Calculation of quasielastic neutrino-nuclear cross sections for neutrino energy $0.2 - 2$ GeV

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For more detail about presented approach see A.B. & S.K., Phys. Rev. C76 (2007) 045502

Motivations

- QE scattering from bound nucleons dominates neutrino-nuclear cross sections in the energy range relevant to neutrino oscillation experiments ($E_\nu = 0.2 - 2.5$ GeV).

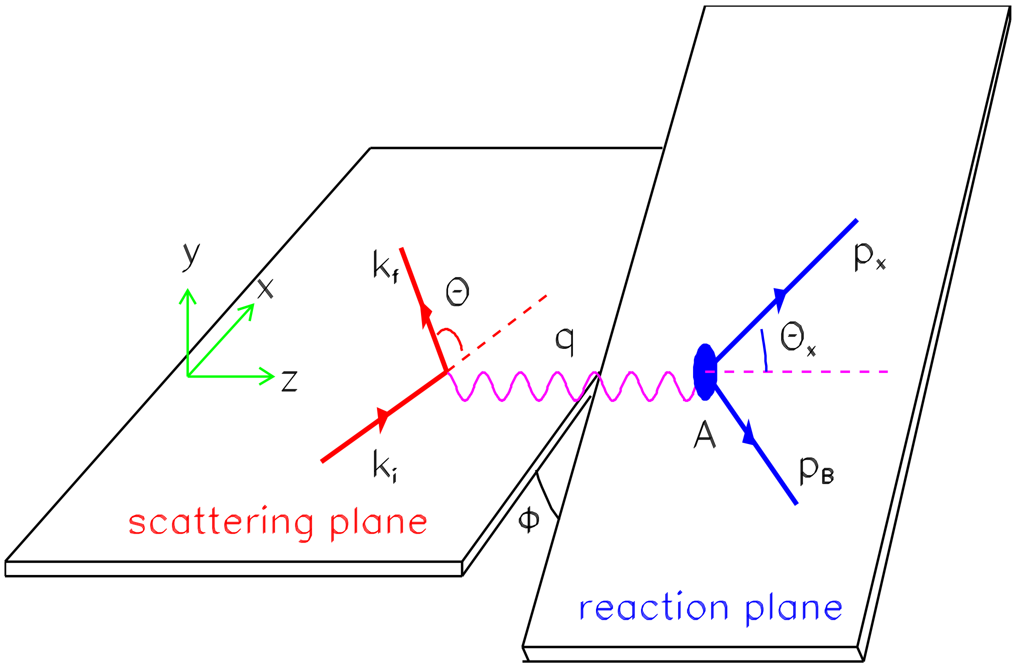
- Important source of systematic uncertainties is related to the treatment of nuclear model dependence of QE cross sections.

- We study QE neutrino charged-current interactions within a realistic model and nuclear model dependence of QE neutrino cross sections:

  - o Relativistic Fermi Gas Model (RFGM)
  - o Plane-Wave Impulse Approximation (PWIA) – no final state interaction (FSI)
  - o Effect of NN correlation in nuclear gr.st.
  - o FSI effects within Relativistic Distorted-Wave Impulse Approximation (RDWIA)

Formalism of QE scattering

$$l(k_i) + A(p_A) \rightarrow l'(k_f) + N(p_x) + B(p_B)$$



Particles' energies and momenta: $k_{i,f} = (\varepsilon_{i,f}, \boldsymbol{k}_{i,f})$, $p_{A,B} = (\varepsilon_{A,B}, \boldsymbol{p}_{A,B})$, $p_x = (\varepsilon_x, \boldsymbol{p}_x)$.
Momentum transfer $q = k_i - k_f = (\omega, \boldsymbol{q})$.

Cross sections

In the lab frame the differential cross section for exclusive electron and (anti)neutrino CC scattering can be written as

$$\frac{d^6\sigma^{el}}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \frac{|\boldsymbol{p}_x|\varepsilon_x}{(2\pi)^3}\frac{\varepsilon_f}{\varepsilon_i}\frac{\alpha^2}{Q^4}L^{(el)}_{\mu\nu}\mathcal{W}^{\mu\nu(el)}$$

$$\frac{d^6\sigma^{cc}}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \frac{|\boldsymbol{p}_x|\varepsilon_x}{(2\pi)^5}\frac{|\boldsymbol{k}_f|}{\varepsilon_i}\frac{G^2\cos^2\theta_C}{2}L^{(cc)}_{\mu\nu}\mathcal{W}^{\mu\nu(cc)},$$

where Ω_f is the solid angle for the lepton momentum, Ω_x is the solid angle for the outgoing nucleon momentum, $\alpha \approx 1/137$ is the fine-structure constant, $G \approx 1.16639 \times 10^{-11}$ MeV^{-2} is the Fermi weak coupling constant, θ_C is the Cabbibo angle ($\cos\theta_C = 0.9749$).

Leptonic & hadronic tensors

The lepton tensor can be written as the sum of the symmetric $L_S^{\mu\nu}$ and antisymmetric $L_A^{\mu\nu}$ tensors

$$L_S^{\mu\nu} = 2\left(k_i^\mu k_f^\nu + k_i^\nu k_f^\mu - g^{\mu\nu} k_i k_f\right)$$

$$L_A^{\mu\nu} = \pm 2i\epsilon^{\mu\nu\alpha\beta}(k_i)_\alpha (k_f)_\beta,$$

where $+(-)$ for positive(negative) lepton helicity, $\epsilon^{\mu\nu\alpha\beta}$, the antisymmetric tensor with $\epsilon^{0123} = -\epsilon_{0123} = 1$.

Unpolarized electron scattering only involves the symmetric tensor.
(Anti)neutrino scattering involves both the symmetric and the antisymmetric parts.

Hadronic tensor

$$\mathcal{W}_{\mu\nu}(p_A, q, p_x) = \sum_\alpha \langle B_\alpha, p_x | J_\mu(0) | A\rangle \langle A | J_\nu^\dagger(0) | B_\alpha, p_x\rangle\, \delta(\varepsilon_A + \omega - \varepsilon_x - \varepsilon_{B_\alpha}),$$

where the sum is taken over undetected states of the recoil nucleus B.

Calculation of hadronic tensor

Nuclear current

$$\langle p, B|J_\mu(0)|A\rangle = \int d^3\boldsymbol{r}\ \exp(i\boldsymbol{q}\cdot\boldsymbol{r})\ \overline{\Psi}^{(-)}_{pB}(\boldsymbol{r})\Gamma_\mu\Phi_{AB}(\boldsymbol{r})$$

$\Psi^{(-)}_{pB}(\boldsymbol{r})$ – outgoing n ucleon w.f. $\Phi_{AB}(\boldsymbol{r})$ – bound state w.f. (overlap matrix element)

Vertex function and form factors:

$$\Gamma^\mu = F_V^{(el)}(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_M^{(el)}(Q^2),$$

$\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, $F_V^{(el)}$ and $F_M^{(el)}$ are the Dirac and Pauli nucleon form factors. We use MMD model of f.f. (Mergel et.al., 1996).

We employ the de Forest prescription for off-shell vertex

$$\tilde{\Gamma}^\mu = F_V^{(el)}(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{\tilde{q}_\nu}{2m}F_M^{(el)}(Q^2),$$

where $\tilde{q} = (\varepsilon_x - \tilde{E}, \boldsymbol{q})$ and the nucleon energy $\tilde{E} = \sqrt{m^2 + (\boldsymbol{p}_x - \boldsymbol{q})^2}$ is placed on shell. The Coulomb gauge is assumed for the single-nucleon current.

The nucleon charged current has the $V{-}A$ structure

$$\Gamma^{\mu(CC)} = \Gamma^\mu_V + \Gamma^\mu_A$$

$$\Gamma^\mu_V = F_V(Q^2)\gamma^\mu + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_M(Q^2),$$

$$\Gamma^\mu_A = F_A(Q^2)\gamma^\mu\gamma_5 + F_P(Q^2)q^\mu\gamma_5.$$

The weak vector form factors F_V and F_M are related to the corresponding electromagnetic ones for proton $F^{(el)}_{i,p}$ and neutron $F^{(el)}_{i,n}$ by the hypothesis of the conserved vector current (CVC)

$$F_i = F^{(el)}_{i,p} - F^{(el)}_{i,n}.$$

The axial F_A and psevdoscalar F_P form factors:

$$F_A(Q^2) = \frac{F_A(0)}{(1 + Q^2/m_A^2)^2}, \quad F_P(Q^2) = \frac{2mF_A(Q^2)}{m_\pi^2 + Q^2},$$

with $F_A(0) = 1.267$, m_π the pion mass, and $m_A \simeq 1.032$ GeV the axial mass.

Plane Wave Impulse Approximation (PWIA)

No FSI assumed: $\Psi^{(-)}_{\boldsymbol{p}_N}(\boldsymbol{r}) = \exp(i\boldsymbol{p}_N \cdot \boldsymbol{r})$
Cross sections have factorized form:

$$\frac{d\,\sigma}{d^3k_f d^3p_x} = K\sigma_N \mathcal{P}(E, \boldsymbol{p})$$

K the kinematic phase-space factor
σ_N the (off-shell) lepton-nucleon cross section.
$\boldsymbol{p} = \boldsymbol{p}_x - \boldsymbol{q}$ the missing momentum and $E = \varepsilon_x(\boldsymbol{p}_x) - \omega$ the missing energy.
The nuclear spectral function:

$$\mathcal{P}(E, \boldsymbol{p}) = \sum_\alpha \left|\langle B_\alpha | a(\boldsymbol{p}) | A\rangle\right|^2 \delta(M_A - M_{B_\alpha} - M + E)$$

describes the probability of removing a nucleon with momentum $\boldsymbol{p}$ and energy E from the nuclear target A and leaving the residual nucleus B in the state B_α.

Nucleon momentum distribution $N(\boldsymbol{p}) = \int dE\ \mathcal{P}(E, \boldsymbol{p})$.

Nuclear spectral function

Mean-field picture

Nucleus in a first approximation can be viewed as a system of protons and neutrons bound to a self-consistent potential (mean field model, MF). Nucleons occupy the MF energy levels according to Fermi statistics and thus distributed over momentum (Fermi motion) and energy states. MF nuclear spectral function

$$\mathcal{P}_{\rm MF}(E,\boldsymbol{p}) = \sum_{\lambda<\lambda_F} n_\lambda |\phi_\lambda(\boldsymbol{p})|^2 \delta(E+\varepsilon_\lambda)$$

where sum is taken over occupied levels with ϕ_λ the wave function and n_λ the occupation number of the level λ (λ_F the Fermi level). MF model is a reasonable approximation if nucleon separation energy and momenta are not high (in nuclear ground state scale, $E < 50\,\text{MeV}$ and $p < 300\,\text{MeV/c}$).

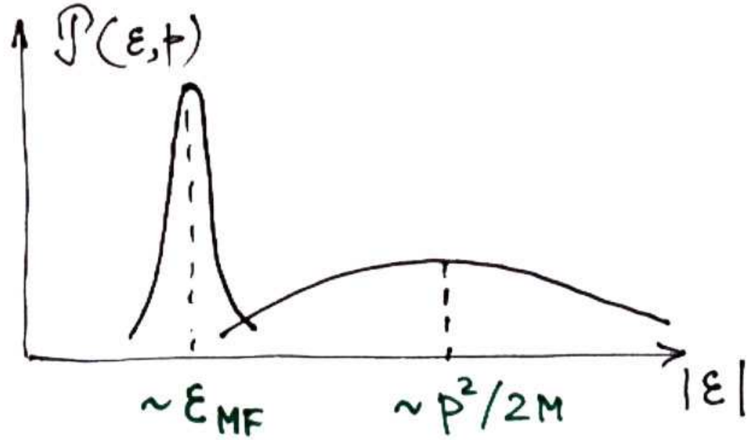
Nuclear spectral function

Nucleon short-range correlation effects

As the separation energy E becomes higher, the MF approximation becomes less accurate. High-energy and high-momentum component of nuclear spectrum can not be described in the MF model. These effects are driven by short-range NN correlations in nuclear ground state.

$$\mathcal{P}_{cor}(E, \boldsymbol{p}) \approx n_{\mathrm{rel}}(\boldsymbol{p}) \left\langle \delta \left(\frac{(\boldsymbol{p} + \boldsymbol{p}_{A-2})^2}{2M} + E_{A-2} - E_A - E \right) \right\rangle_{A-2}$$

The full spectral function can be approximated by a sum of the MF and correlation parts $\mathcal{P} = \mathcal{P}_{\mathrm{MF}} + \mathcal{P}_{\mathrm{cor}}$.



Fermi gas model

In the RFGM the nucleons are described as a system of quasi-free nucleons. This model takes into account the Fermi motion of bound nucleon, Pauli blocking factor and relativistic kinematics. The Fermi gas model provides a simplest form of the spectral function which is given by

$$\mathcal{P}_{FG}(E,|\boldsymbol{p}|) = \frac{3}{4\pi p_F^3}\theta(p_F - |\boldsymbol{p}|)\theta(|\boldsymbol{p}+\boldsymbol{q}| - p_F)\;\delta[(\boldsymbol{p}^2 + M^2)^{1/2} - \varepsilon - E],$$

where p_F is the Fermi momentum and ε is effective binding energy, introduced to account for nuclear binding. For oxygen we use $p_F = 250$ MeV/c and $\varepsilon = 27$ MeV.

The RFGM misses a number of important effects: nuclear shell structure, FSI effect, and the presence of NN-correlations.

Distorted Wave Impulse Approximation (RDWIA)

FSI of outgoing nucleon can be described in terms of a wave function in an optical potential . The distorted wave functions are evaluated using a Dirac equation with a scalar (S) and vector (V) components of an relativistic optical potential.

$$[\alpha \cdot \boldsymbol{p} + \beta(M + S)]\,\Psi = (E - V)\Psi,$$

We use the LEA program of J.Kelly for the numerical calculation of the distorted wave functions with the EDAD1 SV relativistic optical potential.

Reduced cross sections

Reduced cross section (measure the strength of nuclear effects)

$$\sigma_{red} = \frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_x} / K\sigma_{lN}, \tag{1}$$

where $d^5\sigma/d\varepsilon_f d\Omega_f d\Omega_x$ is differential exclusive cross section, ε_f, Ω_f are energy and solid angle of the scattered lepton, Ω_x is solid angle for ejectile nucleon momentum, K is kinematical phase space factors and σ_{lN} is elementary cross section for the lepton scattering from moving free nucleon.

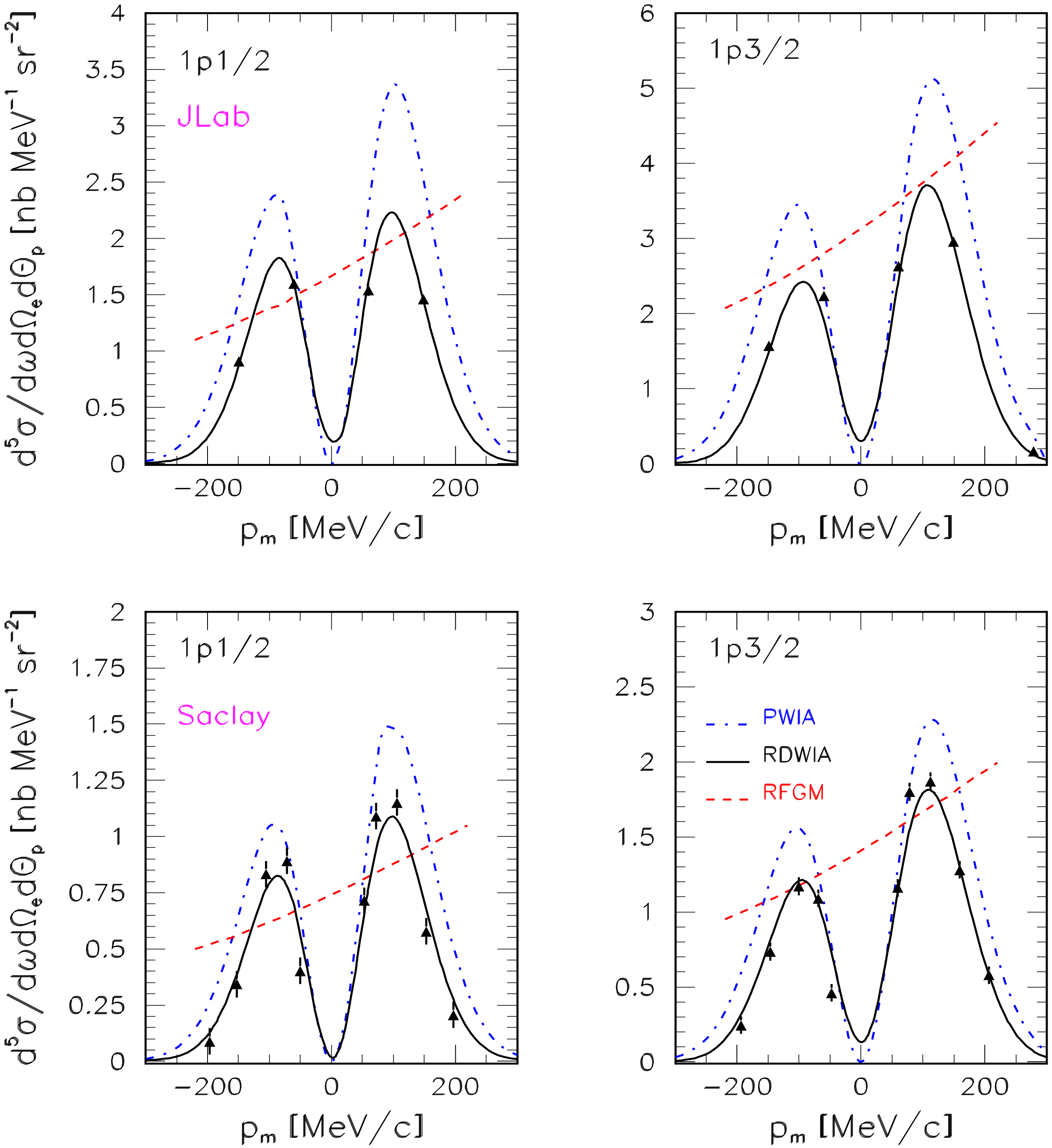
σ_{red} is nucleon momentum distribution in PWIA.

σ_{red} should be similar for electron and neutrino scattering (apart from small differences due to FSI effects for electron and neutrino induced reactions).

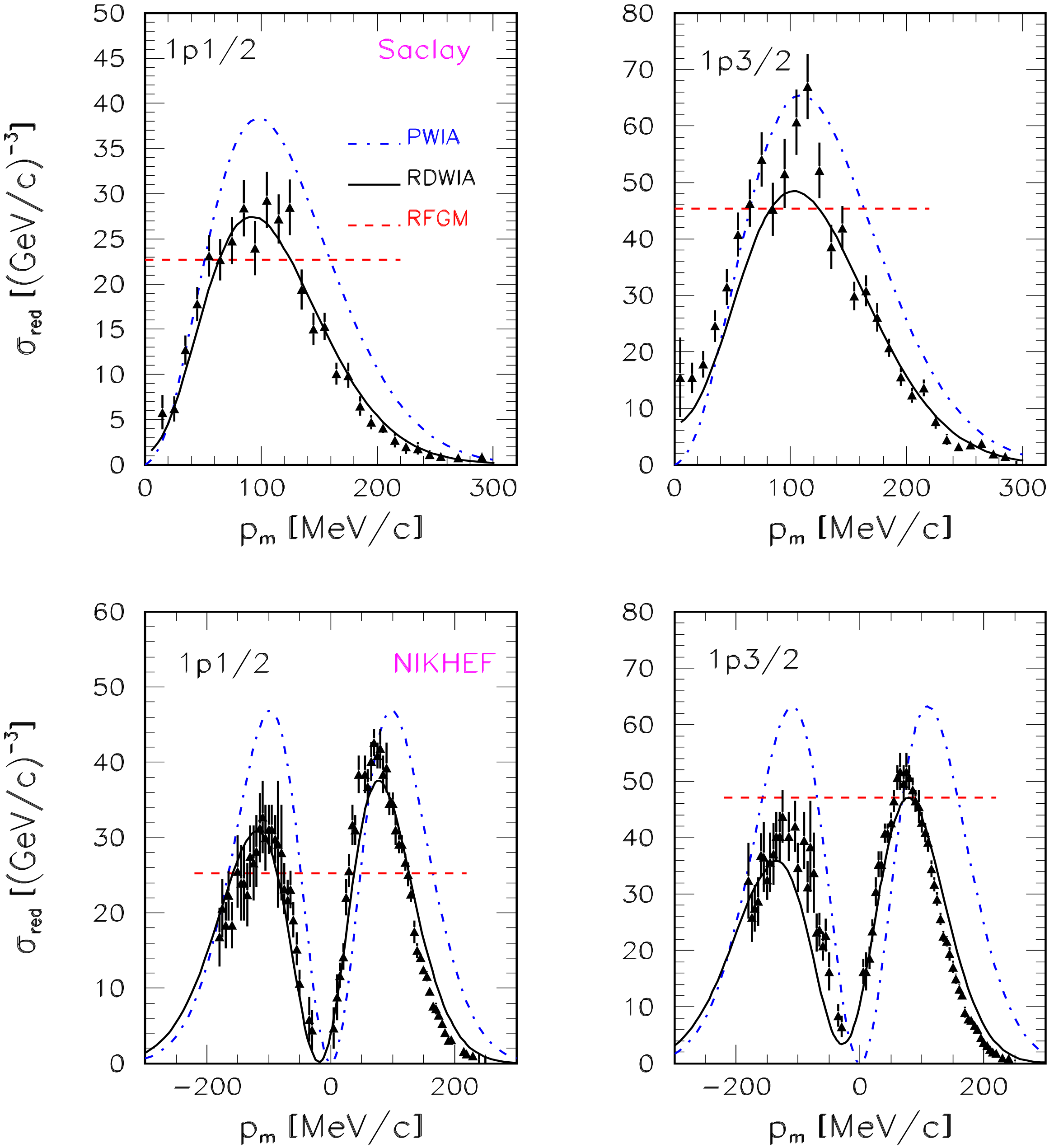
Results

More details about calculation of the cross sections can be found in

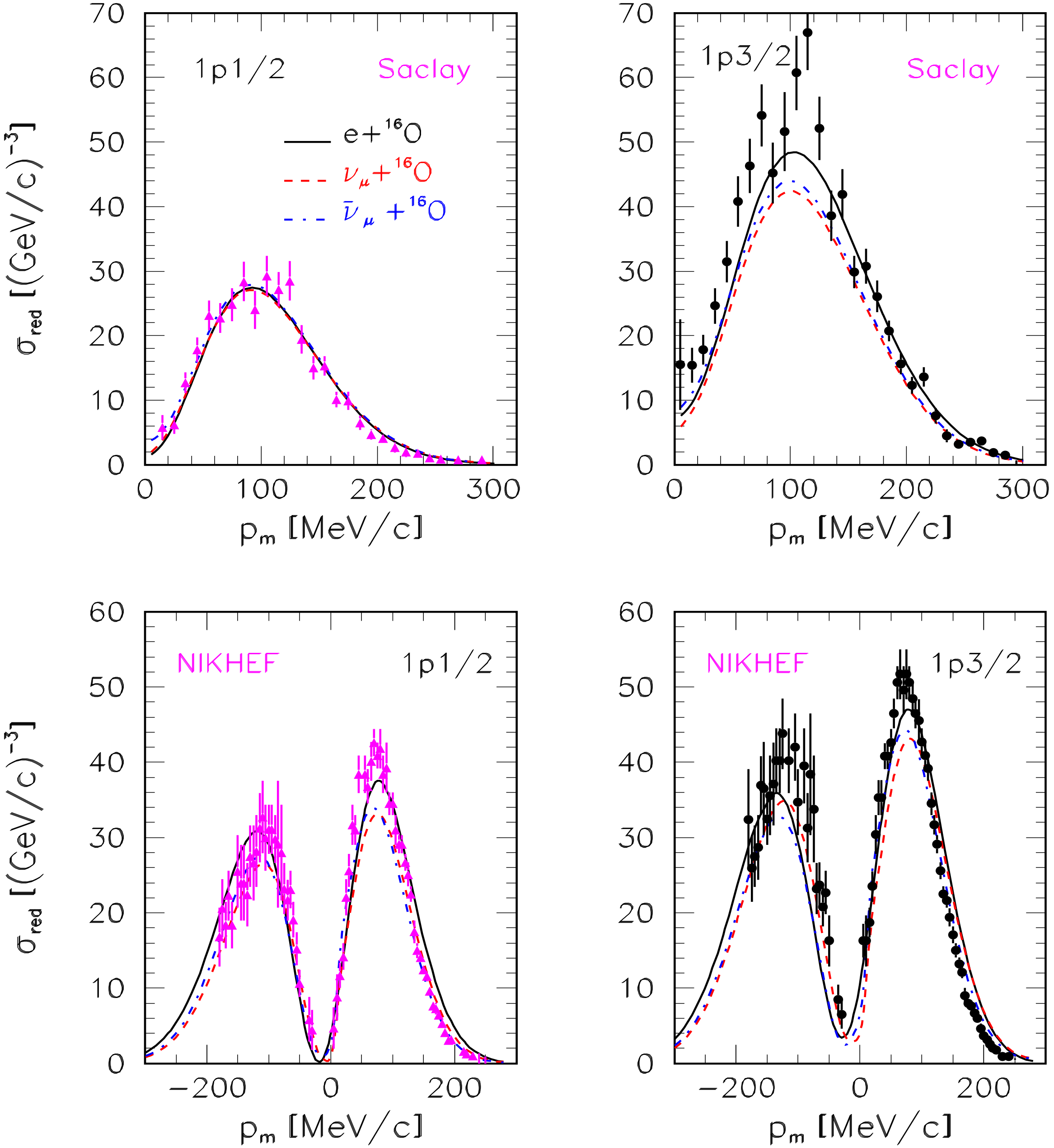
A.Butkevich & S.K., Phys. Rev. C76 (2007) 045502.



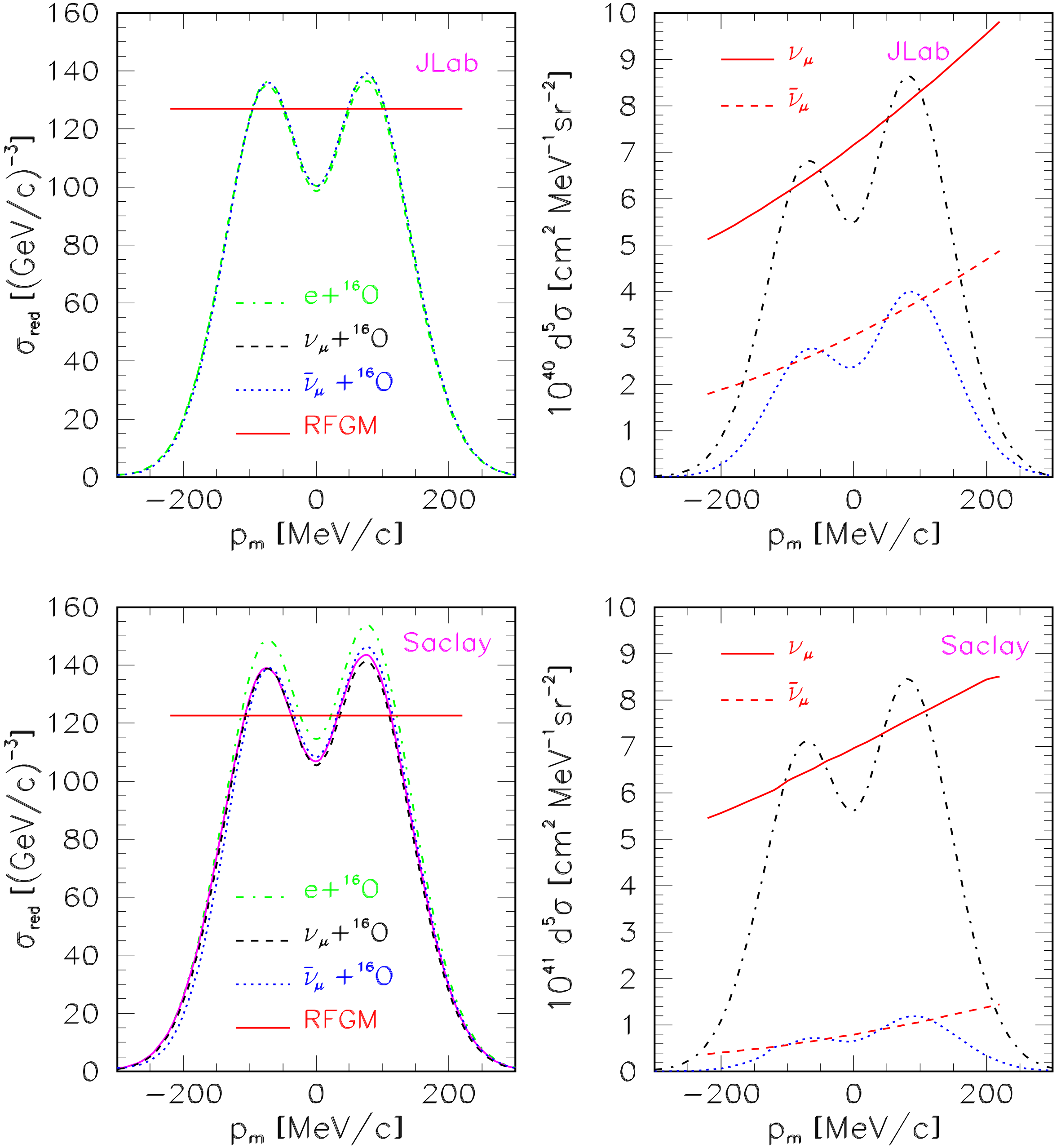
Differential exclusive cross section for the removal of protons from 1p-shell of ^{16}O as a function of missing momentum. The upper panels – JLab data (K.Fissum et.al., PRC70,034606) for electron beam energy E_{beam}=2.442 GeV, proton kinetic energy T_p=427 MeV, and Q^2=0.8 GeV^2. The lower panels – Saclay data (L.Chinitz et.al., PRL67,568) for E_{beam}=580 MeV, T_p=160 MeV, and Q^2=0.3 GeV^2. The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations. Negavite values of p_m correspond to $\pi \le \phi \le 2\pi$.



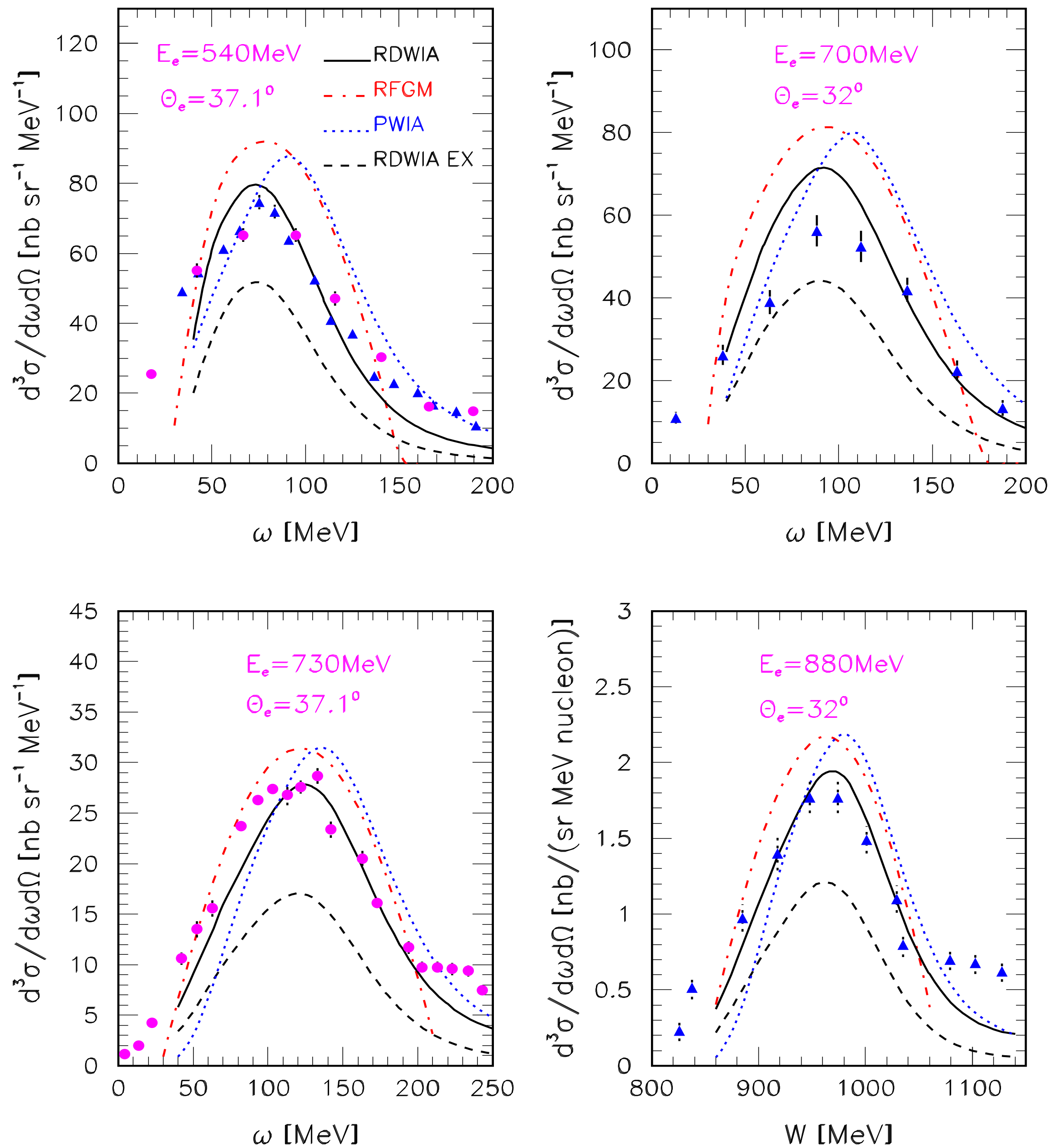
The reduced exclusive cross sections for the removal of protons from 1p-shell of ^{16}O as a function of missing momentum. The upper panels – Saclay data (M.Bernhein et.al., NPA375(1982)381) for electron beam energy E_{beam}=500 MeV, proton kinetic energy T_p=100 MeV, and Q^2=0.3 GeV^2. The lower panels – NIKHEF data (M.Leuschner et.al., PRC49(1994)955) for E_{beam}=521 MeV, T_p=96 MeV, Q^2 is varied. The solid line is the RDWIA calculation while the dashed-dotted and dashed lines are respectively the PWIA and RFGM calculations.



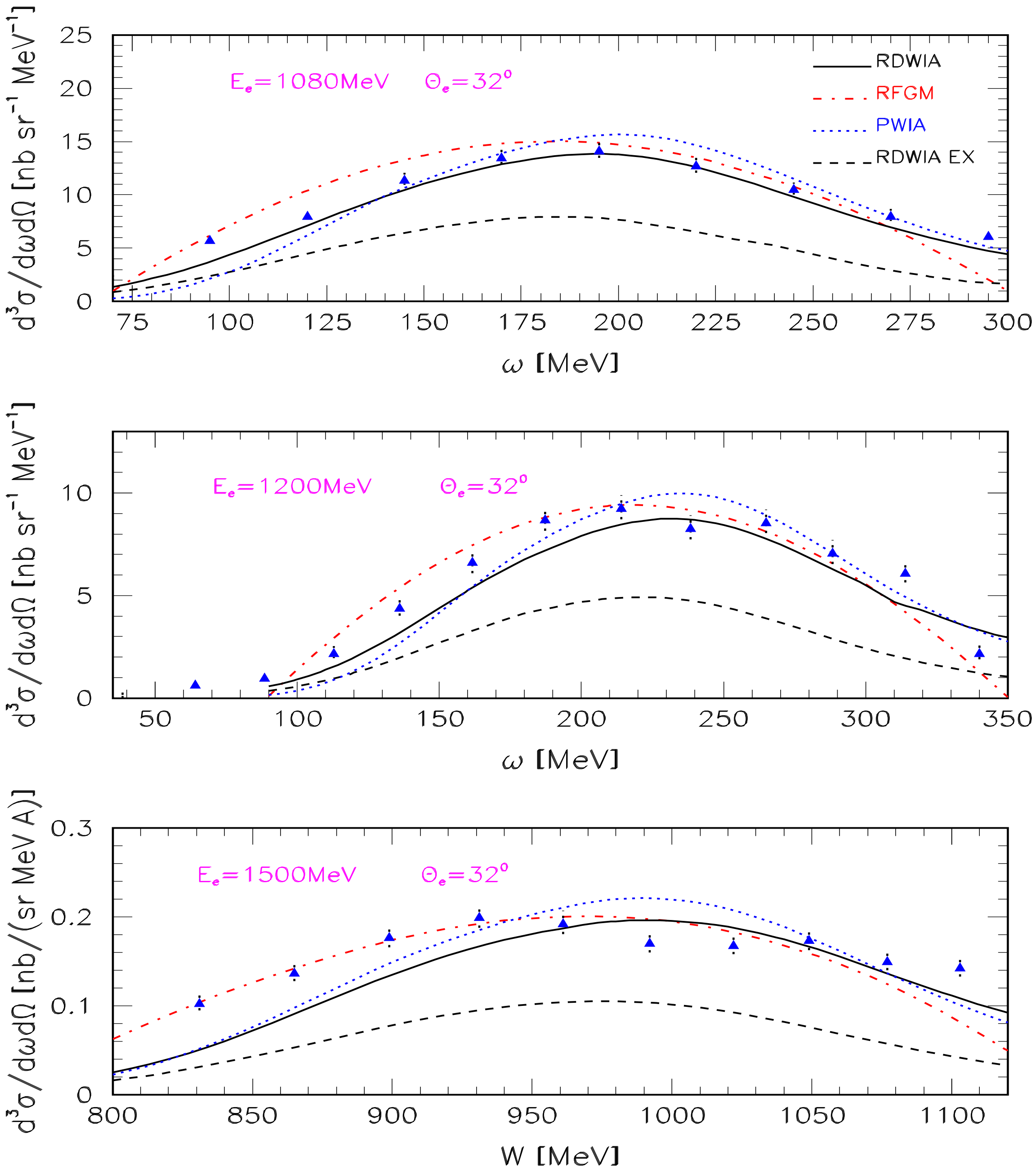
Comparison of the RDWIA electron, neutrino and antineutrino reduced cross sections for the removal of nucleons from 1p-shell of ^{16}O for Saclay (upper panels) and NIKHEF (lower panels) kinematic as functions of missing momentum p_m.



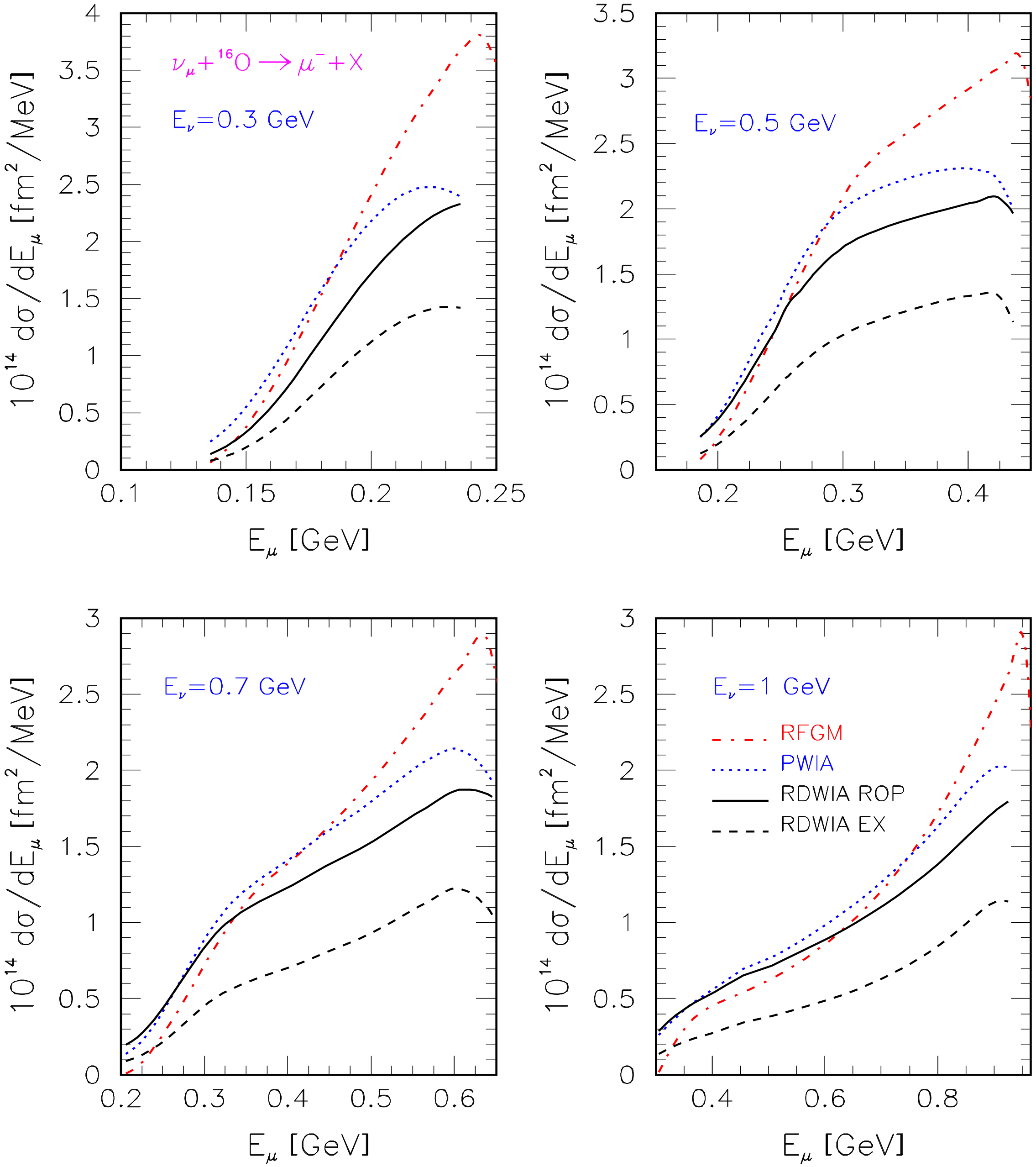
Comparison of the RDWIA and the RFGM calculations for electron, neutrino and antineutrino reduced (left panels) and differential (right panels) cross sections for the removal of nucleons from 1p- and 1s-shells of ^{16}O. The cross sections were calculated for the Jlab and Saclay kinematics. In the left panels, the RDWIA calculations are shown for electron scattering (dashed-dotted line) and neutrino (dashed line) and antineutrino (dotted line) scattering; and the RFGM results are shown for the reduced cross sections (solid line). In the right panels, the RFGM calculations are shown for the neutrino (solid line) and antineutrino (dashed line) differential cross sections; and the RDWIA results are shown for the neutrino (dashed-dotted line) and antineutrino (dotted line) differential cross sections.



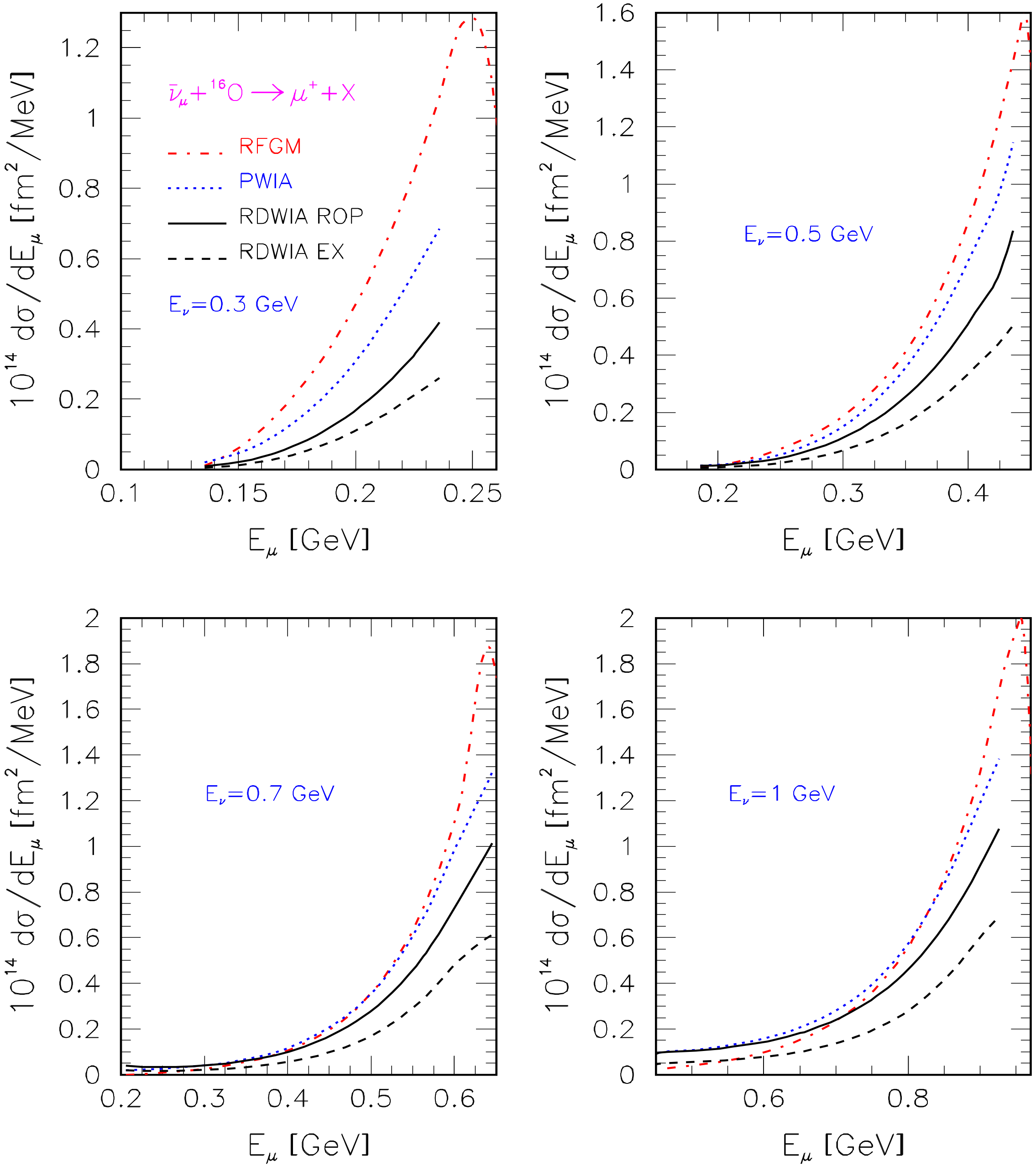
Inclusive cross section versus the energy transfer ω or invariant mass W (lower-right panel) for electron scattering on ^{16}O. The data are from SLAC (J.S.O'Connell et.al., PRC35(1987)1063, filled circles) and Frascati (M.Anghinolfi et.al., NPA602(1996)402, filled triangles). SLAC data are for electron beam energy E_e=540, 730 MeV and scattering angle θ=37.1°. Frascati data are for E_e=540 MeV and θ=37.1°, E_e=700, 880 MeV and θ=32°. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.



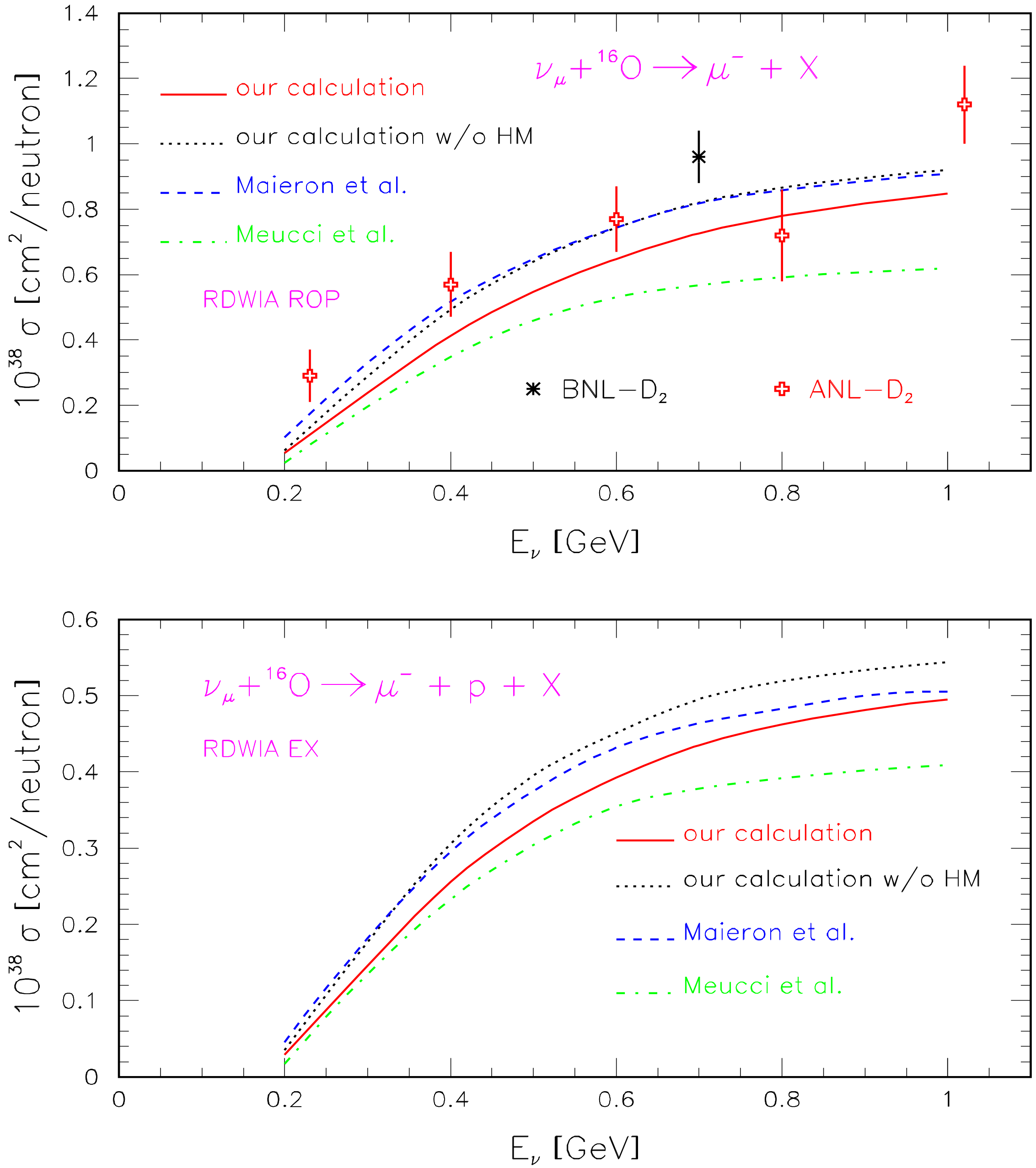
Inclusive cross section versus the energy transfer ω or invariant mass W (lower panel) for electron scattering on ^{16}O. The data are from M.Anghinolfi et.al., NPA602(1996)402 for electron beam energy E_e=1080, 1200, 1500 MeV and scattering angle θ=32.°. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with a complex optical potential.



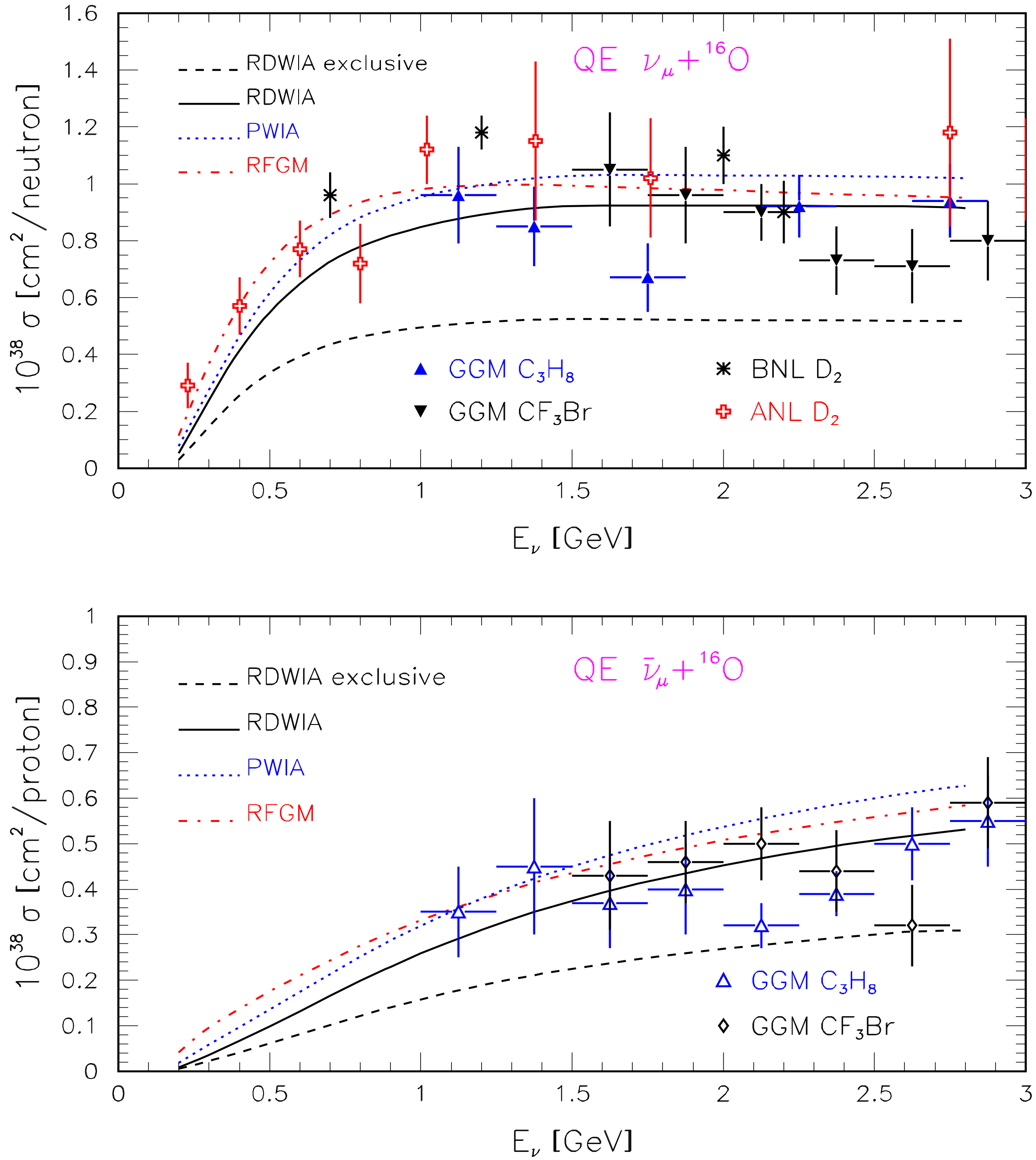
Inclusive cross section versus the muon energy for neutrino scattering on ^{16}O and for the four values of incoming neutrino energy: E_ν=0.3, 0.5, 0.7 and 1 GeV. The solid line is the RDWIA calculation while the dotted and dashed-dotted lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.



Inclusive cross section versus the muon energy for antineutrino scattering on ^{16}O and for the four values of incoming neutrino energy: E_ν=0.3, 0.5, 0.7 and 1 GeV. The solid line is the RDWIA calculation while the dotted and dashed lines are respectively the PWIA and RFGM calculations. The dashed line is the cross section calculated in the RDWIA with complex optical potential.



Total cross section for the CC QE scattering of muon neutrino on ^{16}O as a function of the incoming neutrino energy. The RDWIA results with the real part of optical potential (upper panel) and complex optical potential (lower panel) are shown together with calculations from A.Meucci et.al., NPA739(2004)277 (dashed-dotted line) and C.Maieron et.al.,PRC68(2003)048501 (dashed line). The solid and dotted lines are respectively results obtained in this work with and without contribution of the high-momentum component. Data for the D_2 target (W.Mann et.al., PRL31(1973)844 and N.Baker et al., PRD23(1981)2499).



Total cross section for CC QE scattering of muon neutrino (upper panel) and antineutrino (lower panel) on ^{16}O as a function of incoming (anti)neutrino energy. The solid and dashed lines are respectively the RDWIA results with the real and complex optical potential. The dashed-dotted and dotted lines are respectively the RFGM and PWIA results.

Summary

QE CC $\nu(\bar{\nu})^{16}$O cross sections were studied in different approaches.

- In RDWIA the reduced exclusive cross sections for $\nu(\bar{\nu})$ scattering are similar to those of electron scattering and in a good agreement with data.

- The inclusive and total cross sections were calculated neglecting the imaginary part of relativistic optical potential and taking into account the effect of NN-correlations in the target ground state and tested against ^{16}O(e, e') scattering data.

- FSI effect is important at low neutrino energies. FSI reduces the total cross section for about 30% for $E_\nu = 200$ MeV compared to PWIA and decreases with neutrino energy down to 10% at 1 GeV.

- Effect of NN-correlations further reduces the total cross section for about 15% for $E_\nu = 200$ MeV. This effect also decreases with neutrino energy, down to 8% at 1 GeV.

- The Fermi gas model was tested against e^{16}O data:

  ⇒ In the peak region RFGM overestimates the value of inclusive cross section at low momentum transfer ($|\boldsymbol{q}| < 500$ MeV/c). The discrepancy with data is about 20% at $|\boldsymbol{q}| = 300$ MeV/c and decreases as momentum transfer increases.

  ⇒ RFGM fails completely when compared to exclusive cross section data.

- For total neutrino cross sections RFGM result is about 15% higher than the RDWIA predictions at $E_\nu \sim 1$ GeV.

- Our results show that nuclear-model dependence of the inclusive and total cross sections weakens with neutrino energy but still remains significant for energy $E_\nu \lesssim 1$ GeV.