heating and hampers the recombination. However, the possibility of stimulation of recombination by means of a CO_2 laser is considered by the experimentalists.[3] Hence, the recipe of substantial enhancement of laser-stimulated recombination without increasing the laser power may be of practical interest.

In the process of laser-induced recombination rate the interacting particles are subject to magnetic field of the trap that strongly influences the motion of the positron in the continuum. Most estimations of the recombination rate were carried out without taking this fact into account, or the consideration was limited to estimation of Zeeman splitting for finite discrete states with large $n \sim 10$.[4] In[5] the influence of the magnetic field on the average density of positrons in the vicinity of an antiproton was estimated and shown to be inessential up to the order of magnitude. In any case, the dependence of the transition probability upon the laser frequency was assumed to be monotonic. However, it is known that the joint action of the Coulomb and magnetic field gives rise to quasi-stationary states merged into the continuum. These states manifest themselves in sharp resonances of the ionization cross-section plotted versus the laser frequency.[6] A natural idea is to use these quasi-stationary states for enhancing the laser-induced recombination. Indeed, when the laser is tuned into resonance with a quasi-stationary state, one may expect a substantial increase in the recombination rate. Note, that, as shown in,[7,8] the presence of quasi-stationary states does not affect the rate of spontaneous radiative recombination. In the present paper the laser-induced radiative recombination of antihydrogen is studied theoretically taking into

Further author information: Send correspondence to Vladimir L. Derbov
E-mail: derbov@sgu.ru

Saratov Fall Meeting 2006: Laser Physics and Photonics, Spectroscopy and Molecular Modeling VII, edited by Vladimir L. Derbov, Leonid A. Melnikov, Lev M. Babkov, Proc. of SPIE Vol. 6537, 65370Q, (2007) 0277-786X/07/$18 doi: 10.1117/12.754462

account the quasi-stationary states that arise in the continuum in a strong magnetic field. Section 2 is devoted to the formulation of the model. The ionization cross-section derived here agrees with that from,[9] thus making it possible to use the numerical results of[9] for our estimations. In Section 3 the rate of stimulated recombination is expressed via the ionization cross-section. In Section 4 the typical on-resonance values of the ionization cross-section calculated in[9] are used for estimation of the induced recombination rate. The results are compared with those of,[4] where the rate of laser-induced radiative recombination was calculated without the magnetic field.

2. CROSS-SECTION OF IONIZATION

The evolution of the system is described by the Schrödinger equation

$$i\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left[\hat{H} + V(\mathbf{r},t)\right]\Psi(\mathbf{r},t). \tag{1}$$

$\hat{H}$ describes the motion of the positron in the Coulomb field of the antiproton and the strong permanent magnetic field

$$\hat{H} = \frac{\hat{\mathbf{p}}^2}{2} - \frac{1}{r} - \frac{\gamma}{2}\hat{l}_z + \frac{\gamma^2}{8}\rho^2, \tag{2}$$

where $\gamma = B/B_0$ is the cyclotron frequency, $B_0 = 2.35\times10^5$ Tl, the axis 0z is chosen along the magnetic field. Here and below we use the atomic units. In the dipole approximation for linear polarized laser wave with the frequency ω, its electric field with the amplitude ϵ_0 being parallel to the magnetic field, the potential of interaction with the laser field is

$$V(\mathbf{r},t) = -\epsilon_0 z\cos(\omega t) \tag{3}$$

To provide the comparability with the paper[9] let us derive the expression of the ionization cross section associated with the transition from the initial bound state $\psi_0 \equiv |nlm>$ to the final continuum state $\psi_E \equiv \psi_{En_\rho mp}(z,\rho,\varphi) \equiv |En_\rho mp>$. Here n,l,m are the principal, orbital and magnetic quantum number, respectively. The wave functions of the continuum are characterized by the definite parity p with respect to z, the magnetic quantum number m and the number of the Landau level[10] $n_\rho = 0,...,n_{\rho\max}$, where $n_{\rho\max} = \max_{E_\perp<E} n_\rho$ is the highest number of the open channel, respectively, $n_{\rho\max}+1$ is the number of open channels, $E_\perp = \gamma[n_\rho+(m+|m|+1)/2]$. Their asymptotic expressions have the form

$$\psi_{En_\rho mp}(z\to\pm\infty,\rho,\varphi) = \sum_{n'_\rho=0}^{n_{\rho\max}} \frac{A_{n'_\rho n_\rho}}{\sqrt{2\pi k_{n'_\rho}}}\cos\left[k_{n'_\rho}z + \frac{z}{|z|}\frac{\ln(2k_{n'_\rho}|z|)}{k_{n'_\rho}} + \pi\frac{p-1}{4} + \frac{z}{|z|}\delta_{pn_\rho n'_\rho}\right]\phi_{n'_\rho m}(\rho,\varphi) \tag{4}$$

where $\phi_{n_\rho m}(\rho,\varphi)$ is the eigenfunction of 2D circular oscillator, $\delta_{pn_\rho n'_\rho}$ is the phase,
$k_{n_\rho} = \sqrt{2\{E-\gamma[n_\rho+(m+|m|+1)/2]\}}$ is the longitudinal momentum in the channel. The functions satisfy the normalization condition

$$\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{2\pi}\psi^*_{E'n'_\rho m'p'}(z,\rho,\varphi)\psi_{En_\rho mp}(z,\rho,\varphi)dz\rho d\rho d\varphi = \delta_{n'_\rho n_\rho}\delta_{m'm}\delta_{p'p}\delta(E'-E). \tag{5}$$

Note, that there are $n_{\rho\max}+1$ linearly independent solutions that satisfy the asymptotic and normalization conditions, labelled by n_ρ. Particular form of these solutions is not important at the present stage of our consideration, and we will return to this question in the next Section.

In weak laser field the wave function of the system in the process of ionization may be presented as

$$\Psi(\mathbf{r},t) = \psi_0\exp(-iE_0t) + C_E(t)\psi_E\exp(-iEt).$$

Within the first-order approximation of the perturbation theory

$$C_E(t) = i\epsilon_0 < En_\rho mp|z|nlm > \int_0^t \exp[i(E-E_0)t']\cos(\omega t')dt'. \tag{6}$$

Taking into account that for large t

$$\left|\int_0^t \exp[i(E-E_0)t']\cos(\omega t')dt'\right|^2 = \frac{\pi}{2}t\delta(E-E_0-\omega), \tag{7}$$

we arrive at the following expression for the rate of the transition to a state with the energy E:

$$\lambda(E,\omega) = \frac{\pi}{2}\epsilon_0^2 \sum_{n_\rho=0}^{n_{\rho\,\max}} |<En_\rho mp|z|nlm>|^2\delta(E-E_0-\omega). \tag{8}$$

Expressing the electric field strength ϵ_0 in terms of the photon flow density $j_\gamma = (c\epsilon_0^2)/(8\pi\omega)$ and integrating over the final state energy

$$\lambda(\omega) = \int_0^\infty \lambda(E,\omega)dE$$

we get the ionization cross-section

$$\sigma_{\text{ion}}(\omega) = \frac{4\pi^2\omega}{c}\sum_{n_\rho=0}^{n_{\rho\,\max}} |<En_\rho mp|z|nlm>|^2, \tag{9}$$

that agrees with that of the paper,[9] so that we can use the numerical results of this paper in our estimates.

3. RECOMBINATION RATE

In the process of recombination under the action of a weak laser field the state of the positron may be described by the wave function

$$\Psi(\mathbf{r},t) = \psi_k\exp(-iEt) + C(t)\psi_0\exp(-iE_0t),$$

where the initial state of the continuum $\psi_k \equiv \psi^{(+)}_{kn_\rho m}(z,\rho,\varphi) \equiv |kn_\rho m>$ is a sum of the incident wave with the unit "linear density" and the wave number $k = \pm k_{n_\rho}$, related to the channel n_ρ, and the wave outgoing in both directions in all channels. Such states are normalized by the condition

$$\int_{-\infty}^\infty\int_0^\infty\int_0^{2\pi}\psi^{(+)*}_{k'n'_\rho m'}(z,\rho,\varphi)\psi^{(+)}_{kn_\rho m}(z,\rho,\varphi)dz\rho d\rho d\varphi = 2\pi\delta(k'-k)\delta_{n'_\rho n_\rho}\delta_{m'm}. \tag{10}$$

Carrying out the transformations similar to those of the pervious Section, we get the expression for the rate of recombination form the continuum state with the energy E into the bound state $|nlm>$

$$\lambda_{n_\rho m}(E,\omega) = \frac{4\pi^2 I}{c}|<nlm|z|kn_\rho m>|^2\delta(E-E_0-\omega), \tag{11}$$

where $I = (c\epsilon_0^2)/(8\pi)$ is the intensity of the laser radiation. The wave functions considered here may be expressed in terms of the positive- and negative-parity functions of the previous Section, that may be defined so that

$$\psi^{(+)}_{\pm|k|n_\rho m}(z,\rho,\varphi) = \sqrt{2\pi k_{n_\rho}}(\psi_{En_\rho m1}e^{i\delta_{1n_\rho n_\rho}} \pm i\psi_{En_\rho m-1}e^{i\delta_{-1n_\rho n_\rho}}). \tag{12}$$

For the matrix elements we have

$$|<nlm|z|kn_\rho m>|^2 = 2\pi k_{n_\rho}|<En_\rho mp|z|nlm>|^2,$$

and, hence, the recombination rate may be written as

$$\lambda_{n_\rho m}(E,\omega) = \frac{4\pi^2 I}{c}2\pi k_{n_\rho}|<En_\rho mp|z|nlm>|^2\delta(E-E_0-\omega).$$

In the case of one open channel, the simplest and the most interesting here, the comparison of the latter expression with Eq. (9) yields

$$\lambda_{0m}(E,\omega) = \frac{2\pi k_0 I}{\omega}\sigma_{\rm ion}(\omega)\delta(E - E_0 - \omega). \tag{13}$$

Given the initial positron velocity $v = \pm k_0 = \pm\sqrt{2(E-\gamma/2)}$, the initial Landau level $n_\rho = n_{\rho\,\max} = 0$ and the angular momentum projection m, the recombination rate is expressed as

$$\lambda_{0m}(v,\omega) = \frac{2\pi I}{\omega}\sigma_{\rm ion}(\omega)[\delta(v - v_0) + \delta(v + v_0)], \tag{14}$$

where $v_0 = \sqrt{2(E_0 + \omega - \gamma/2)}$.

To calculate the recombination rate in plasma it is necessary to average over the initial states. First, let us find the probability that the positron possesses the angular momentum projection m. Classical consideration yields the following expression for the angular momentum projection of a positron moving in the magnetic field far from the nucleus

$$l_z = \frac{1}{2}\gamma\left(r_0^2 - \frac{v_\perp^2}{\gamma^2}\right),$$

where γ is the cyclotron frequency, r_0 is the distance between the z-axis passing through the nucleus and the axis of the spiral trajectory of the positron in the magnetic field, $v_\perp$ is the transverse component of the velocity. The cross-section of the projection of the angular momentum being less than m is

$$\sigma(l_z \le m) = \pi r_0^2 = \frac{2\pi m}{\gamma} + \frac{\pi v_\perp^2}{\gamma^2},$$

while for the projection equal to m it is

$$\sigma_m = \frac{2\pi}{\gamma}. \tag{15}$$

This result is completely the same as the quantum mechanical one.[11] Using this expression, let us sum the rates of the laser-induced radiative recombination over all parameters of the initial state

$$\lambda_{\rm SRR}(\omega) = n_e\sigma_m \sum_{n_\rho=0}^{n_{\rho\,\max}} \int_{-\infty}^{\infty} f_{||}(v) f_\perp(n_\rho)\lambda_{n_\rho m}(v,\omega)dv, \tag{16}$$

where n_e is the concentration of positrons, $f_{||}(v)$ is the Maxwell distribution over the longitudinal velocity of the positrons, $f_\perp(n_\rho)$ is the Boltzmann distribution over the Landau levels

$$f_{||}(v) = \frac{1}{\sqrt{2\pi T}}e^{-v^2/2T}; \quad f_\perp(n_\rho) = (1 - e^{-\gamma/T})e^{-\gamma n_\rho/T}. \tag{17}$$

For us the most interesting case is one open channel, i.e., $n_{\rho\,\max} = 0$. First, in this case the highest resonance peaks of the ionization cross section take place. Second, only in this case the simple relation between the ionization cross-section and the recombination rate (14) exists. In this case

$$\lambda_{\rm SRR}(\omega) = 8\pi^2 n_e I f_{||}(v_0) f_\perp(0)\gamma^{-1}\omega^{-1}\sigma_{\rm ion}(\omega), \tag{18}$$

Where the longitudinal and transverse temperatures are expressed in atomic energy units. In the limit case of small cyclotron frequency $\gamma \ll T$ we get

$$\lambda_{\rm SRR}(\omega) = 2^{5/2}\pi^{3/2} n_e T^{-3/2} I\omega^{-1}\sigma_{\rm ion}(\omega). \tag{19}$$

In the low-temperature limit and in strong magnetic fields ($\gamma \gg T$)

$$\lambda_{\rm SRR}(\omega) = 2^{5/2}\pi^{3/2} n_e T^{-1/2} e^{-E_{\|}/T} I\gamma^{-1}\omega^{-1}\sigma_{\rm ion}(\omega), \tag{20}$$

where $E_{\|} = E - \gamma/2$. The exponential dependence on $E_{||}$ means that for the sake of increasing the recombination rate one has to choose the resonances with $E_{\|} \ll T$.

In all previous considerations we assumed that the centers of mass of all atoms are fixed. However, this is not quite true in real experimental situation. Although the atomic center-of-mass velocities are much less than those of the positrons, one can not neglect the resulting Doppler shift, because the Doppler broadening is large compared with very small homogeneous width of the cross-section resonances. Therefore, the recombination rate should be averaged over all values of the projection of the center-of-mass velocity onto the direction of the laser beam.

$$< \lambda_{\mathrm{SRR}}(\omega) >= \int_{-\infty}^{\infty} f_p(V)\lambda_{\mathrm{SRR}}[(1 + V/c)\omega]dV. \tag{21}$$

Here $f_p(V)$ is the Maxwell velocity distribution for antiprotons, V being the velocity projection onto the laser beam direction,

$$f_p(V) = \sqrt{\frac{m_p}{2\pi T}} e^{-m_p V^2/2T}, \tag{22}$$

and m_p is the mass of antiproton.

4. RESULTS

Using the relation (18) and the values of the cross-section $\sigma_{\mathrm{ion}}(\omega)$ of ionization from the state $3s$, calculated in,[12] we estimate the rate of the laser-stimulated resonance recombination.

For calculations we choose the parameters, typical for positron-antiproton plasma in magnetic traps used for antihydrogen recombination, namely, the temperature of the plasma $T = 4$ K, the positron density $n_e = 1 \times 10^8$ cm^{-3}, the magnetic induction $B = 6.1$ Tl. We consider the recombination into the state $n = 3$, $l = 0$, $m = 0$, that may be stimulated by a titanium-sapphire laser. In[4] the intensity of laser radiation was estimated for which the ratio of the induced recombination rate to the spontaneous one λ_{RR} without the magnetic field at 4 K is equal to 1. In particular, for $n = 3$ this intensity is $I = 24$ W/cm^2. In our calculations we used this value of the laser intensity. Fig. 1 shows the dependence of the laser-stimulated recombination rate per one antiproton upon the initial energy of the positron $E = E_{nlm} + \omega$. In the same figure for comparison the horizontal dashed line dispays the rate of the spontaneous radiative recombination into the state $n = 3$, which at the intensity considered is equal to the rate of the laser-stimulated recombination without the magnetic field.

Obviously, there are narrow resonances for which the rate of recombination into the state with fixed $l = 0$, $m = 0$ is appreciably higher than the rate of recombination into all nine states with different l and m possible for $n = 3$. However, the account for the Doppler broadening due to the thermal motion of antiprotons, the situation changes drastically. Fig. 2 demonstrates the rate of the laser-stimulated recombination in the presence of the Doppler broadening due to the Maxwell distribution of the antiproton velocities. For convenience it is divided by the rate of spontaneous recombination.

It is seen that even the highest peaks of the rate o laser-stimulated recombination are lower than that of the spontaneous recombination by an order of magnitude. The comparison of Figs. 1 and 2 confirms the idea that the laser-stimulated recombination in a magnetic field may be used for obtaining monoenergetic atoms,[13] or, to be more precise, the atoms with the fixed projection of velocity onto the direction of the laser beam. As to the total outcome of antiatoms in low-lying states, the conclusion is that at moderate laser intensities the stimulated recombination can be hardly expected to provide better results than the spontaneous one (Fig. 2). However, basing on the facts that on average the magnetic field does not affect the recombination rate[5] and that the dependence of the recombination rate on the strength of the magnetic field is rather nontrivial, one can suppose that at some values of the magnetic field and for some final states there may be "trains" of closely spaced high peaks that, due to Doppler broadening, merge into a "super-peak", in which the stimulated recombination rate is substantially higher than that in the absence of the magnetic field. Checking this hypothesis will require the calculation of the dependence of the recombination rate upon both the laser frequency and the magnetic field strength. The solution of this time-consuming problem will be a subject of our further studies.

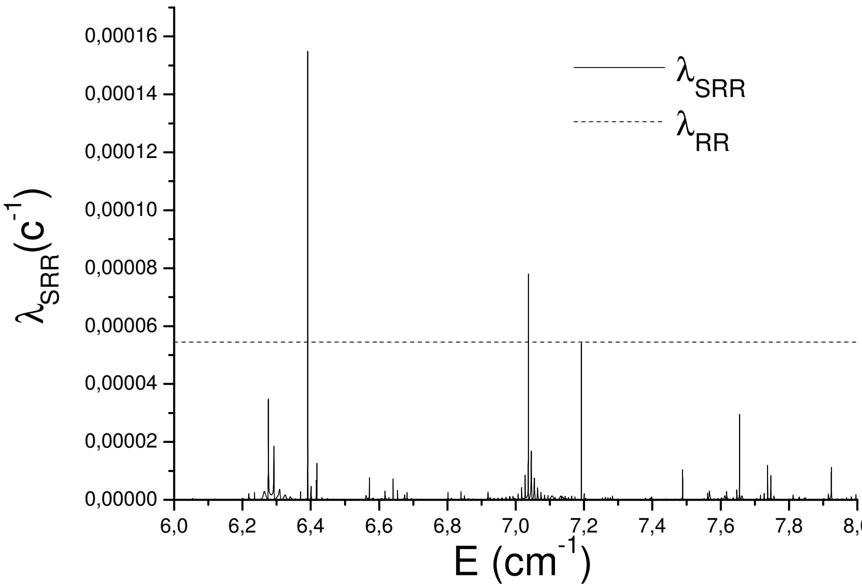

Figure 1. Laser-stimulated radiative recombination rate into the bound state $n = 3$, $l = 0$, $m = 0$ versus the energy of initially free positron

5. ACKNOWLEDGMENTS

This work was carried out within the framework of the Russian Analytical Departmental Objective Program "Development of the Scientific Potential of the Higher School)" (2006-2008), project RNP.2.1.1.4473 "Mesooptics", and supported by the grant CRDF BRHE REC-006 SR-006-X1/B75M06 Y3-P-06-08.

REFERENCES

1. M.Amoretti , C.Amsler , G.Bonomi, A.Bouchta, P.Bowe, C.Carraro, C.L.Cesar, M.Charlton, M.J.T.Collier, M.Doser, V.Filippini, K.S.Fine, A.Fontana, M.C.Fujiwara, R.Funakoshi, P.Genova, J.S.Hangst, R.S.Hayano, M.H.Holzscheiter, L.V.Jørgensen, V.Lagomarsino, R.Landua, D.Lindelöf, E.Lodi Rizzini, M.Macri, N.Madsen, G.Manuzio, M.Marchesotti, P.Montagna, H.Pruys, C.Regenfus, P.Riedler, J.Rochet, A.Rotondi, G.Rouleau, G.Testera, A.Variola, T.L.Watson, D.P.van der Werf, *Nature*, **419**, p. 456, 2002.
2. G.Gabrielse, N.S.Bowden, P.Oxley, A.Speck, C.H.Storry, J.N.Tan, M.Wessels, D.Grzonka, W.Oelert, G.Schepers, T.Sefzick, J.Walz, H.Pittner, T.W.Hänsch, E.A.Hessels, *Phys. Rev. Lett.* **89**, p. 213401, 2004.
3. C.H.Storry, A.Speck, D.Le Sage, N.Guise, G.Gabrielse, D.Grzonka, W.Oelert, G.Schepers, T.Sefzick, H.Pittner, M.Herrmann, J.Walz, T.W.Hänsch, D.Comeau, E.A.Hessels, *Phys. Rev. Lett.*, **93**, p. 263401, 2004.
4. M.V.Ryabinina, L..Melnikov, *Nuclear Instruments and Methods in Physical Research B*, **214**, p. 35, 2004.
5. L.I. Menshikov, R. Landua, *Rus. Usp. Fiz. Nauk*, **173**, p. 233, 2003 (Russian); *Phys. Uspekhi*, **46**, p. 233, 2003.
6. Iu Chun-ho, G.R.Welch, M.M.Kash, D.Kleppner, D.Delande, J.C.Gay, *Phys. Rev. Lett.*, **66**, p. 145, 1991.
7. V.V.Serov, V.P.Kadjaeva, V.L.Derbov, *Proc. SPIE*, **5773**, p. 195, 2004.
8. V.V.Serov, V.P.Kadjaeva, V.L.Derbov, S.I.Vinitsky, *Izvestiya Saratovskogo Universiteta, ser. Fiz.*, **5**, p. 84-91, 2005 (Russian).

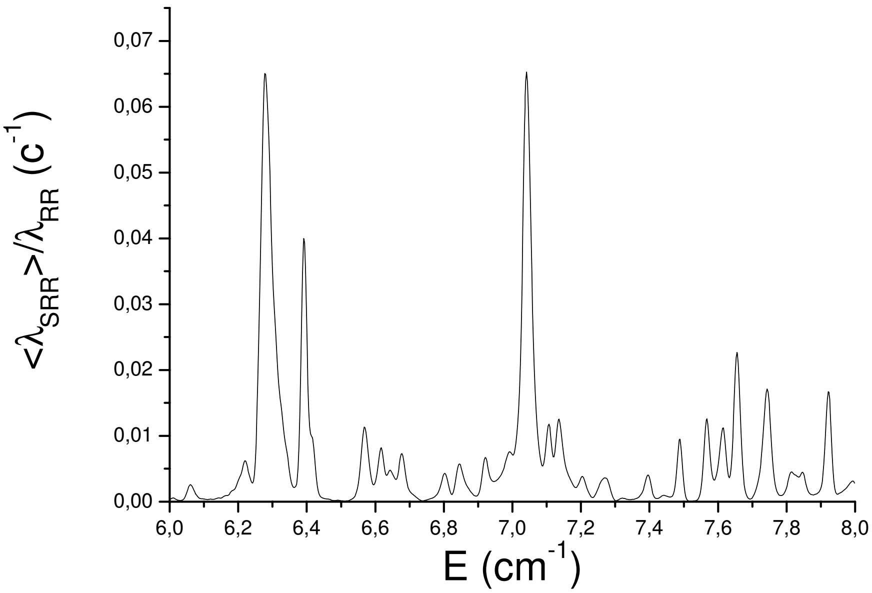

Figure 2. The ratio of laser-stimulated/spontaneous recombination with Dopper broadening due to thermal motion of antiprotons taken into account versus the energy of initially free positron

9. D.Delande, A.Bommier , J.C.Gay, *Phys. Rev. Lett.*, **66**, p. 141, 1991.
10. L.D.Landau and E.M.Lifshitz. *Quantum Mechanics. Non-Relativistic Theory* , Butterworth-Heinemann, 1981.
11. Z.Zarcone, M.R.C.McDowell, F.H.M.Faisal, *J. Phys. B: At. Mol. Phys.*, **16**, p. 4005-4014, 1983.
12. O.Chuluunbaatar, A.A.Gusev, V.P.Gerdt, S.I.Vinitsky, A.G.Abrashkevich, M.S.Kaschiev, and V.V.Serov, "POTHMF: A program for computing potential curves and matrix elements of the coupled adiabatic radial equations for a Hydrogen-like atom", submitted to *Computer Physics Communication*, 2007.
13. V.V.Serov, V.L.Derbov, S.I.Vinitsky, "Laser-stimulated radiative recombination of antihydrogen in magnetic field via a quasistationary state". *Optika i Spektroskopiya*, **102**, No. 4, 2007 (Russian, to be published)