Progress in the description of νA cross sections at high energy

Sergey Kulagin (INR, Moscow)

Talk at the workshop *Neutrino physics at accelerators*, Dubna, Jan 24, 2007

Outline

- Major mechanisms of lepton-nuclear scattering at high energy ($E_{\text{lepton}} \gg M_{\text{nucleon}}$).
- Description of charged-lepton nuclear DIS data and development of a quantitative model of nuclear structure functions.
- Description of νA differential cross sections.

Scales

At high energy and momentum transfer $q_0 \gg M$ and $|\boldsymbol{q}| \gg M$ naive estimates suggest that the scattering process proceeds at a small time $t \sim 1/q_0$ and small distance $z \sim 1/q_z$. This in turn suggests that nuclear DIS should be (to a very good approx.) incoherent scattering off bound nucleons.

However, this is not true!

Typical regions in configuration space which contribute to DIS hadronic tensor:

- $t^2 - z^2 \sim Q^{-2}$ DIS proceeds near the light cone.
- $t \sim z \sim L = (Mx)^{-1}$ NOT small in hadronic scale (in the target rest frame)
  $\Rightarrow$ the reason for nuclear corrections to survive even at high Q^2.

In order to understand mechanisms of nuclear scattering L has to be compared with average distance between bound nucleons $d = (3/4\pi\rho)^{1/3} \sim 1.2\,\mathrm{Fm}$ (central region of heavy nuclei). One should distinguish two different regions:

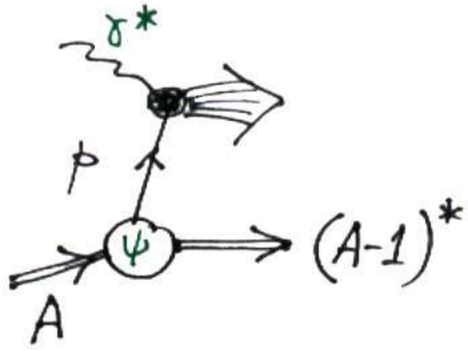
- $L < d$ $\Rightarrow$ Nuclear DIS $\approx$ incoherent sum of contributions from bound nucleons.
- $L \gg d$ $\Rightarrow$ Coherent effects of interactions with a few nucleons are important.

Bound nucleon contribution to nuclear DIS

In incoherent scattering (impulse) approximation the nuclear structure function (SF) can be written as convolution of nuclear spectral function and bound nucleon SFs:

$$F_2^A(x,Q^2) = \int \mathrm{d}^4k\, \mathcal{P}_A(k) \left(1+\frac{k_z}{M}\right) F_2^N(x',Q^2,k^2),$$

$$x = \frac{Q^2}{2p\cdot q}, \quad x' = \frac{Q^2}{2p\cdot q} = \frac{x}{1+(\varepsilon+k_z)/M}$$



Major nuclear effects are due to smering with $\mathcal{P}_A(k)$ which describes probability to find a bound nucleon with momentum $\boldsymbol{k}$ and energy $k_0 = M + \varepsilon$ (Fermi motion and nuclear binding effect).

$$\mathcal{P}_A(k) = \sum_n |\psi_n(\boldsymbol{k})|^2 \delta(\varepsilon + E_n(A-1) - E_0(A)).$$

Nucleon off-shell effect

Bound nucleons are off-mass-shell $p^2 = (M + \varepsilon)^2 - \boldsymbol{p}^2 < M^2$ and a correction associated with analytical continuation of the nucleon structure functions to off-shell region should be addressed. The virtuality parameter $v = (p^2 - M^2)/M^2$ (average virtuality $\langle v \rangle = 2 \langle V \rangle /M \sim -0.15$ for iron).

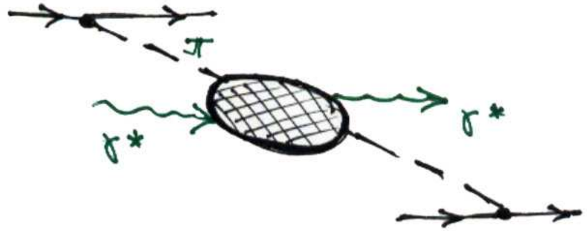
$$F_2^N(x, p^2) = F_2^N(x)\big(1 + v\,\delta f(x)\big)$$

Below we discuss phenomenological determination of $\delta f(x)$ from data on the ratios of nuclear structure functions (S.K. & R.Petti).

Nuclear pion effect

Leptons can scatter on nuclear meson field which mediate interaction between bound nucleons. This process generate a correction to nuclear sea quark distribution

$$\delta q^{\pi/A}(x,Q^2) = \int_x \frac{\mathrm{d}y}{y} f_{\pi/A}(y) q^\pi(\frac{x}{y},Q^2)$$



• Contribution from nuclear pions (mesons) is important to balance nuclear light-cone momentum $\langle y\rangle_\pi + \langle y\rangle_N = 1$.

• The nuclear pion distribution function is localized in a region of $y < p_F/M \sim 0.3$. For this reason the pion correction to nuclear (anti)quark distributions is localized at $x < 0.3$.

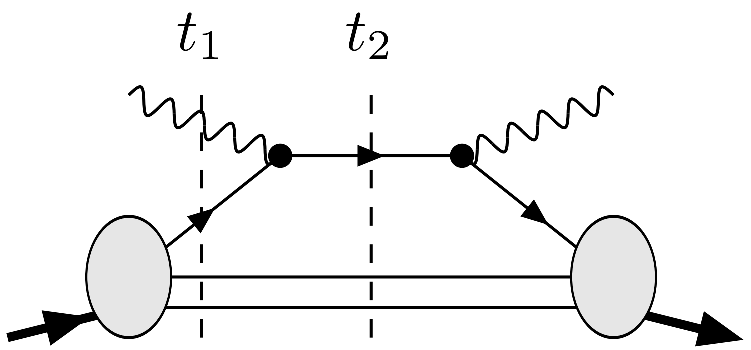
• The magnitude of the correction is driven by average number of "pions" $n_\pi = \int \mathrm{d}y\, f_{\pi/A}(y)$. By order of magnitude $n_\pi/A \sim 0.1$ for a heavy nucleus like ^{56}Fe.

• Nuclear pion correction effectively leads to enhancement of nuclear sea quark distribution and does not affect the valence quark distribution (for isoscalar nuclear target).

Coherent nuclear corrections

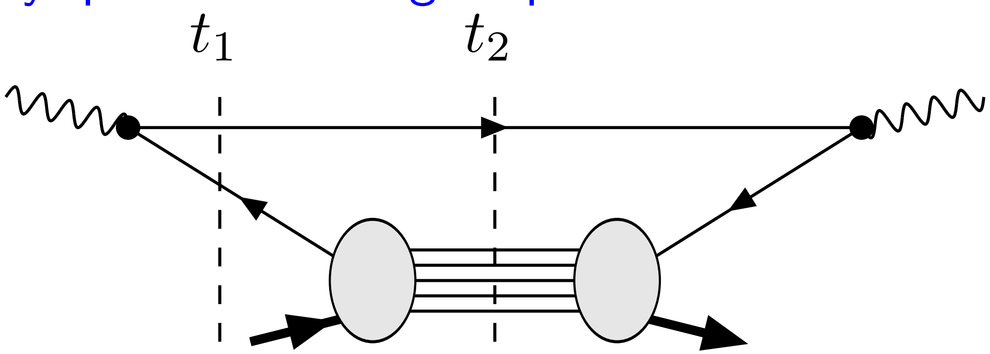
Two different mechanisms of DIS:

(I) QE scattering off bound quark. This process dominates at intermediate and large values of x and the structure functions are determined by quark wave functions.



Nuclear effects arise because of averaging with nucleon distributions in a nucleus.

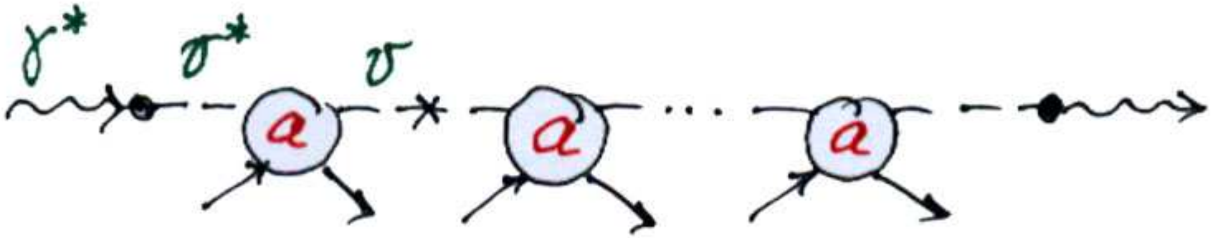
(II) Conversion $\gamma^* \to q\bar{q}$. Then $q\bar{q}$ state propagates in a target. This process dominates at small x since life time of $q\bar{q}$ state grows as $1/(Mx)$. The structure functions are determined by quark scattering amplitudes.



Nuclear effects arise due to interaction of intermediate states during propagation in matter. Relative correction to a single scattering term $\delta\mathcal{R} = \delta F_T^A/F_T^N \Rightarrow \delta\sigma_T^{\mathrm{mult.sc.}}/\sigma_T$

Nuclear shadowing

Multiple scattering Glauber series



The series can be summed up in a compact form in optical approx. (suitable for large A)

$$\delta\mathcal{R} = \mathrm{Im}\left[i a^2 \mathcal{C}_2^A(a)\right] / \,\mathrm{Im}\, a,$$

$$\mathcal{C}_2^A(a) = \int_{z_1<z_2} \mathrm{d}^2\boldsymbol{b}\, \mathrm{d}z_1 \mathrm{d}z_2\, \rho_A(\boldsymbol{b}, z_1)\rho_A(\boldsymbol{b}, z_2) \exp\left[i \int_{z_1}^{z_2} \mathrm{d}z' \left(a\, \rho_A(\boldsymbol{b}, z') - k_L\right)\right].$$

$a = \sigma(i+\alpha)/2$ is (effective) scattering amplitude in forward direction ($\alpha = \mathrm{Re}\, a/\,\mathrm{Im}\, a$), $k_L = Mx(1 + m_v^2/Q^2)$ is longitudinal momentum transfer in the process $v^* \to v$ which accounts for finite life time of intermediate $q\bar{q}$ state.

Phenomenology of nuclear DIS

Motivation: the development of a quantitative model providing predictions of nuclear cross sections (structure functions) and corresponding uncertainties to be used in the analyses of present and future lepton scattering data from nuclear targets.
Model: S.K. & R.Petti, hep-ph/0412425

$$F_i^A = F_i^{p/A} + F_i^{n/A} + F_i^{\pi/A} + \delta F_i^{\rm coh},$$

Hadronic/nuclear input to the model:
- Proton and neutron SFs computed in NNLO pQCD + TMC + HT using phenomenological PDFs and HTs from fits to DIS data (Alekhin).
- Realistic two-component nuclear spectral function (mean-field + correlated part) is used to calculate SF in impulse approx.
- Mesonic correction is calculated using pion PDFs extracted from fits to Drell–Yan data and the nuclear pion distribution .
- Coherent nuclear corrections are calculated using multiple scattering theory and nuclear number densities $\rho_A(\boldsymbol{r})$ from elastic electron scattering data.

Phenomenological parameters

Off-shell corrections to the nucleon structure functions and effective scattering amplitude of hadronic component of virtual photon off the nucleon are treated phenomenologically.

Off-shell correction function $\delta f_2(x) = C_N(x - x_1)(x - x_0)(h - x)$.

From preliminary studies we observe that h is fully correlated with x_0, i.e. $h = 1 + x_0$. The nuclear valence number normalization helps to fix $x_1 = 0.05$. C_N and x_0 are fit parameters.

Effective amplitude

$$\bar{a}_T = \bar{\sigma}_T(i + \alpha)/2$$

$$\bar{\sigma}_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

Parameters $\sigma_0 = 27\,\mathrm{mb}$ and $\alpha = -0.2$ were fixed in order to match the VMD model at low Q. Parameter $\sigma_1 = 0$ (preferred by preliminary fits and fixed in final fit). Q_0^2 is adjustable parameter controlling transition between low and high Q regions.

Analysis

We use the data from electron and muon DIS in the form of ratios $\mathcal{R}_2(A/B) = F_2^A/F_2^B$ for a variaty of targets. The data are available for A/D and $A/^{12}$C ratios.

We perform a fit and minimize $\chi^2 = \sum_{\text{data}}(\mathcal{R}_2^{exp} - \mathcal{R}_2^{th})^2/\sigma^2(\mathcal{R}_2^{exp})$ with σ the experimental uncertainty. In the fit we use data with $Q^2 > 1\,\mathrm{GeV}^2$ (overall about 560 points). Then we validate the predictions for $Q^2 < 1\,\mathrm{GeV}^2$.

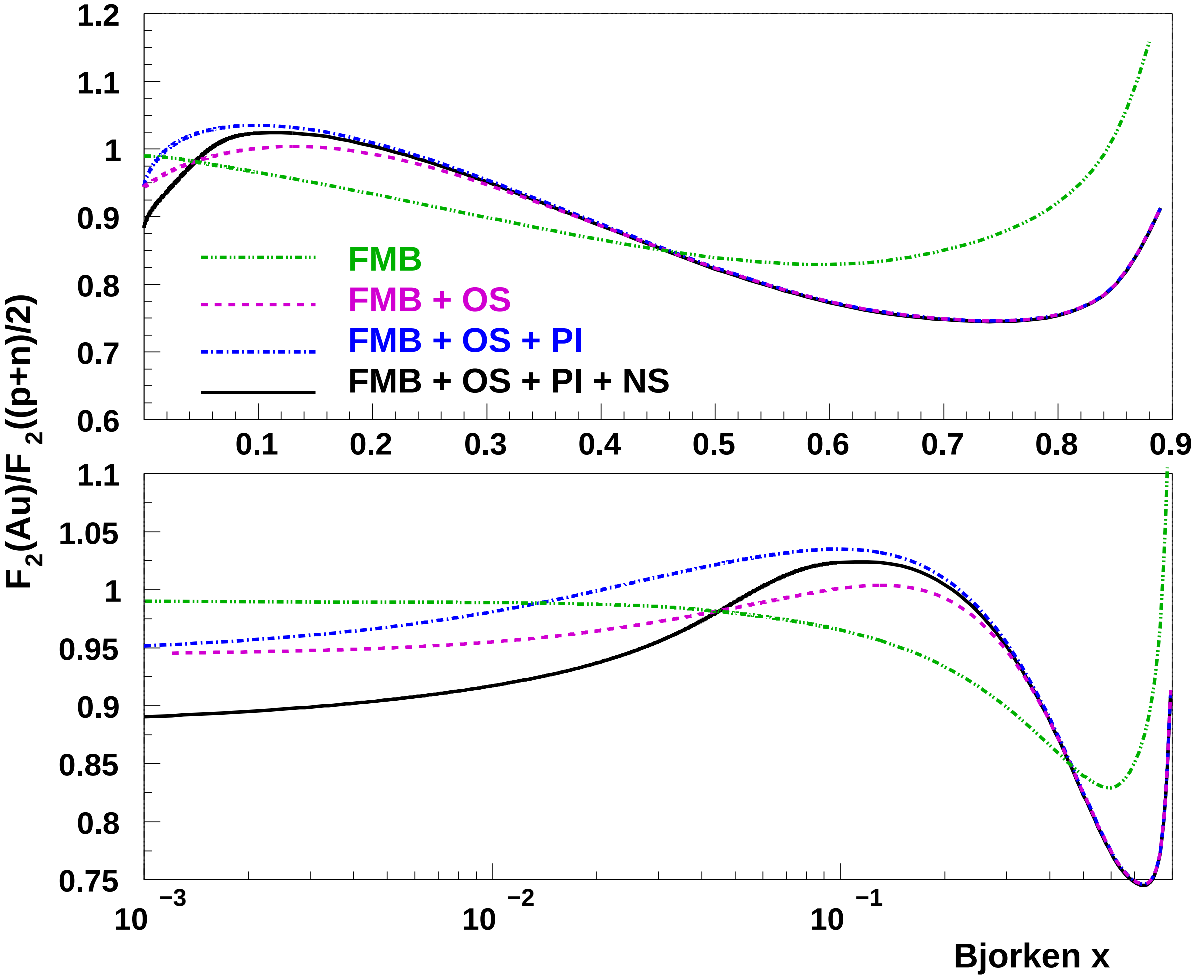
Results

- Very good agreement with data (both for x and Q^2 dependence) for entire kinematical region

$$\chi^2/\text{d.o.f} = 459/556$$

- Phenomenological parameters from global fit (statistical + syst./theoretical uncertainties)

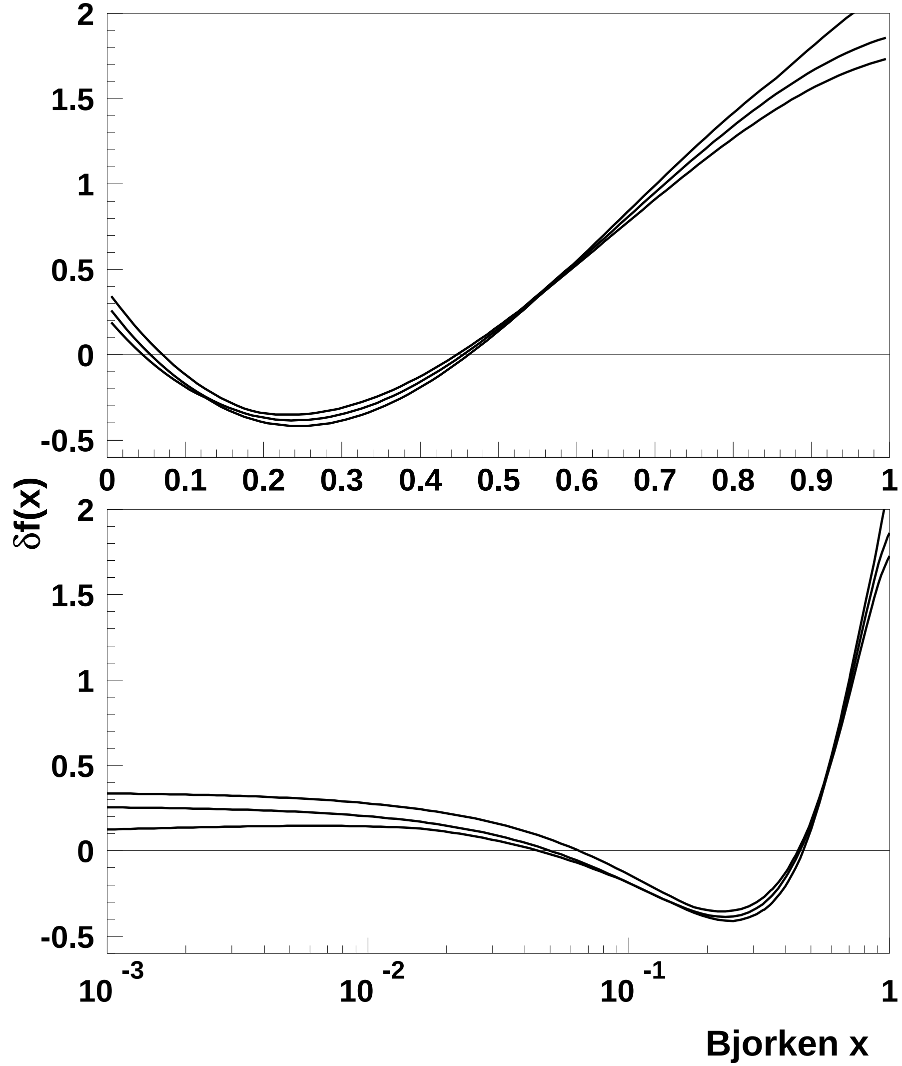
$$C_N = 8.1 \pm 0.3 \pm 0.5$$
$$x_0 = 0.448 \pm 0.005 \pm 0.007$$
$$Q_0^2 = 1.43 \pm 0.06 \pm 0.2\ \text{GeV}^2$$



The EMC ratio for gold and the isoscalar nucleon calculated at $Q^2 = 10\ \text{GeV}^2$. The labels on the curves mark the effects included in turn: Fermi motion and nuclear binding (FMB), off-shell correction (OS), nuclear pion excess (PI) and corrections from coherent nuclear processes (NS).

Off-shell function

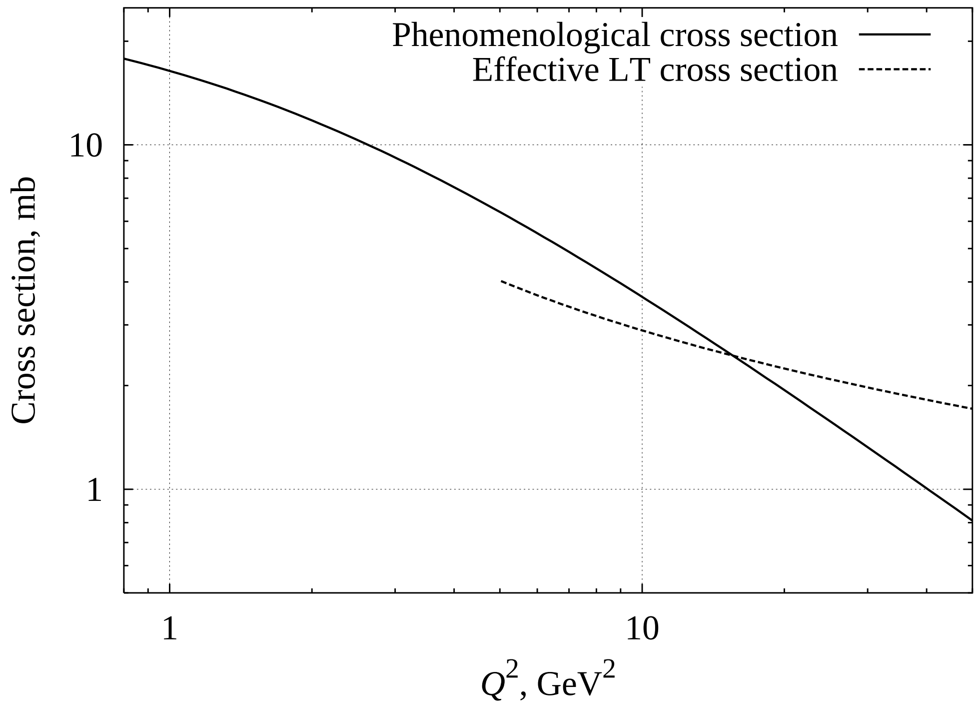
- The function $\delta f(x)$ provides a measure of modification of quark distributions in bound nucleon.
- The off-shell effect results in the enhancement of the structure function for $x_1 < x < x_0$ and depletion for $x < x_1$ and $x > x_0$.
- The phenomenological function $\delta f(x)$ suggests the increase in the radius of the bound nucleon valence region (in Fe by $\sim 10\%$).



Effective cross section

- The monopole form $\bar{\sigma} = \sigma_0/(1 + Q^2/Q_0^2)$ provides a good fit to existing data on nuclear shadowing for $Q^2 < 20\,\mathrm{GeV}^2$ (ratio $F_2(\mathrm{Sn})/F_2(\mathrm{C})$ from NMC).

- This does not necessarily mean that $\bar{\sigma}$ vanish as $1/Q^2$ at high Q. Effective cross section at high Q can be calculated from the normalization condition of the valence quark distribution $\delta N_{\mathrm{val}}^{\mathrm{off-shell}} + \delta N_{\mathrm{val}}^{\mathrm{shad}} = 0$.



Application to neutrino scattering

Neutrino scattering is affected by both vector (V) and axial-vector (A) currents.

$$
\begin{aligned}
VV, AA &\quad\Longrightarrow\quad F_{1,2} \quad (\text{or } F_L,\ F_T) \\
VA &\quad\Longrightarrow\quad F_3 \text{ (not present for CL scattering)}
\end{aligned}
$$

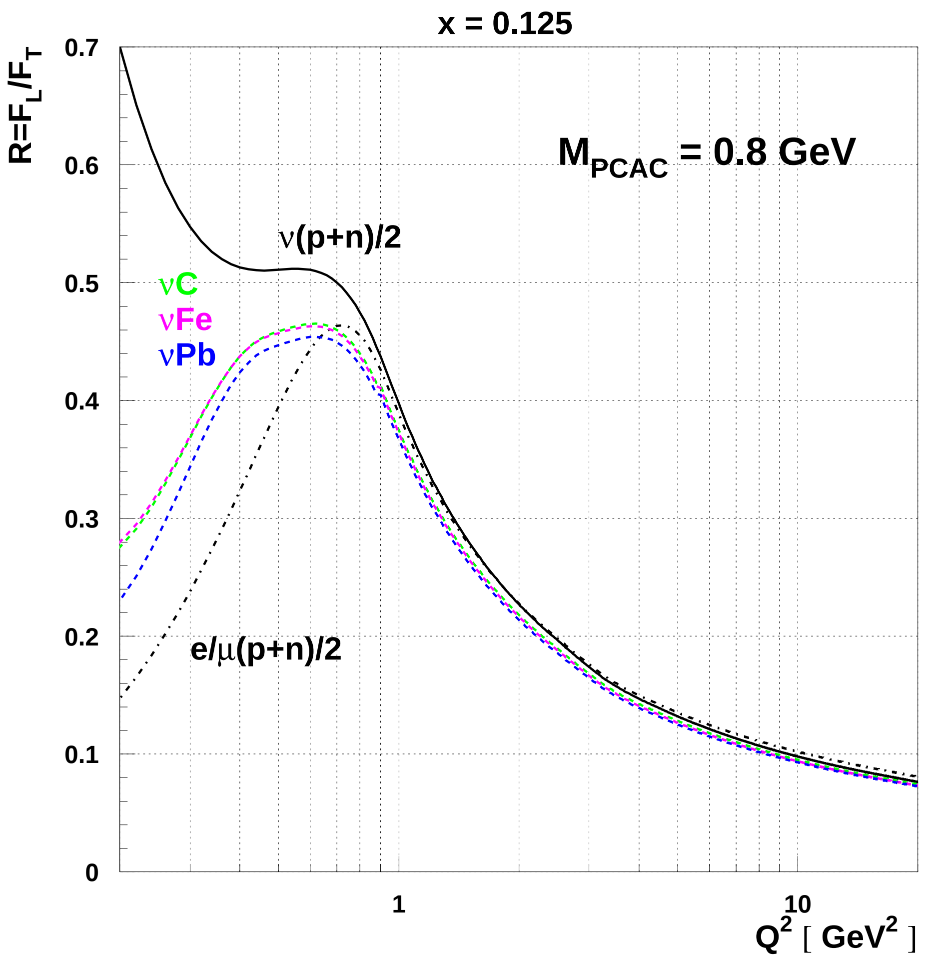
Axial current is not conserved and dominates SF and cross sections at low Q^2 (Adler 1966).

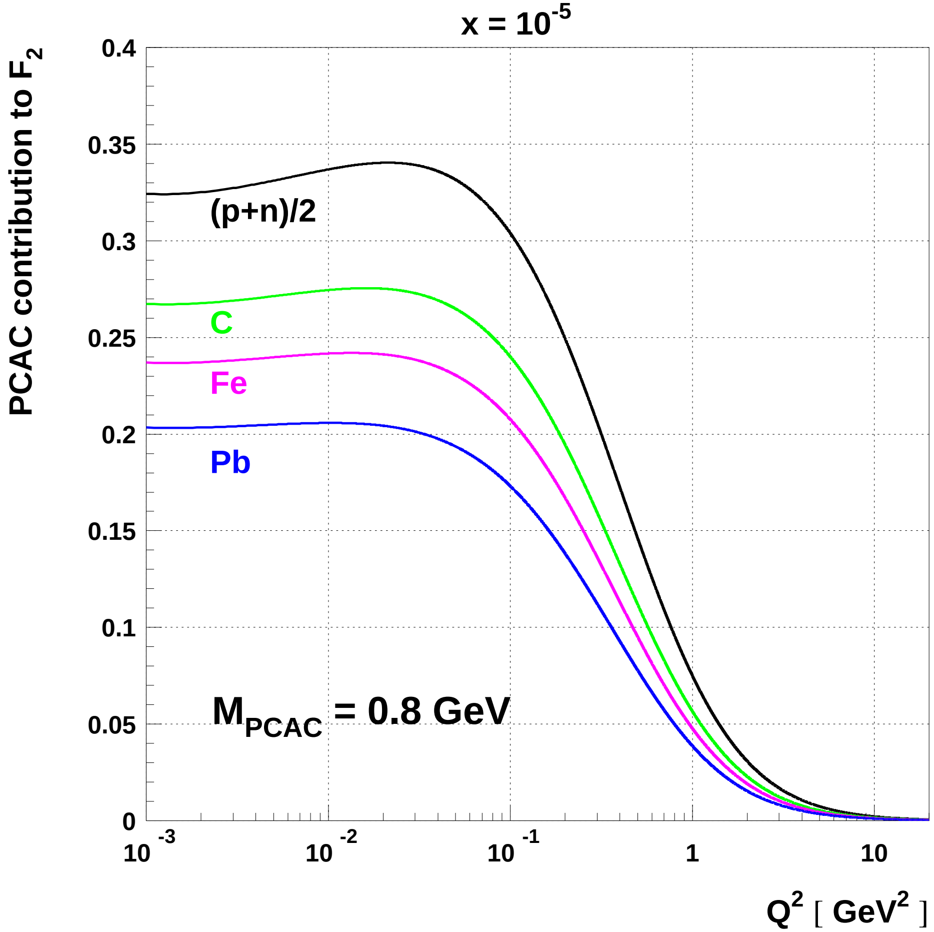
$$
\text{PCAC:} \quad \partial A = f_\pi m_\pi^2 \varphi \quad \Longrightarrow \quad F_L = \frac{f_\pi^2 \sigma_\pi}{\pi} + \mathcal{O}(Q^2)
$$

Transition scale between low and high Q^2 is NOT m_π^2 but rather $m_a^2 \sim 1\,\mathrm{GeV}^2$ (direct contribution from the pion current $\partial_\mu \varphi$ cancels out). Model (S.K. and R. Petti):

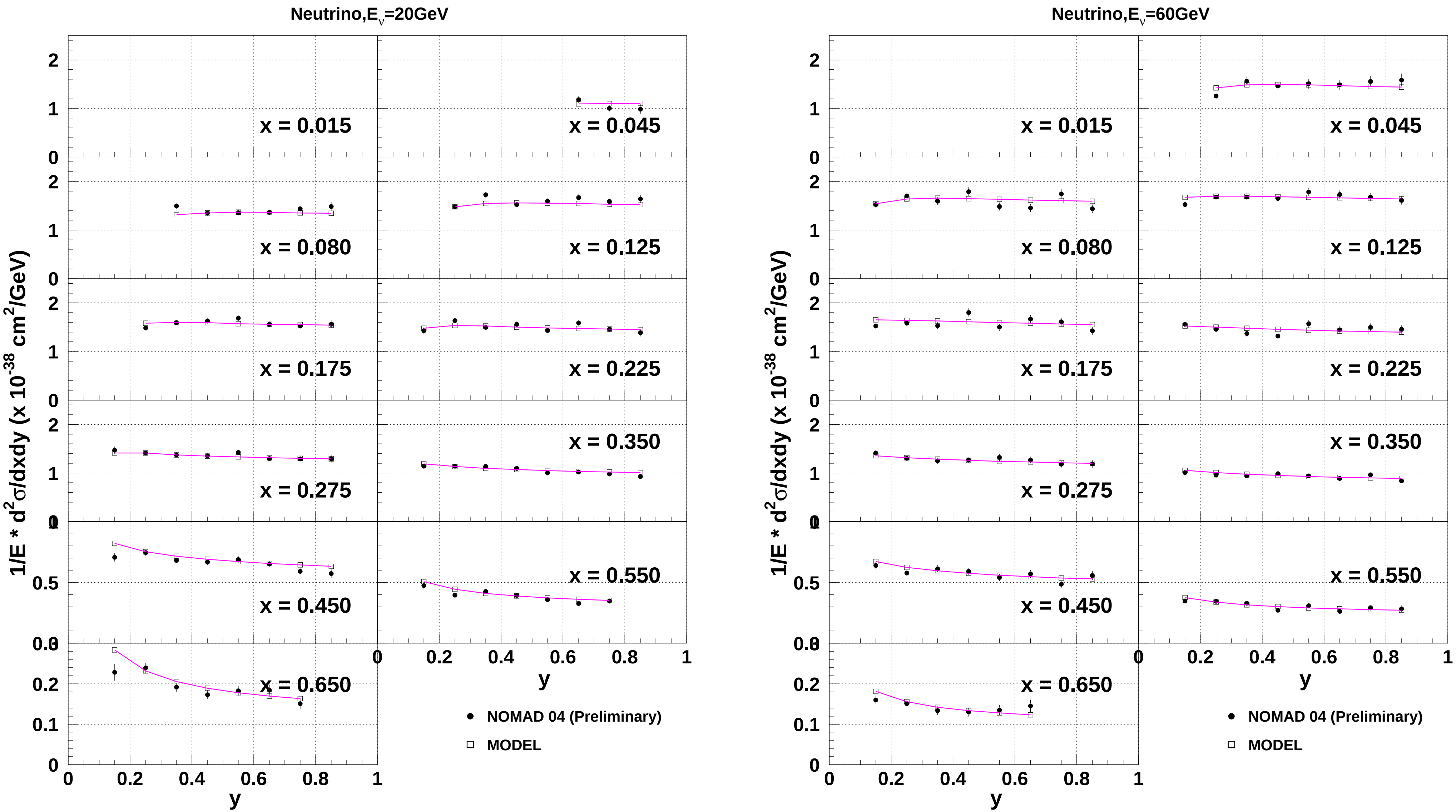
$$
F_L = \frac{f_\pi^2 \sigma_\pi}{\pi}(1 + Q^2/m_a^2)^{-2} + \widetilde{F}_L
$$

$$
\widetilde{F}_L = \begin{cases} F_L^{\mathrm{QCD}} & , \ Q^2 > 1\,\mathrm{GeV}^2, \\ \propto Q^4 & , \ Q^2 \to 0 \end{cases}
$$


x = 0.125
R=F_L/F_T
M_PCAC = 0.8 GeV
ν(p+n)/2
νC
νFe
νPb
e/μ(p+n)/2
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
10
Q² [GeV²]

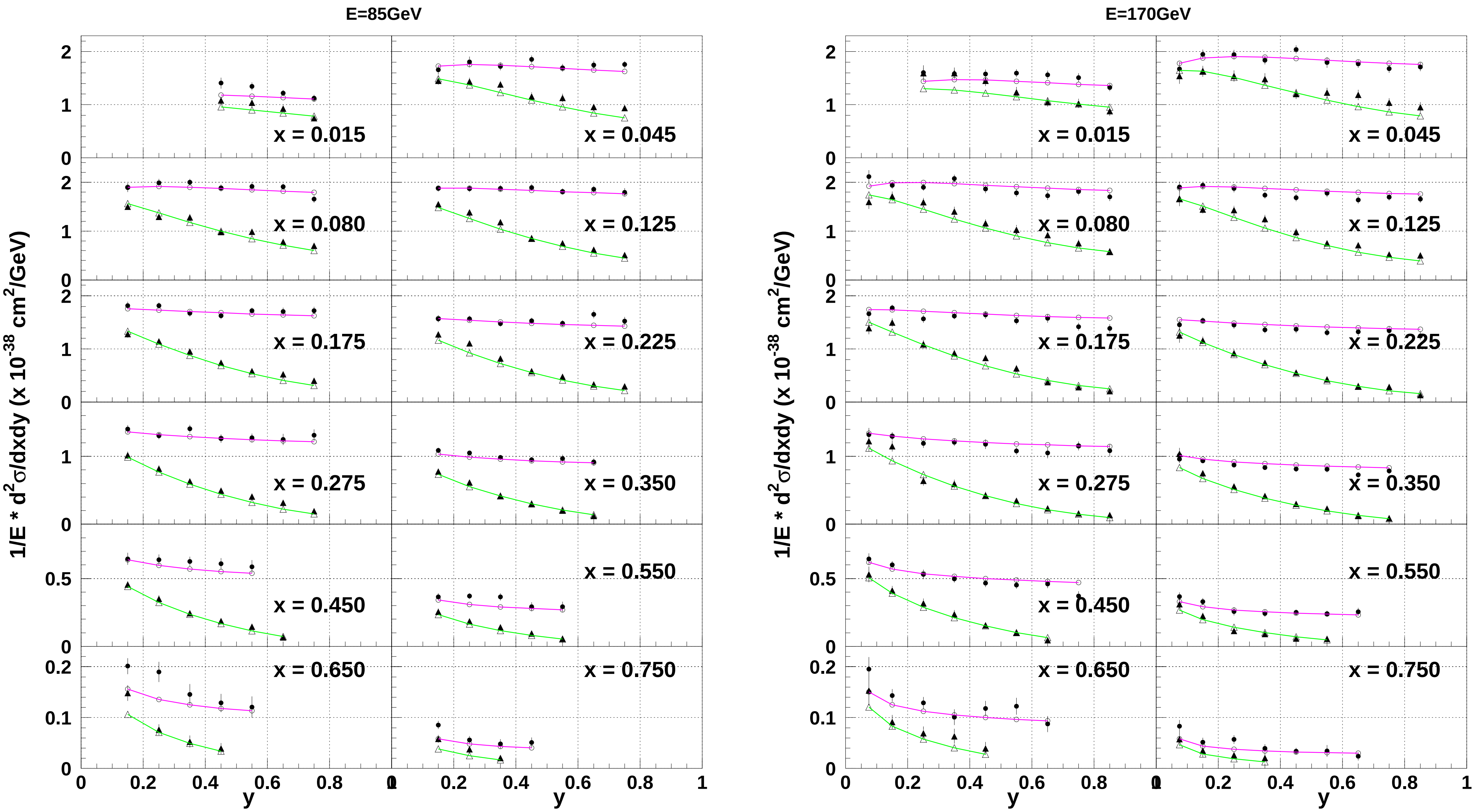

x = 10^-5
PCAC contribution to F_2
(p+n)/2
C
Fe
Pb
M_PCAC = 0.8 GeV
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
10^-3
10^-2
10^-1
1
10
Q² [GeV²]

Comparison with NOMAD νC cross sections



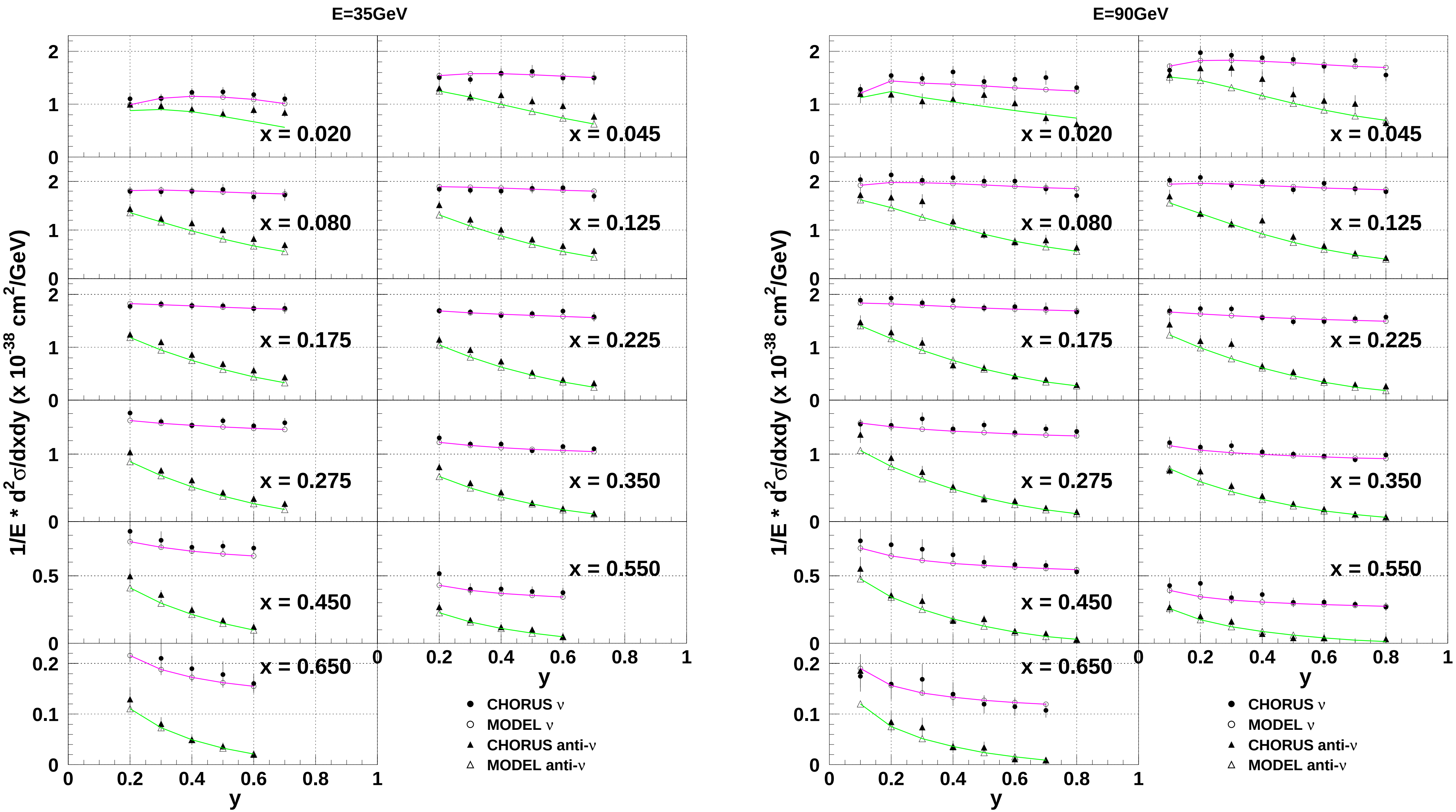
Data: R. Petti (Presented at NuInt05).

Comparison with NuTeV $\nu(\overline{\nu})$Fe cross sections



Data: M. Tzanov (Presented at DIS05).

Comparison with CHORUS $\nu(\overline{\nu})$Pb cross sections



Data: J. Panman (Private communication March 2005).

Summary

A detailed quantitative study of nuclear charged-lepton DIS data has been performed in a wide kinematical region of x and Q^2.

A model was developed which includes the QCD treatment of the nucleon structure functions and addresses a number of nuclear effects including nuclear shadowing, Fermi motion and nuclear binding, nuclear pions and off-shell corrections to bound nucleon structure functions.

The model was applied to calculate the charged-current neutrino-nucleus differential DIS cross sections and shows a very good agreement with available data.

Other applications

(not discussed today, work in progress in collaboration with Sergey Alekhin and Roberto Petti)

- Studies of the Gross–Llewellyn-Smith and Adler sum rules for nuclei. Analysis of neutrino data of NOMAD Collaboration (^{12}C target) aiming to extract the weak mixing angle ($\sin^2\theta_W$) from NOMAD data by comparing NC/CC neutrino cross sections.

- Futher studies of the C parity and isospin dependence of nuclear effects (how different are nuclear corrections for u- and d-quark distributions, for quarks and antiquarks? Extraction of d/u at $x \to 1$).

- Extention of the approach to low Q^2 using the current conservation and low-energy theorems for vector and axial-vector currents. Tests of duality for CL and neutrino scattering off nuclei.

- Global QCD fit to charged-lepton and neutrino DIS, DY pair production cross sections including data on nuclear targets.